CHAPTER - 1
INTEGERS

EXERCISE 1 (A)

Question 1.
Evaluate:

1. **427 x 8 + 2 x 427**
2. **394 x 12 + 394 x (-2)**
3. **558 x 27 + 3 x 558**

Solution:

1. 427 x 8 + 2 x 427 = 427 x (8 + 2) (Distributive property)
   = 427 x 10
   = 4270
2. 394 x 12 + 394 x (-2) = 394 x (12-2) (Distributive property)
   = 394 x 10
   = 3940
3. 558 x 27 + 3 x 558 = 558 x (27 + 3) (Distributive property)
   = 558 x 30
   = 16740

Question 2.
Evaluate:

1. **673 x 9 + 673**
2. **1925 x 101 – 1925**

Solution:

1. 673 x 9 + 673 = 673 x (9 + 1) (Distributive property) = 673 x 10 = 6730
2. 1925 x 101 – 1925 = 1925 x (101 – 1) (Distributive property) = 1925 x 100 = 192500

Question 3.
Verify:

1. **37 x {8 +(-3)} = 37 x 8 + 37 x – (3)**
2. **(-82) x {(-4) + 19} = (-82) x (-4) + (-82) x 19**
3. **{7 – (-7)} x 7 = 7 x 7 – (-7) x 7**
4. **{(-15) – 8} x -6 = (-15) x (-6) – 8 x (-6)**

Solution:

1. **37 x {8 + (-3)} = 37 x 8 + 37 x – (3)**
   L.H.S. = 37 x {8 + (-3)}
   = 37 x {8-3}
   = 37 x {5}
   = 37 x 5
   = 185
   R.H.S. = 37 x 8 + 37 – 3
   = 37 x (8 – 3)
   = 37 x 5
   = 185
   Hence, L.H.S. = R.H.S.
2. **(-82) x {(-4) + 19} = (-82) x (-4) + (-82) x 19**
   L.H.S. = (-82) x {(_4) + 19}
   = (-82) x {-4 + 19}
   = (-82)x {15}
   = -82 x 15
   =-1230
   R.H.S. = (-82) x (-4) + (-82) x 19
   = -82 x (-4 + 19)
   = -82 x 15
   =-1230
   Hence, L.H.S. = R.H.S.
3. **{7 – (-7)}. x 7 = 7 x 7 – (-1) x 7**
   L.H.S. = {7 – (-7)} x 7
   = {7 + 7} x 7
   = {14} x 7
   = 14 x 7
   = 98
   R.H.S. = 7 x 7 – (-7) x 7
   =7 x 7+7 x 7 =
   7 x (7 + 7)
   = 7 x (14)
   = 98
   Hence, L.H.S. = R.H.S.
4. **{(-15) – 8} x -6 = (-15) x (-6) – 8 x (-6)**
   L.H.S. = {(-15)-8} x-6
   = {-15-8} x-6
   = {-23} x-6
   = -23 x- 6
   = 138
   R.H.S. = (-15) x (-6) – 8 x (-6)
   = -6 x (-15-8)
   = -6 x -23

= 138
Hence, L.H.S. = R.H.S.

Question 4.
Evaluate:

1. **15 x 8**
2. **15 x (-8)**
3. **(-15) x 8**
4. **(-15) x -8**

Solution:

1. 15 x 8= 120
2. 15 x (-8) = -120
3. (-15) x 8 = -120
4. (-15) x -8 = 120
   (Since the number of negative integers in the product is even)

Question 5.
Evaluate:

1. **4 x 6 x 8**
2. **4 x 6 x (-8)**
3. **4 x (-6) x 8**
4. **(-4) x 6 x 8**
5. **4 x (-6) x (-8)**
6. **(-4) x (-6) x 8**
7. **(-4) x 6 x (- 8)**
8. **(-4) x (-6) x (-8)**

Solution:

1. 4 x 6 x 8 = 192
2. 4 x 6 x (-8) = -192
   (It have one negative factor)
3. 4 x (-6) x 8 = -192
   (It have one negative factor)
4. (-4)x 6 x 8 = -192
   (It have one negative factor)
5. 4 x (-6) x (-8) = 192
   (It have two negative factors)
6. (-4) x (-6) x 8 = 192
   (It have two negative factors)

7. (-4) x 6 x (-8) = 192
   (It have two negative factors)
8. (-4) x (-6) x (-8) = -192
   (It have three negative factors)

Question 6.
Evaluate:

1. **2 x 4 x 6 x 8**
2. **2 x (-4) x 6 x 8**
3. **(-2) x 4 x (-6) x 8**
4. **(-2) x (-4) X 6 x (-8)**
5. **(-2) x (-4) x (-6) x (-8)**

Solution:

1. 2 x 4 x 6 x 8 = 384
2. 2 x (-4) x 6 x 8 = -384
   (Number of negative integer in the product is odd)
3. (-2) x 4 x (-6) x 8 = 384
   (Number of negative integer in the product is even)
4. (-2) x (-4) x 6 x (-8) = -384
   (Number of negative integer in the product is odd)
5. (-2) x (-4) x (-6) x (-8) = 384
   (Number of negative integer in the product is even)

Question 7.
Determine the integer whose product with '-1' is:

1. **-47**
2. **63**
3. **-1**
4. **0**

Solution:

1. -1 x 47 = -47
   Hence, integer is 47
2. -1 x -63 = 63
   Hence, integer is -63
3. -1 x 1 = -1
   Hence, integer is 1
4. -1 x 0 = 0
   Hence, integer is 0

Question 8.
Eighteen integers are multiplied together. What will be the sign of their product, if:

1. **15 of them are negative and 3 are positive?**
2. **12 of them are negative and 6 are positive?**
3. **9 of them are positive and the remaining are negative?**
4. **all are negative?**

Solution:

1. Since out of eighteen integers, 15 of them are negative, which is odd number. Hence, sign of product will be negative (-).
2. Since out of eighteen integers 12 of them are negative, which is even number. Hence sign of product will be positive (+).
3. Since out of eighteen integers 9 of them are negative, which is odd number. Hence, sign of product will be negative (-).
4. Since all are negative, which is even number. Hence sign of product will be positive (+).

Question 9.
Find which is greater?

1. **(8 + 10) x 15 or 8 + 10 x 15**
2. **12 x (6 – 8) or 12 x 6 – 8**
3. **{(-3) – 4} x (-5) or (-3) – 4 x (-5)**

Solution:

1. (8 + 10) x 15 or 8 + 10 x 15
   (8 + 10) x 15 = 18 x 15 = 270
   8 + 10 x 15 = 8 + 150 = 158
   ∴(8 + 10) x 15 > 8 + 10 x 15
2. 12 x (6 – 8) or 12 x 6 – 8
   12 x (6 – 8) = 12 (-2) = -24
   12 x 6 – 8 = 72 – 8 = 64
   ∴12 x 6 – 8 > 12 x (6-8)
3. {(-3) – 4} x (-5) or (-3) – 4 x (-5)
   {(-3) – 4} x (-5) = {-3 – 4} x (-5) = -7 x -5 = 35
   (-3) – 4 x (-5) = -7 x (-5) = 35
   ∴{(-3) – 4} x (-5) = (-3) – 4 x (-5)

Question 10.
State, true or false :

1. **product of two integers can be zero.**
2. **product of 120 negative integers and 121 positive integers is negative.**
3. **a x (b + c) = a x b + c**
4. **(b – c) x a=b – c x a**

Solution:

1. False.
2. False.
   Correct : Since 120 integers are even numbers, hence product will be positive and for 121 integers are positive in numbers, hence product will be positive.
3. False.
   Correct :a x (b + c) ≠ a x b + c
   ab + ac ≠ ab + c
4. False.
   Correct: (b – c) x a ≠ b – c x a
   ab – ac ≠ b – ca

EXERCISE 1 (B)

Question 1.
Divide:
(i) 117 by 9
(ii) (-117) by 9
(iii) 117 by (-9)
(iv) (-117) by (-9)
(v) 225 by (-15)
(vi) (-552) ÷ 24
(vii) (-798) by (-21)
(viii) (-910) ÷ – 26

Solution :

$$(i)\ 117 \text{ by } 9 = \frac{117}{9} = \frac{13\times 9}{9} = 13$$

$$(ii)\ (-117) \text{ by } 9 = \frac{-117}{9} = \frac{-13\times 9}{9} = -13$$

$$(iii)\ 117 \text{ by } (-9) = \frac{117}{-9} = \frac{13\times 9}{-9} = -13$$

(iv) (-117) by (-9)

$$= \frac{-117}{-9} = \frac{117}{9} = \frac{13\times 9}{9} = 13$$

$$(v)\ 225 \text{ by } (-15) = -\frac{225}{15} = -\frac{15\times 15}{15} = -15$$

$$(vi)\ (-552) \div 24 = -\frac{552}{24} = -\frac{23\times 24}{24} = -23$$

(vii) (-798) by (-21)

$$= \frac{-798}{-21} = \frac{798}{21} = \frac{38\times 21}{21} = 38$$

$$(viii)\ (-910) \div 26 = -\frac{910}{26} = -\frac{35\times 26}{26} = -35$$

Question 2.

Evaluate:

(i) (-234) ÷ 13
(ii) 234 ÷ (-13)
(iii) (-234) ÷ (-13)
(iv) 374 ÷ (-17)
(v) (-374) ÷ 17
(vi) (-374) ÷ (-17)
(vii) (-728) ÷ 14
(viii) 272 ÷ (-17)

Solution:

$$(i)\ (-234) \div 13 = \frac{-234}{13} = \frac{-18 \times 13}{13} = -18$$

$$(ii)\ 234 \div (-13) = \frac{234}{-13} = \frac{18 \times 13}{-13} = -18$$

$$(iii)\ (-234) \div (-13)$$

$$= \frac{-234}{-13} = \frac{234}{13} = \frac{18 \times 13}{13} = 18$$

$$(iv)\ 374 \div (-17) = \frac{374}{-17} = \frac{22 \times 17}{-17} = -22$$

$$(v)\ (-374) \div 17 = -\frac{374}{17} = -\frac{22 \times 17}{17} = -22$$

$$(vi)\ (-374) \div (-17)$$

$$= \frac{-374}{-17} = \frac{374}{17} = \frac{22 \times 17}{17} = 22$$

$$(vii)\ (-728) \div 14 = -\frac{728}{14} = -\frac{52 \times 14}{14} = -52$$

$$(viii)\ 272 \div (-17) = -\frac{16 \times 17}{17} = -16$$

Question 3.

Find the quotient in each of the following divisions:

(i) 299 ÷ 23

(ii) 299 ÷ (-23)

(iii) (-384) ÷ 16

(iv) (-572) ÷ (-22)

(v) 408 ÷ (-17)

Solution:

$$(iv)\ (-572) \div (-22) = \frac{-572}{-22}$$

$$= \frac{572}{22} = \frac{26 \times 22}{22} = 26$$

$$(v)\ 408 \div (-17) = -\frac{408}{17} = -\frac{24 \times 17}{17} = -24$$

$$(i)\ 299 \div 23 = \frac{299}{23} = \frac{23 \times 13}{23} = 13$$

$$(ii)\ 299 \div (-23) = -\frac{299}{23} = -\frac{23 \times 13}{23} = -13$$

$$(iii)\ (-384) \div 16 = -\frac{384}{16} = -\frac{24 \times 16}{16} = -24$$

Question 4.
Divide:
(i) 204 by 17
(ii) 152 by-19
(iii) 0 by 35
(iv) 0 by (-82)
(v) 5490 by 10
(vi) 762800 by 100

Solution:

$$(i)\ 204 \text{ by } 17 = \frac{204}{17} = \frac{12 \times 17}{17} = 12$$

$$(ii)\ 152 \text{ by } -19 = -\frac{152}{19} = -\frac{8 \times 19}{19} = -8$$

$$(iii)\ 0 \text{ by } 35 = \frac{0}{35} = 0$$

$$(iv)\ 0 \text{ by } (-82) = -\frac{0}{82} = 0$$

$$(v)\ 5490 \text{ by } 10 = \frac{5490}{10} = \frac{549 \times 10}{10} = 549$$

$$(vi)\ 762800 \text{ by } 100 = \frac{762800}{100} = 7628$$

Question 5.
State, true or false :

1. **0 ÷ 32 = 0**
2. **0 ÷ (-9) = 0**
3. **(-37) ÷ 0 = 0**
4. **0 ÷ 0 = 0**

Solution:

1. True.
2. True.
3. False.
   Correct: It is not meaningful (defined)
4. False.
   Correct: It is not defined.

Question 6.
Evaluate:
(i) 42 ÷ 7 + 4
(ii) 12+18 ÷ 3
(iii) 19 – 20 ÷ 4
(iv) 16 – 5 x 3+4
(v) 6 – 8 – (-6) ÷ 2
(vi) 13 -12 ÷ 4 x 2
(vii) 16 + 8 ÷ 4- 2 x 3
(viii) 16 ÷ 8 + 4 – 2 x 3
(ix) 16 – 8 + 4 ÷ 2 x 3
(x) (-4) + (-12) ÷ (-6)
(xi) (-18) + 6 ÷ 3 + 5
(xii) (-20) x (-1) + 14 – 7

Solution:

(*i*) $42 \div 7 + 4$

$= \frac{42}{7} + 4 = 6 + 4 = 10$

(*ii*) $12 + 18 \div 3$

$= 12 + \frac{18}{3} = 12 + 6 = 18$

(*iii*) $19 - 20 \div 4$

$= 19 - \frac{20}{4} = 19 - 5 = 14$

(*iv*) $16 - 5 \times 3 + 4$

$= 16 - 15 + 4 = 16 - 19 = -3$

(*v*) $6 - 8 - (-6) \div 2$

$= 6 - 8 - \left(\frac{-6}{2}\right) = 6 - 8 - (-3)$

$= 6 - 8 + 3 = 9 - 8 = 1$

(*vi*) $13 - 12 \div 4 \times 2$

$= 13 - \frac{12}{4} \times 2 = 13 - 3 \times 2$

$= 13 - 6 = 7$

(*vii*) $16 + 8 \div 4 - 2 \times 3$

$= 16 + \frac{8}{4} - 2 \times 3 = 16 + 2 - 2 \times 3$

$= 16 + 2 - 6 = 18 - 6 = 12$

(*viii*) $16 \div 8 + 4 - 2 \times 3$

$= \frac{16}{8} + 4 - 2 \times 3 = 2 + 4 - 6$

$= 6 - 6 = 0$

(ix) $16 - 8 + 4 \div 2 \times 3$

$= 16 - 8 + \frac{4}{2} \times 3 = 16 - 8 + 2 \times 3$

$= 16 - 8 + 6$

$= 16 + 6 - 8 = 22 - 8 = 14$

(x) $(-4) + (-12) \div (-6)$

$= (-4) + \left(\frac{-12}{-6}\right) = (-4) + 2$

$= -4 + 2 = -2$

(xi) $(-18) + 6 \div 3 + 5$

$= (-18) + \frac{6}{3} + 5 = (-18) + 2 + 5$

$= -18 + 7 = -11$

(xii) $(-20) \times (-1) + 14 \div 7$

$= (-20) \times (-1) + \frac{14}{7} = (-20) \times (-1) + 2$

$= 20 + 2 = 22$

EXERCISE 1 (C)

Question 1.
Evaluate:
18-(20- 15 ÷ 3)
Solution:
18-(20- 15 ÷ 3)
= 18 – $\left(20 - \frac{15}{5}\right)$
= 18 – (20 – 5)
= 18 – 20 + 5
= 18 + 5 – 20
= 23 – 20
= 3

Question 2.
-15+ 24÷ (15-13)
Solution:
-15+ 24÷ (15- 13)
= -15 + 24 ÷ 2
= -15 + 12
= -3

Question 3.
35 – [15 + {14-(13 + $\overline{2-1+3}$)}]
Solution:
35- [15 + {14-(13 + $\overline{2-1+3}$)}]
= 35-[15+ 14-(13+4)]
= 35 — [15 + 14 – (13 + 4}]
= 35-{15 + 14-17]
= 35-15-14+ 17
= 35 + 17-15-14
= 52 – 29
= 23

Question 4.
27- [13 + {4-(8 + 4 – $\overline{1+3}$)}]
Solution:
27- [13 + {4-(8 + 4 – $\overline{1+3}$)}]
= 27-[13 +{4-(8+ 4-4)}]
= 27-[13 + {4-8}]
= 27 – [13 + (-4)]
= 21 – [9]
= 27-9
= 18

Question 5.
32 – [43-{51 -(20 – $\overline{18-7}$)}]
Solution:
32 – [43 – {51 – (20 – $\overline{18-7}$)}]
= 32-[43 – {51 -(20- 11)}]
= 32-[43-{51 -9}]
= 32-[43 -42]
= 32-1
=31

Question 6.
46-[26-{14-(15-4÷ 2 x 2)}]
Solution:
46 – [26 – {14 – (15 – 4 ÷ 2 x 2)}]

= 46-[26- {14-(15-2 x 2)}]
= 46-[26- {14-(15 -4)}]
= 46-[26- {14- 11}]
= 46 – [26 – 3]
= 46 – 23
= 23

Question 7.
45 – [38 – {60 ÷ 3 – (6 – 9 ÷ 3) ÷ 3}]
Solution:
45 – [38 – {60 ÷ 3 – (6 – 9 ÷ 3) ÷ 3}]
= 45-[38- {60 ÷ 3-(6-3)÷ 3}]
= 45-[38 -{20-3 ÷ 3}]
= 45-[38- {20-1}]
= 45-[38- 19]
= 45-19
= 26

Question 8.
17- [17 — {17 — (17 – $\overline{17-17}$)}]
Solution:
17- [17-{17-(17 –$\overline{17-17}$)}]
= 17-[17-{17-(17-0)}]
= 17 – [17 – {17 — 17}]
= 17 — [17 — 0]
= 17-17
= 0

Question 9.
2550 – [510 – {270 – (90 – $\overline{80+7}$)}]
Solution:
2550- [510-{270-(90-$\overline{80+7}$)}]
= 2550 – [510 – {270 – (90 – 87)}]
= 2550 -[510- {270 -3}]
= 2550-[510-267]
= 2550 – 243
= 2307

Question 10.
30+ [{-2 x (25-$\overline{13-3}$)}]
Solution:
30+ [{-2 x (25-$\overline{13-3}$)}]
= 30 + [{-2 x (25 – 10)}]
= 30 + [{-2 x 15}]
= 30 + [-30]

= 30-30
= 0

Question 11.
88-{5-(-48)+ (-16)}
Solution:
88- {5-(-48)+ (-16)}
=88 – $\left\{5-\frac{(-48)}{-16}\right\}$
= 88 – {5-3}
= 88 – 2
= 86

Question 12.
9 x (8-$\overline{3+2}$) – 2 (2 + $\overline{3+3}$)
Solution:
9 x (8-$\overline{3+2}$) -2(2 + $\overline{3+3}$)
= 9 x (8 – 5) – 2(2 + 6)
= 9 x 3 – 2 x 8
= 27- 16
= 11

Question 13.
2 – [3 – {6 – (5 – $\overline{4-3}$)}]
Solution:
2 – [3 – {6 – (5 – $\overline{4-3}$)}]
⇒ 2 – [3 – {6 – (5 – 1)}]
⇒ 2 – [3 – {6 – 4}]
⇒2 – (3 – 2)
⇒2-1 = 1

EXERCISE 1 (D)

Question 1.
The sum of two integers is -15. If one of them is 9, find the other.
Solution:
Sum of two integers = -15
One integer = 9
∴ Second integer = -15 – 9
= -(15 + 9)
= -24

Question 2.
The difference between an integer and -6 is -5. Find the values of x.

Solution:
The difference between an integer
= x-(-6) = -5
∴ Value of
⇒ x – (-6) = -5
⇒ x + 6 = -5
x = -5 – 6
x = -11

Question 3.
The sum of two integers is 28. If one integer is -45, find the other.
Solution:
Sum of two integers = 28
One integer = -45
∴ Second integer = 28 – (-45)
= 28 + 45
= 73

Question 4.
The sum of two integers is -56. If one integer is -42, find the other.
Solution:
Sum of two integers = -56
One integer = -42
∴Second integer = -56 – (-42)
= -56+ 42
=-14

Question 5.
The difference between an integer x and (-9) is 6. Find all possible values ofx.
Solution:
The difference between an integer x – (-9) = 6 or -9 – x = 6
∴ Value of x
⇒ x – (-9) = 6 or ⇒ -9 – x = 6
⇒ x + 9 = 6 or Answer-x = 6 + 9
⇒ x = 6 – 9 or ⇒ -x = 15
⇒x = -3 or ⇒ x = -15
Hence, possible values ofx are -3 and -15.

Question 6.
Evaluate:

1. **(-1) x (-1) x (-1) x ….60 times.**
2. **(-1) x (-1) x (-1) x (-1) x …. 75 times.**

Solution:

1. 1 (because (-1) is multiplied even times.)
2. -1 (because (-1) is multiplied odd times.)

Question 7.
Evaluate:

1. **(-2) x (-3) x (-4) x (-5) X (-6)**
2. **(-3) x (-6) x (-9) x (-12)**
3. **(-11) x (-15) + (-11) x (-25)**
4. **10 x (-12) + 5 x (-12)**

Solution:

1. (-2) x (-3) x (-4) x (-5) x (-6)
   ⇒ 6 x 20 x (-6) = 120 x (-6)
   = -720
2. (-3) x (-6) x (-9) x (-12)
   ⇒ 18 x 108
   = 1944
3. (-11) x (-15) + (-11) x (-25)
   ⇒ 165 + 275
   = 440
4. 10 x (-12) + 5 x (-12)
   ⇒ -120-60
   = -180

Question 8.

1. **If x x (-1) = -36, is x positive or negative?**
2. **If x x (-1) = 36, is x positive or negative?**

Solution:

1. x x (-1) = -36
   -lx = -36
   x = $\frac{-36}{-1}$
   x = 36
   ∵ x = 36
   ∴ It is a positive integer.
2. x x (-1) = 36
   -1x = 36

$x = \frac{36}{-1}$
x = -36
∵x = -36
∴It is a negative integer.

Question 9.
Write all the integers between -15 and 15, which are divisible by 2 and 3.
Solution:
The integers between -15 and 15 are :
-12, -6, 0, 6 and 12
That are divisible by 2 and 3.

Question 10.
Write all the integers between -5 and 5, which are divisible by 2 or 3.
Solution:
The integers between -5 and 5 are :
-4, -3, -2, 0, 0, 2, 3 and 4
That are divisible by 2 or 3.

Question 11.
Evaluate:

1. **(-20) + (-8) ÷ (-2) x 3**
2. **(-5) – (-48) ÷ (-16) + (-2) x 6**
3. **16 + 8 ÷ 4- 2 x 3**
4. **16 ÷ 8 x 4 – 2 x 3**
5. **27 – [5 + {28 – (29 – 7)}]**
6. **48 – [18 – {16 – (5 – $\overline{4 + 1}$)}]**
7. **-8 – {-6 (9 – 11) + 18 = -3}**
8. **(24 ÷ $\overline{12 - 9}$ – 12) – (3 x 8 ÷ 4 + 1)**

Solution:
We know that, if these type of expressions that has more than one fundamental operations, we use the rule of DMAS i.e., First of all we perform D (division), then M (multiplication), then A (addition) and in the last S (subtraction).

1. **(-20) + (-8) ÷ (-2) x 3**
   ⇒ -20 + 4 x 3
   ⇒ -20+ 12
   =-8
2. **(-5) – (-48) ÷ (-16) + (-2) x 6**
   ⇒ (-5) – 3 + (-2) x 6
   ⇒ -5 – 3 – 12

⇒ -8- 12
= -20

3. **16 + 8 ÷ 4 – 2 x 3**
⇒ 16 + 2 – 2 x 3
⇒16 + 2 – 6
⇒ 18-6
= 12
4. **16 ÷ 8 x 4 – 2 x 3**
⇒ 2 x 4 – 2 x 3
⇒ 8 – 6
= 2
5. **27 – [5 + {28 – (29 – 7)}]**
⇒ 27 – [5 + {28 – 22}]
⇒ 27 – [5 + 6]
⇒ 27 — 11
= 16
6. **48-[18-{16-(5 – $\overline{4+1}$)}]**
⇒ 48-[18-{16-(5-5)}]
⇒ 48-[18- {16-0)}]
⇒ 48-[18- 16]
⇒ 48 – 2
= 46
7. **-8 – {-6 (9 – 11) + 18 ÷ -3}**
⇒ -8 – {-6 (-2) – 6}
⇒ -8- {12-6}
⇒ -8 – {6}
⇒ -8-6
= -14
8. **(24 ÷ $\overline{12-9}$ – 12) – (3 x 8 = 4 + 1)**
⇒ (24 ÷ 3-12)-(3 x 2 + 1)
⇒ (8- 12)-(6+ 1)
⇒ —4 — 7
= —11

Question 12.
Find the result of subtracting the sum of all integers between 20 and 30 from the sum of all integers from 20 to 30.
Solution:
Required number = (Sum of all integers between 20 and 30 – Integers between 20 and 30)
(20 + 21 + 22 + 23 + 24 + 25 + 26 + 27 + 28 + 29 + 30) – (21 + 22 + 23 + 24 + 25 + 26 + 27 + 28 + 29)
⇒ 20 + 30 = 50
∴ Required number = 50

Question 13.

Add the product of (-13) and (-17) to the quotient of (-187) and 11.

Solution:

(-13) x (-17)+ (-187- 11)
⇒ (-13) x (-17) + (-17)
⇒ 221 – 17 = 204

Question 14.

The product of two integers is-180. If one of them is 12, find the other.

Solution:

The product of two integers = -180 One integer = 12
∴ Second integer = -180 – 12 = -15

Question 15.

1. **A number changes from -20 to 30. What is the increase or decrease in the number?**
2. **A number changes from 40 to -30. What is the increase or decrease in the number?**

Solution:

1. ∵A number changes from = -20 to 30
   ⇒ -20 – 30 = -50
   ∴-50, it will be increases.
2. ∵A number changes from = 40 to -30
   ⇒ 40 – (-30)
   40 + 30 = 70
   ∴70, it will be decreases

CHAPTER - 2
RATIONAL NUMBERS

EXERCISE 2 (A)

Question 1.
Write down a rational number whose numerator is the largest number of two digits and denominator is the smallest number of four digits.

Solution:
Largest two digit = 99
Smallest, number of four digit = 1000 Now numerator = 99 and denominator = 1000
∴ Rational number = $\frac{99}{1000}$

Question 2.
Write the numerator of each of the following rational numbers:

(i) $\frac{-125}{127}$ (ii) $\frac{37}{-137}$

(iii) $\frac{-85}{93}$ (iv) 2

(v) 0

Solution:

(i) $\frac{-125}{127}$

Numerator = –125

(ii) $\frac{37}{-137}$

Numerator = 37

(iii) $\frac{-85}{93}$

Numerator = –85

(iv) $2 = \frac{2}{1}$

Numerator = 2

(v) $0 = \frac{0}{1}$

Numerator = 0

Question 3.
Write the denominator of each of the following rational numbers:

(i) $\frac{7}{-15}$ (ii) $\frac{-18}{29}$

(iii) $\frac{-3}{4}$ (iv) -7

(v) 0

Solution:

(i) $\frac{7}{-15}$

Denominator $= -15$

(ii) $\frac{-18}{29}$

Denominator $= 29$

(iii) $\frac{-3}{4}$

Denominator $= 4$

(iv) $-7 = \frac{-7}{1}$

Denominator $= 1$

(v) $0 = \frac{0}{1}$

Denominator $= 1$

Question 4.
Write down a rational number numerator (-5) x (-4) and denominator (28 – 27) x (8 – 5).

Solution:

Numerator = $(-5) \times (-4) = 20$

Denominator = $(28 - 27) \times (8 - 5)$

$= (1) \times (3) = 3$

$\therefore$ Rational number $= \frac{20}{5} = \frac{4}{1} = 4$

Question 5.

(i) $\frac{-15}{1}$ in integer form is ....

(ii) $\frac{23}{-1}$ in integer form is ....

(iii) If $18 = \frac{18}{a}$ then $a =$ ....

(iv) If $-57 = \frac{57}{a}$ then $a =$ ....

Solution:

(i) $\frac{-15}{1}$ in integer form is $= \mathbf{-15}$

(ii) $\frac{23}{-1}$ in integer form is $= \mathbf{-23}$

(iii) If $18 = \frac{18}{a}$ then $a = \mathbf{\frac{18}{18}} = \mathbf{1}$

(iv) If $-57 = \frac{57}{a}$ then $a = \mathbf{\frac{57}{-57}} = \mathbf{-1}$

Question 6.
Separate positive and negative rational numbers from the following :

$$\frac{-3}{5}, \frac{3}{-5}, \frac{-3}{-5}, \frac{3}{5}, 0, \frac{-13}{-3}, \frac{15}{-8}, \frac{-15}{8}$$

Solution:

Positive rational numbers are $\frac{-3}{-5} = \frac{3}{5}$

(as both or negative)

$\frac{-13}{-3} = \frac{13}{3}$ (as both are negative)

Negative rational numbers $= \frac{-3}{5}, \frac{3}{-5}, \frac{15}{-8}$

and $\frac{-15}{8}$

0 is neither positive nor negative integer.

Question 7.
Find three rational numbers equivalent to

(*i*) $\frac{3}{5}$ (*ii*) $\frac{4}{-7}$

(*iii*) $\frac{-5}{9}$ (*iv*) $\frac{8}{-15}$

Solution:

(i) $\frac{3}{5} = \frac{3\times 2}{5\times 2} = \frac{6}{10}, \frac{3\times 3}{5\times 3} = \frac{9}{15}$ and $\frac{3\times 4}{5\times 4} = \frac{12}{20}$

Hence, $\frac{6}{10}, \frac{9}{15}$ and $\frac{12}{20}$ are rational numbers equivalent to the given rational number $\frac{3}{5}$.

(ii) $\frac{4}{-7} = \frac{4\times 2}{-7\times 2} = \frac{8}{-14}, \frac{4\times 3}{-7\times 3} = \frac{12}{-21}$ and $\frac{4\times 4}{-7\times 4} = \frac{16}{-28}$

Hence $\frac{8}{-14}, \frac{12}{-21}$ and $\frac{16}{-28}$ are rational numbers equivalent to given rational number $\frac{4}{-7}$.

(iii) $\frac{-5}{9} = \frac{-5\times 2}{9\times 2} = \frac{-10}{18}, \frac{-5\times 3}{9\times 3} = \frac{-15}{27}$ and $\frac{-5\times 4}{9\times 4} = \frac{-20}{36}$

Hence, $\frac{-10}{18}, \frac{-15}{27}$ and $\frac{-20}{36}$ are rational numbers equivalent to given rational number $\frac{-5}{9}$.

(iv) $\frac{8}{-15} = \frac{8 \times 2}{-15 \times 2} = \frac{16}{-30}$,

$\frac{8 \times 3}{-15 \times 3} = \frac{24}{-45}$ and $\frac{8 \times 4}{-15 \times 4} = \frac{32}{-60}$

Hence, $\frac{16}{-30}, \frac{24}{-45}$ and $\frac{32}{-60}$ are rational numbers equivalent to given rational number $\frac{8}{-15}$.

Question 8.
Which of the following are not rational numbers :

(i) -3 (ii) 0

(iii) $\frac{0}{4}$ (iv) $\frac{8}{0}$

(v) $\frac{0}{0}$

Solution:

(i) $-3 = \frac{-3}{1}$ is a rational number.

(ii) $0 = \frac{0}{1}$ is a rational number.

(iii) $\frac{0}{4}$ is a rational number.

(iv) $\frac{8}{0}$ is not a rational number.

(v) $\frac{0}{0}$ is not a rational number as numerator and denominator both are zero.

Question 9.
Express each of the following integers as a rational number with denominator 7 :
(i) 5
(ii) -8
(iii) 0
(iv) -16
(v) 7

Solution:

(i) $5 = \frac{5 \times 7}{7} = \frac{35}{7}$

(ii) $-8 = \frac{-8 \times 7}{7} = \frac{-56}{7}$

(iii) $0 = \frac{0 \times 7}{7} = \frac{0}{7}$

(iv) $-16 = \frac{-16 \times 7}{7} = \frac{-112}{7}$

(v) $7 = \frac{7 \times 7}{7} = \frac{49}{7}$

Question 10.
Express $\frac{3}{5}$ as a rational number with denominator:

Solution:

(i) 20

$$\frac{3}{5} = \frac{3\times 4}{5\times 4} = \frac{12}{20}$$

(ii) –20

$$\frac{3}{5} = \frac{3\times -4}{5\times -4} = \frac{-12}{-20}$$

(iii) 45

$$\frac{3}{5} = \frac{3\times 9}{5\times 9} = \frac{27}{45}$$

(iv) 25

$$\frac{3}{5} = \frac{3\times 5}{5\times 5} = \frac{15}{25}$$

(v) –35

$$\frac{3}{5} = \frac{3\times -7}{5\times -7} = \frac{-12}{-35}$$

Question 11.
Express $\frac{4}{7}$ as a rational number with numerator :

Solution:
(i) 12

$$\frac{4}{7}=\frac{4\times3}{7\times3}=\frac{12}{21}$$

(ii) –12

$$\frac{4}{7}=\frac{4\times-3}{7\times-3}=\frac{-12}{-21}$$

(iii) –16

$$\frac{4}{7}=\frac{4\times-4}{7\times-4}=\frac{-16}{-28}$$

(iv) –20

$$\frac{4}{7}=\frac{4\times-5}{7\times-5}=\frac{-20}{-35}$$

(v) 20

$$\frac{4}{7}=\frac{4\times5}{7\times5}=\frac{20}{35}$$

Question 12.
Find x, such that:

(i) $\frac{-2}{3}=\frac{6}{x}$ (ii) $\frac{7}{-4}=\frac{x}{8}$

(iii) $\frac{3}{7}=\frac{x}{-35}$ (iv) $\frac{-48}{x}=6$

(v) $\frac{36}{x}=3$ (vi) $\frac{-27}{x}=9$

Solution:

(i) $\frac{-2}{3} = \frac{6}{x}$

$-2x = 6 \times 3$

$x = \frac{6\times3}{-2} = -3 \times 3 = -9$

$\therefore\ \frac{-2}{3} = \frac{6}{-9} = -\frac{6}{9}$

OR

$\frac{-2}{3} = \frac{6}{x}$

$\Rightarrow\ \frac{-2\times3}{-3\times3} = \frac{6}{x}$

$\Rightarrow\ \frac{6}{-9} = \frac{6}{x}$

$\Rightarrow x = -9$

(ii) $\frac{7}{-4} = \frac{x}{8}$

$= -4x = 7 \times 8$

$x = \frac{7\times8}{-4} = -7 \times 2$

$x = -14$

OR

$\Rightarrow\ \frac{-7\times2}{4\times2} = \frac{x}{8}$

$\Rightarrow\ \frac{-14}{8} = \frac{x}{8}$

$x = -14$

(iii) $\frac{3}{7} = \frac{x}{-35}$

$\Rightarrow\ \frac{3\times-5}{7\times-5} = \frac{x}{-35}$

$\Rightarrow\ \frac{-15}{-35} = \frac{x}{-35}$

$\Rightarrow x = -15$

(*iv*) $\frac{-48}{x} = 6$

$\Rightarrow \frac{-48}{x} = \frac{6}{1}$

$\Rightarrow \frac{-48}{x} = \frac{6 \times -8}{1 \times -8}$

$\Rightarrow \frac{-48}{x} = \frac{-48}{-8}$

$\Rightarrow x = -8$

(*v*) $\frac{36}{x} = 3$

$\Rightarrow \frac{36}{x} = \frac{3}{1}$

$\Rightarrow \frac{36}{x} = \frac{3 \times 12}{1 \times 12}$

$\Rightarrow \frac{36}{x} = \frac{36}{12}$

$\Rightarrow x = 12$

(*vi*) $\frac{-27}{x} = 9$

$\Rightarrow \frac{-27}{x} = \frac{9}{1}$

$\Rightarrow \frac{-27}{x} = \frac{9 \times (-3)}{1 \times (-3)}$

$\Rightarrow \frac{-27}{x} = \frac{-27}{-3}$

$\Rightarrow x = -3$

Question 13.
Express each of the following rational numbers to the lowest terms :

(i) $\frac{12}{15}$ (ii) $\frac{-120}{144}$

(iii) $\frac{-48}{-72}$ (iv) $\frac{14}{-56}$

Solution:

(i) $\frac{12}{15}$

$$\begin{array}{l} 12)\overline{15}(1 \\ \quad 12 \\ \quad\overline{\ \ 3)12(4} \\ \qquad\ 12 \\ \qquad\ \overline{\ \times\ } \end{array}$$

(Dividing by 3, H.C.F. of 12 and 15)

$$\Rightarrow \frac{12 \div 3}{15 \div 3} = \frac{4}{5}$$

(*ii*) $\dfrac{-120}{144}$

$$\begin{array}{l}
120\overline{)144}(1 \\
\quad\;\; \underline{120} \\
\quad\;\; 24\overline{)120}(5 \\
\qquad\quad\;\; \underline{120} \\
\qquad\quad\;\; \underline{\;\times\;}
\end{array}$$

(Dividing by 24, H.C.F. of −120 and 144)

$$\Rightarrow \frac{-120 \div 24}{144 \div 24} = \frac{-5}{6}$$

(*iii*) $\dfrac{-48}{-72}$

$$\begin{array}{l}
48\overline{)72}(1 \\
\quad\; \underline{48} \\
\quad\; 24\overline{)48}(2 \\
\qquad\;\; \underline{48} \\
\qquad\;\; \underline{\;\times\;}
\end{array}$$

(Dividing by 24, H.C.F. of −48 and −72)

$$\Rightarrow \frac{-48 \div 24}{-72 \div 24} = \frac{-2}{-3} = \frac{2}{3}$$

(*iv*) $\dfrac{14}{-56}$

$$\begin{array}{l}
14\overline{)56}(4 \\
\quad\; \underline{56} \\
\quad\; \underline{\;\times\;}
\end{array}$$

(Dividing by 14, H.C.F. of 14 and −56)

$$\Rightarrow \frac{14 \div 14}{-56 \div 14} = \frac{1}{-4} \text{ or } \frac{-1}{4}$$

Question 14.
Express each of the following rational numbers in the standard form.

(i) $\frac{-7}{-8}$ (ii) $\frac{5}{-12}$

(iii) $\frac{-7}{-20}$ (iv) $\frac{4}{-9}$

Solution:

We know that, a rational number is said to be in standard form, if its denominator is positive in lowest term.

(i) $\frac{-7}{-8} = \frac{7}{8}$

(ii) $\frac{5}{-12} = \frac{-5}{12}$

(iii) $\frac{-7}{-20} = \frac{7}{20}$

(iv) $\frac{4}{-9} = \frac{-4}{9}$

EXERCISE 2 (B)

Question 1.

Mark the following pairs of rational numbers on the separate number lines :

(i) $\frac{3}{4}$ and $-\frac{1}{4}$

(ii) $\frac{2}{5}$ and $\frac{-3}{5}$

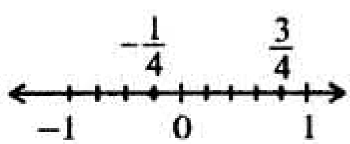
(iii) $\frac{5}{6}$ and $-\frac{2}{3}$

(iv) $\frac{2}{5}$ and $-\frac{4}{5}$

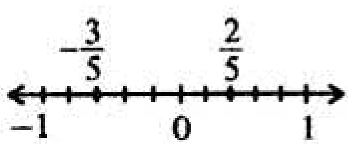
(v) $\frac{1}{4}$ and $-\frac{5}{4}$

Solution:

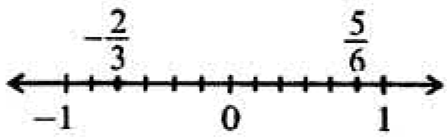
(i) $\frac{3}{4}$ and $-\frac{1}{4}$



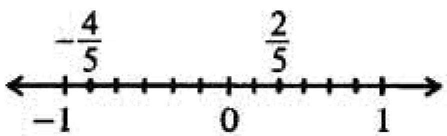
(ii) $\frac{2}{5}$ and $\frac{-3}{5}$



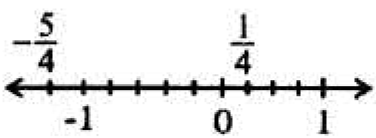
(iii) $\frac{5}{6}$ and $-\frac{2}{3}$



(iv) $\frac{2}{5}$ and $-\frac{4}{5}$



(v) $\frac{1}{4}$ and $-\frac{5}{4}$



Question 2.
Compare:

(i) $\frac{3}{5}$ and $\frac{5}{7}$

(ii) $\frac{-7}{2}$ and $\frac{5}{2}$

(iii) -3 and $2\frac{3}{4}$

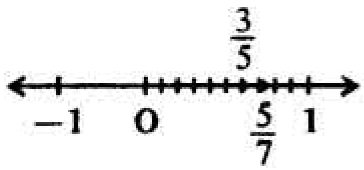
(iv) $-1\frac{1}{2}$ and 0

(iv) 0 and $\frac{3}{4}$

(vi) 3 and -1

Solution:

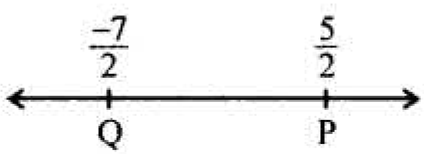
(i) $\frac{3}{5}$ and $\frac{5}{7}$



Since, $\frac{5}{7}$ is on the right side of the number line.

$\therefore\ \frac{5}{7} > \frac{3}{5}$

(ii) $\frac{-7}{2}$ and $\frac{5}{2}$



Since, P is on the right of Q.

$\Rightarrow\ \frac{5}{2} > \frac{-7}{2}$

(iii) -3 and $2\frac{3}{4}$ or $\frac{11}{4}$

Since, P is on the right of Q.

$\Rightarrow\ \frac{11}{4} > -3$ or $2\frac{3}{4} > -3$

(*iv*) $-1\frac{1}{2}$ and 0 or $\frac{-3}{2}$ and 0

Since, P is on the right of Q

$\Rightarrow 0 > \frac{-3}{2}$

(*iv*) 0 and $\frac{3}{4}$

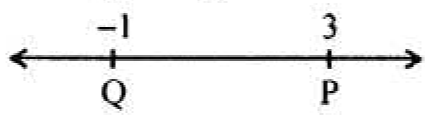
Since, P is on the right of Q

$\Rightarrow \frac{3}{4} > 0$

(*vi*) 3 and –1

Since, P is on the right of Q

$\Rightarrow 3 > -1$



Question 3.
Compare:

(i) $-\frac{1}{4}$ and 0 (ii) $\frac{1}{4}$ and 0

(iii) $-\frac{3}{8}$ and $\frac{2}{5}$ (iv) $\frac{-5}{8}$ and $\frac{7}{-12}$

(v) $\frac{5}{-9}$ and $\frac{-5}{-9}$ (vi) $\frac{-7}{8}$ and $\frac{5}{-6}$

(vii) $\frac{2}{7}$ and $\frac{-3}{-8}$ (viii) $\frac{-5}{8}$ and $\frac{7}{-12}$

Solution:

(i) $-\frac{1}{4}$ and 0

Since, $-\frac{1}{4}$ is a negative rational number and always less than 0.

$\therefore\ -\frac{1}{4} < 0$

(ii) $\frac{1}{4}$ and 0

Since, $\frac{1}{4}$ is a positive rational number and always greater than 0.

$\therefore\ \frac{1}{4} > 0$

(iii) $-\frac{3}{8}$ and $\frac{2}{5}$

-3×5 and 2×8

$$\left(\because \frac{a}{b} \text{ and } \frac{c}{d} \Rightarrow a \times d \text{ and } b \times c\right)$$

$-15 < 16 \qquad (\because a \times d < b \times c)$

$\therefore \; -\dfrac{3}{8} < \dfrac{2}{5}$

(*iv*) $\dfrac{-5}{8}$ and $\dfrac{7}{-12}$

-5×-12 and 7×8

$$\left(\because \frac{a}{b} \text{ and } \frac{c}{d} \Rightarrow a \times d \text{ and } b \times c\right)$$

$60 > 56 \qquad (\because a \times d > b \times c)$

$\therefore \; \dfrac{-5}{8} > \dfrac{7}{-12}$

(*v*) $\dfrac{5}{-9}$ and $\dfrac{-5}{-9}$

5×-9 and -5×-9

$$\left(\because \frac{a}{b} \text{ and } \frac{c}{d} \Rightarrow a \times d \text{ and } b \times c\right)$$

$-45 < 45 \qquad (\because a \times d < b \times c)$

$\therefore \; \dfrac{5}{-9} < \dfrac{-5}{-9}$

(vi) $\frac{-7}{8}$ and $\frac{5}{-6}$

-7×-6 and 5×8

$$\left(\because \frac{a}{b} \text{ and } \frac{c}{d} \Rightarrow a \times d \text{ and } b \times c\right)$$

$42 > 40$ $\quad (\because a \times d > b \times c)$

$\therefore \frac{-7}{8} > \frac{5}{-6}$

(vii) $\frac{2}{7}$ and $\frac{-3}{-8}$

2×8 and 7×-3

$$\left(\because \frac{a}{b} \text{ and } \frac{c}{d} \Rightarrow a \times d \text{ and } b \times c\right)$$

$16 > -21$ $\quad (\because a \times d > b \times c)$

$\therefore \frac{2}{7} > \frac{-3}{-8}$

(viii) $\frac{-5}{8}$ and $\frac{7}{-12}$

-5×-12 and 7×8

$60 > 56$

$\therefore \frac{-5}{8} > \frac{7}{-12}$

Question 4.

Arrange the given rational numbers in ascending order :

(i) $\frac{7}{10}, \frac{-11}{-30}$ and $\frac{5}{-15}$

(ii) $\frac{4}{-9}, \frac{-5}{12}$ and $\frac{2}{-3}$

Solution:

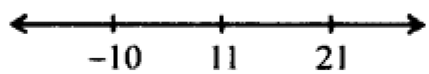
(*i*) $\frac{7}{10}, \frac{-11}{-30}$ and $\frac{5}{-15}$

$= \frac{7}{10}, \frac{11}{30}$ and $\frac{-5}{15}$

$= \frac{7\times3}{10\times3}, \frac{11}{30}$ and $\frac{-5\times2}{15\times2}$

($\because$ LCM of 10, 30 and 15 = 30)

$= \frac{21}{30}, \frac{11}{30}, \frac{-10}{30}$



Since, $-10 < 11 < 21$

$\therefore \frac{-10}{30} < \frac{11}{30} < \frac{21}{30}$

$\Rightarrow \frac{5}{-15} < \frac{-11}{-30} < \frac{7}{10}$

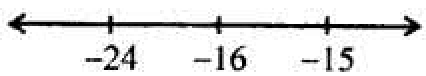
(*ii*) $\frac{4}{-9}, \frac{-5}{12}$ and $\frac{2}{-3}$

$$= \frac{-4}{9}, \frac{-5}{12} \text{ and } \frac{-2}{3}$$

$$= \frac{-4 \times 4}{9 \times 4}, \frac{-5 \times 3}{12 \times 3} \text{ and } \frac{-2 \times 12}{3 \times 12}$$

($\because$ LCM of 9, 12 and 3 = 36)

$$= \frac{-16}{36}, \frac{-15}{36} \text{ and } \frac{-24}{36}$$



Since, $-24 < -16 < -15$

$$\therefore \frac{-24}{36} < \frac{-16}{36} < \frac{-15}{36}$$

$$\Rightarrow \frac{2}{-3} < \frac{4}{-9} < \frac{-5}{12}$$

Question 5.
Arrange the given rational numbers in descending order:

(*i*) $\frac{5}{8}, \frac{13}{-16}$ and $\frac{-7}{12}$

(*ii*) $\frac{3}{-10}, \frac{-13}{30}$ and $\frac{8}{-20}$

Solution:

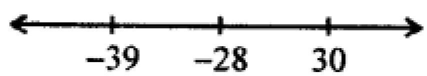
(i) $\frac{5}{8}, \frac{13}{-16}$ and $\frac{-7}{12}$

$= \frac{5}{8}, \frac{-13}{16}$ and $\frac{-7}{12}$

$= \frac{5\times 6}{8\times 6}, \frac{-13\times 3}{16\times 3}$ and $\frac{-7\times 4}{12\times 4}$

($\because$ LCM of 8, 16 and 12 = 48)

$= \frac{30}{48}, \frac{-39}{48}$ and $\frac{-28}{48}$



Since, $30 > -28 > -39$

$\therefore \frac{30}{48} > \frac{-28}{48} > \frac{-39}{48}$

$\Rightarrow \frac{5}{8} > \frac{-7}{12} > \frac{-13}{16}$

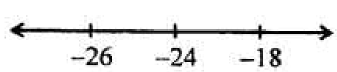
(ii) $\frac{3}{-10}, \frac{-13}{30}$ and $\frac{8}{-20}$

$= \frac{-3}{10}, \frac{-13}{30}$ and $\frac{-8}{20}$

$= \frac{-3\times 6}{10\times 6}, \frac{-13\times 2}{30\times 2}$ and $\frac{-8\times 3}{20\times 3}$

($\because$ LCM of 10, 20 and 30 = 60)

$= \frac{-18}{60}, \frac{-26}{60}$ and $\frac{-24}{60}$



Since, $-18 > -24 > -26$

$\therefore \frac{-18}{60} > \frac{-24}{60} > \frac{-26}{60}$

$\Rightarrow \frac{-3}{10} > \frac{-8}{20} > \frac{-13}{30}$

Question 6.
Fill in the blanks :

(i) $\frac{5}{8}$ and $\frac{3}{10}$ are on the .... side of zero.

(ii) $-\frac{5}{8}$ and $\frac{3}{10}$ are on the .... sides of zero.

(iii) $-\frac{5}{8}$ and $-\frac{3}{10}$ are on the .... side of zero.

(iv) $\frac{5}{8}$ and $-\frac{3}{10}$ are on the .... sides of zero.

Solution:

(i) $\frac{5}{8}$ and $\frac{3}{10}$ are on the **right** side of zero.

(ii) $-\frac{5}{8}$ and $\frac{3}{10}$ are on the **opposite** sides of zero.

(iii) $-\frac{5}{8}$ and $-\frac{3}{10}$ are on the **same/left** side of zero.

(iv) $\frac{5}{8}$ and $-\frac{3}{10}$ are on the **opposite** sides of zero.

EXERCISE 2 (C)

Question 1.

Add:

(i) $\frac{7}{5}$ and $\frac{2}{5}$ (ii) $\frac{-4}{9}$ and $\frac{2}{9}$

(iii) $\frac{5}{-12}$ and $\frac{1}{12}$ (iv) $\frac{4}{-15}$ and $\frac{-7}{-15}$

(v) $\frac{-7}{25}$ and $\frac{9}{-25}$ (vi) $\frac{-7}{26}$ and $\frac{7}{-26}$

Solution:

(i) $\frac{7}{5}$ and $\frac{2}{5} = \frac{7}{5} + \frac{2}{5}$

$= \frac{7+2}{5} = \frac{9}{5}$

(ii) $\frac{-4}{9}$ and $\frac{2}{9} = \frac{-4}{9} + \frac{2}{9}$

$= \frac{-4+2}{9} = \frac{-2}{9}$

(iii) $\frac{5}{-12}$ and $\frac{1}{12} = \frac{-5}{12} + \frac{1}{12}$

$= \frac{-5+1}{12} = \frac{-4}{12} = -\frac{4}{12}$

(iv) $\frac{4}{-15}$ and $\frac{-7}{-15} = \frac{-4}{15} + \frac{7}{15}$

$= \frac{-4+7}{15} = \frac{3}{15}$

(v) $\frac{-7}{25}$ and $\frac{9}{-25} = \frac{-7}{25} + \frac{-9}{25}$

$= \frac{(-7)+(-9)}{25} = \frac{-16}{25}$

(vi) $\frac{-7}{26}$ and $\frac{7}{-26} = \frac{-7}{26} + \frac{-7}{26}$

$= \frac{(-7)+(-7)}{26} = \frac{-14}{26}$

Question 2.
Add:

(i) $\frac{-2}{5}$ and $\frac{3}{7}$ (ii) $\frac{-5}{6}$ and $\frac{4}{9}$

(iii) -3 and $\frac{2}{3}$ (iv) $\frac{-5}{9}$ and $\frac{7}{18}$

(v) $\frac{-7}{24}$ and $\frac{-5}{48}$ (vi) $\frac{1}{-18}$ and $\frac{5}{-27}$

(vii) $\frac{-9}{25}$ and $\frac{1}{-75}$ (viii) $\frac{13}{-16}$ and $\frac{-11}{24}$

(ix) $\frac{-9}{-16}$ and $\frac{-11}{8}$

Solution:

(*i*) $\frac{-2}{5}$ and $\frac{3}{7}$

$= \frac{-2\times7}{5\times7} + \frac{3\times5}{7\times5}$

($\because$ L.C.M. of 5 and 7 = 35)

$= \frac{-14}{35} + \frac{15}{35}$

$= \frac{-14+15}{35} = \frac{1}{35}$

(*ii*) $\frac{-5}{6}$ and $\frac{4}{9} = \frac{-5}{6} + \frac{4}{9}$

$= \frac{-5\times6}{6\times6} + \frac{4\times4}{9\times4}$

($\because$ L.C.M. of 6 and 9 = 36)

$$= \frac{-30}{36} + \frac{16}{36}$$

$$= \frac{-30+16}{36} = \frac{-14}{36}$$

(*iii*) -3 and $\frac{2}{3} = \frac{-3}{1} + \frac{2}{3}$

$$= \frac{-3\times 3}{1\times 3} + \frac{2\times 1}{3\times 1}$$

($\because$ L.C.M. of 1 and 3 = 3)

$$= \frac{-9}{3} + \frac{2}{3}$$

$$= \frac{-9+2}{3} = \frac{-7}{3}$$

(*iv*) $\frac{-5}{9}$ and $\frac{7}{18} = \frac{-5}{9} + \frac{7}{18}$

$$= \frac{-5\times 2}{9\times 2} + \frac{7\times 1}{18\times 1}$$

($\because$ L.C.M. of 9 and 18 = 18)

$$= \frac{-10}{18} + \frac{7}{18} = \frac{-10+7}{18} = \frac{-3}{18}$$

(*v*) $\frac{-7}{24}$ and $\frac{-5}{48}$

$$= \frac{-7\times 2}{24\times 2} + \frac{-5\times 1}{48\times 1}$$

($\because$ L.C.M. of 24 and 48 = 48)

$$= \frac{-14}{48} + \frac{-5}{48}$$

$$= \frac{(-14)+(-5)}{48} = \frac{-14-5}{48} = \frac{-19}{48}$$

(*vi*) $\frac{1}{-18}$ and $\frac{5}{-27} = \frac{-1}{18} + \frac{-5}{27}$

$$= \frac{-1\times 3}{18\times 3} + \frac{-5\times 2}{27\times 2}$$

(∵ L.C.M. of 18 and 27 = 54)

$$= \frac{-3}{54} + \frac{-10}{54}$$

$$= \frac{(-3)+(-10)}{54} = \frac{-3-10}{54} = \frac{-13}{54}$$

(*vii*) $\frac{-9}{25}$ and $\frac{1}{-75} = \frac{-9}{25} + \frac{-1}{75}$

$$= \frac{-9\times 3}{25\times 3} + \frac{-1\times 1}{75\times 1}$$

(∵ L.C.M. of 25 and 75 = 75)

$$= \frac{-27}{75} + \frac{-1}{75}$$

$$= \frac{(-27)+(-1)}{75} = \frac{-27-1}{75} = \frac{-28}{75}$$

(*viii*) $\frac{13}{-16}$ and $\frac{-11}{24} = \frac{-13}{16} + \frac{-11}{24}$

$$= \frac{-13\times 3}{16\times 3} + \frac{-11\times 2}{24\times 2}$$

(∵ L.C.M. of 16 and 24 = 48)

$$= \frac{-39}{48} + \frac{-22}{48}$$

$$= \frac{(-39)+(-22)}{48} = \frac{-39-22}{48} = \frac{-61}{48}$$

(ix) $\frac{-9}{-16}$ and $\frac{-11}{8} = \frac{9}{16} + \frac{-11}{8}$

$= \frac{9\times1}{16\times1} + \frac{-11\times2}{8\times2}$

($\because$ L.C.M. of 16 and 8 = 16)

$= \frac{9}{16} + \frac{-22}{16}$

$= \frac{9+(-22)}{16} = \frac{9-22}{16} = \frac{-13}{16}$

Question 3.
Evaluate:

(i) $\frac{-2}{5} + \frac{3}{5} + \frac{-1}{5}$

(ii) $\frac{-8}{9} + \frac{4}{9} + \frac{-2}{9}$

(iii) $\frac{5}{-24} + \frac{-1}{8} + \frac{3}{16}$

(iv) $\frac{-7}{6} + \frac{4}{-15} + \frac{-4}{-30}$

(v) $-2 + \frac{2}{5} + \frac{-2}{15}$

(vi) $\frac{-11}{12} + \frac{5}{16} + \frac{-3}{8}$

Solution:

(i) $\frac{-2}{5}+\frac{3}{5}+\frac{-1}{5}$

$=\frac{-2+3-1}{5}=\frac{0}{5}=0$

(ii) $\frac{-8}{9}+\frac{4}{9}+\frac{-2}{9}$

$=\frac{-8+4-2}{9}=\frac{-10+4}{9}=\frac{-6}{9}$

(iii) $\frac{5}{-24}+\frac{-1}{8}+\frac{3}{16}$

$=\frac{-5\times2}{24\times2}+\frac{-1\times6}{8\times6}+\frac{3\times3}{16\times3}$

($\because$ L.C.M. of 8, 16, 24 = 48)

$=\frac{-10}{48}+\frac{-6}{48}+\frac{9}{48}$

$=\frac{-10-6+9}{48}=\frac{-16+9}{48}=\frac{-7}{48}$

(iv) $\dfrac{-7}{6} + \dfrac{4}{-15} + \dfrac{-4}{-30}$

$= \dfrac{-7}{6} + \dfrac{-4}{15} + \dfrac{4}{30}$

$= \dfrac{-7\times 5}{6\times 5} + \dfrac{-4\times 2}{15\times 2} + \dfrac{4\times 1}{30\times 1}$

($\because$ L.C.M. of 6, 15 and 30 = 30)

$= \dfrac{-35}{30} + \dfrac{-8}{30} + \dfrac{4}{30}$

$= \dfrac{-35-8+4}{30} = \dfrac{-43+4}{30} = \dfrac{-39}{30}$

(v) $-2 + \dfrac{2}{5} + \dfrac{-2}{15}$

$= \dfrac{-2}{1} + \dfrac{2}{5} + \dfrac{-2}{15}$

$= \dfrac{-2\times 15}{1\times 15} + \dfrac{2\times 3}{5\times 3} + \dfrac{-2\times 1}{15\times 1}$

($\because$ L.C.M. of 1, 5 and 15 = 15)

$= \dfrac{-30}{15} + \dfrac{6}{15} + \dfrac{-2}{15}$

$= \dfrac{-30+6-2}{15} = \dfrac{-32+6}{15} = \dfrac{-26}{15}$

(vi) $\frac{-11}{12} + \frac{5}{16} + \frac{-3}{8}$

$= \frac{-11 \times 4}{12 \times 4} + \frac{5 \times 3}{16 \times 3} + \frac{-3 \times 6}{8 \times 6}$

($\because$ L.C.M. of 8, 12 and 16 = 48)

$= \frac{-44}{48} + \frac{15}{48} + \frac{-18}{48}$

$= \frac{-44 + 15 - 18}{48} = \frac{-62 + 15}{48} = \frac{-47}{48}$

Question 4.

Evaluate:

(i) $-\frac{11}{18} + \frac{-3}{9} + \frac{2}{-3}$

(ii) $\frac{-9}{4} + \frac{13}{3} + \frac{25}{6}$

(iii) $-5 + \frac{5}{-8} + \frac{-5}{-12}$

(iv) $-\frac{2}{3} + \frac{5}{2} + 2$

(v) $5 + \frac{-3}{4} + \frac{-5}{8}$

Solution:

(i) $-\frac{11}{18}+\frac{-3}{9}+\frac{2}{-3}$

$=\frac{-11}{18}+\frac{-3}{9}+\frac{-2}{3}$

$=\frac{-11\times 1}{18\times 1}+\frac{-3\times 2}{9\times 2}+\frac{-2\times 6}{3\times 6}$

($\because$ L.C.M. of 3, 9 and 18 = 18)

$=\frac{-11}{18}+\frac{-6}{18}+\frac{-12}{18}$

$=\frac{-11-6-12}{18}=\frac{-29}{18}$

(ii) $\frac{-9}{4}+\frac{13}{3}+\frac{25}{6}$

$=\frac{-9\times 6}{4\times 6}+\frac{13\times 8}{3\times 8}+\frac{25\times 4}{6\times 4}$

($\because$ L.C.M. of 4, 3 and 6 = 24)

$=\frac{-54}{24}+\frac{104}{24}+\frac{100}{24}$

$=\frac{-54+104+100}{24}=\frac{150}{24}=\frac{25}{6}$

(*iii*) $-5 + \dfrac{5}{-8} + \dfrac{-5}{-12}$

$= \dfrac{-5}{1} + \dfrac{-5}{8} + \dfrac{5}{12}$

$= \dfrac{-5 \times 24}{1 \times 24} + \dfrac{-5 \times 3}{8 \times 3} + \dfrac{5 \times 2}{12 \times 2}$

($\because$ L.C.M. of 1, 8 and 12 = 24)

$= \dfrac{-120}{24} + \dfrac{-15}{24} + \dfrac{10}{24}$

$= \dfrac{-120 - 15 + 10}{24} = \dfrac{-125}{24}$

(*iv*) $-\dfrac{2}{3} + \dfrac{5}{2} + 2$

$= -\dfrac{2}{3} + \dfrac{5}{2} + \dfrac{2}{1}$

$= \dfrac{-2 \times 2}{3 \times 2} + \dfrac{5 \times 3}{2 \times 3} + \dfrac{2 \times 6}{1 \times 6}$

($\because$ L.C.M. of 3, 2 and 1 = 6)

$= \dfrac{-4}{6} + \dfrac{15}{6} + \dfrac{12}{6}$

$= \dfrac{-4 + 15 + 12}{6} = \dfrac{23}{6}$

$(v)\ 5 + \frac{-3}{4} + \frac{-5}{8}$

$= \frac{5}{1} + \frac{-3}{4} + \frac{-5}{8}$

$= \frac{5\times8}{1\times8} + \frac{-3\times2}{4\times2} + \frac{-5\times1}{8\times1}$

($\because$ L.C.M. of 1, 4 and 8 = 8)

$= \frac{40}{8} + \frac{-6}{8} + \frac{-5}{8}$

$= \frac{40-6-5}{8} = \frac{40-11}{8} = \frac{29}{8}$

Question 5.
Subtract :

$(i)\ \frac{2}{9}$ from $\frac{5}{9}$ $\quad (ii)\ \frac{-6}{11}$ from $\frac{-3}{-11}$

$(iii)\ \frac{-2}{15}$ from $\frac{-8}{15}$ $\quad (iv)\ \frac{11}{18}$ from $\frac{-5}{18}$

$(v)\ \frac{-4}{11}$ from -2

Solution:

(i) $\frac{2}{9}$ from $\frac{5}{9}$

$= \frac{5}{9} - \frac{2}{9} = \frac{5-2}{9} = \frac{3}{9} = \frac{1}{3}$

(ii) $\frac{-6}{11}$ from $\frac{-3}{-11}$

$= \frac{3}{11} - \left(-\frac{6}{11}\right)$

$= \frac{3}{11} + \frac{6}{11} = \frac{3+6}{11} = \frac{9}{11}$

(iii) $\frac{-2}{15}$ from $\frac{-8}{15}$

$= \frac{-8}{15} - \left(\frac{-2}{15}\right)$

$= \frac{-8}{15} + \frac{2}{15} = \frac{-8+2}{15} = \frac{-6}{15} = \frac{-2}{5}$

(iv) $\frac{11}{18}$ from $\frac{-5}{18}$

$= \frac{-5}{18} - \frac{11}{18} = \frac{-5-11}{18} = \frac{-16}{18} = \frac{-8}{9}$

(v) $\frac{-4}{11}$ from -2

$= \frac{-2}{1} - \left(\frac{-4}{11}\right) = \frac{-2\times11}{1\times11} + \frac{4\times1}{11\times1}$

$= \frac{-22}{11} + \frac{4}{11} = \frac{-22+4}{11} = \frac{-18}{11}$

Question 6.
Subtract :

(i) $-\frac{3}{10}$ from $\frac{1}{5}$ (ii) $\frac{-6}{25}$ from $\frac{-8}{5}$

(iii) $\frac{-7}{4}$ from -2 (iv) $\frac{-16}{21}$ from 1

(v) $\frac{-8}{15}$ from 0 (vi) 0 from $\frac{-3}{8}$

(vii) -2 from $\frac{-3}{10}$ (viii) $\frac{5}{8}$ from $\frac{-5}{16}$

(ix) 4 from $-\frac{3}{13}$

Solution:

(i) $-\frac{3}{10}$ from $\frac{1}{5}$

$= \frac{1}{5} - \left(-\frac{3}{10}\right)$

$= \frac{1\times 2}{5\times 2} + \frac{3}{10}$

$= \frac{2}{10} + \frac{3}{10} = \frac{2+3}{10} = \frac{5}{10}$

(ii) $\frac{-6}{25}$ from $\frac{-8}{5}$

$= \frac{-8}{5} - \left(\frac{-6}{25}\right)$

$= \frac{-8\times 5}{5\times 5} + \frac{6}{25} = \frac{-40}{25} + \frac{6}{25}$

$= \frac{-40+6}{25} = \frac{-34}{25}$

(iii) $\frac{-7}{4}$ from -2

$$= \frac{-2}{1} - \left(\frac{-7}{4}\right)$$

$$= \frac{-2 \times 4}{1 \times 4} + \frac{7}{4} = \frac{-8}{4} + \frac{7}{4}$$

$$= \frac{-8+7}{4} = \frac{-1}{4}$$

(*iv*) $\frac{-16}{21}$ from 1

$$= \frac{1}{1} - \left(\frac{-16}{21}\right)$$

$$= \frac{1}{1} + \frac{16}{21} = \frac{1 \times 21 + 16}{21}$$

$$= \frac{21+16}{21} = \frac{37}{21}$$

(*v*) $\frac{-8}{15}$ from 0

$$= 0 - \left(\frac{-8}{15}\right)$$

$$= 0 + \frac{8}{15} = \frac{8}{15}$$

(*vi*) 0 from $\frac{-3}{8}$

$$= \frac{-3}{8} - 0 = \frac{-3}{8}$$

(*vii*) -2 from $\frac{-3}{10}$

$= \frac{-3}{10} - \left(\frac{-2}{1}\right)$

$= \frac{-3}{10} + \frac{2}{1} = \frac{-3+2\times 10}{10} = \frac{17}{10}$

(*viii*) $\frac{5}{8}$ from $\frac{-5}{16}$

$= \frac{-5}{16} - \left(\frac{5}{8}\right)$

$= \frac{-5}{16} - \frac{5\times 2}{8\times 2} = \frac{-5}{16} - \frac{10}{16}$

$= \frac{-5-10}{16} = \frac{-15}{16}$

(*ix*) 4 from $-\frac{3}{13}$

$= -\frac{3}{13} - \frac{4}{1} = \frac{-3-4\times 13}{13}$

$= \frac{-3-52}{13} = \frac{-55}{13}$

Question 7.
The sum of two rational numbers is $\frac{11}{24}$. If one of them is $\frac{3}{8}$, find the other.

Solution:

$\because$ Sum of two rational number $= \frac{11}{24}$

and one of them $= \frac{3}{8}$

$\therefore$ The other rational number $= \frac{11}{24} - \frac{3}{8}$

$$= \frac{11}{24} - \frac{3\times3}{8\times3} = \frac{11}{24} - \frac{9}{24}$$

$$= \frac{11-9}{24} = \frac{2}{24}$$

Question 8.
The sum of two rational numbers is $\frac{-7}{11}$. If one of them is $\frac{13}{24}$, find the other.
Solution:

$\because$ Sum of two rational number $= \frac{-7}{12}$

One of them $= \frac{13}{24}$

$\therefore$ Other rational number $= \frac{-7}{12} - \frac{13}{24}$

$$= \frac{-7\times2}{12\times2} - \frac{13}{24}$$

$$= \frac{-14}{24} - \frac{13}{24}$$

$$= \frac{-14-13}{24} = \frac{-27}{24}$$

Question 9.
The sum of two rational numbers is -4. If one of them is $-\frac{13}{12}$, find the other.

Solution:

$\because$ Sum of two rational number = -4

and one of them = $-\frac{13}{12}$

$\therefore$ Other rational number $= -4 - \left(-\frac{13}{12}\right)$

$= -4 + \frac{13}{12}$

$= \frac{-4 \times 12 + 13}{12} = \frac{-48 + 13}{12} = \frac{-35}{12}$

Question 10.
What should be added to $-\frac{3}{6}$ to get $-\frac{11}{24}$?

Solution:

Let the required rational number be x

Other number = $-\frac{3}{16}$

Sum of two number = $\frac{11}{24}$

According to question,

$$-\frac{3}{16} + x = \frac{11}{24}$$

$$\Rightarrow x = \frac{11}{24} + \frac{3}{16}$$

$$x = \frac{11 \times 2}{24 \times 2} + \frac{3 \times 3}{16 \times 3}$$

($\because$ L.C.M. of 16 and 24 = 48)

$$x = \frac{22}{48} + \frac{9}{48}$$

$$x = \frac{22 + 9}{48} = \frac{31}{48}$$

Question 11.
What should be added to $\frac{-3}{5}$ to get 2?

Solution:
Let the required rational number be x

Other number = $\frac{-3}{5}$

Sum of two number = 2

According to question,

$$\frac{-3}{5} + x = 2$$

$$\Rightarrow x = 2 + \frac{3}{5}$$

$$= \frac{2 \times 5 + 3}{5} = \frac{10 + 3}{5} = \frac{13}{5}$$

Question 12.
What should be subtracted from $\frac{-4}{5}$ to get 1?

Solution:
Let the required rational number = x

Other number = $\frac{-4}{5}$

Difference of two number = 1

According to question,

$$\therefore \frac{-4}{5} - x = 1$$

$$\Rightarrow \frac{-4}{5} - 1 = x$$

$$\Rightarrow x = \frac{-4 - 1 \times 5}{5} = \frac{-4 - 5}{5} = \frac{-9}{5}$$

Question 13.
The sum of two numbers is $-\frac{6}{5}$. If one of them is -2, find the other.

Solution:

$\because$ Sum of two rational number $= -\frac{6}{5}$

and one of them $= -2$

$\therefore$ Other rational number $= -\frac{6}{5} - \left(-\frac{2}{1}\right)$

$= \frac{-6+2\times5}{5} = \frac{-6+10}{5} = \frac{4}{5}$

Question 14.
What should be added to $\frac{-7}{12}$ to get $\frac{3}{8}$?

Solution:

Let the required rational number be $= x$

Other number $= \frac{-7}{12}$

Sum of two numbers $= \frac{3}{8}$

$\therefore \frac{-7}{12} + x = \frac{3}{8}$

$\Rightarrow x = \frac{3}{8} - \frac{-7}{12}$

$= \frac{3\times3}{8\times3} + \frac{7\times2}{12\times2}$

($\because$ L.C.M. of 8 and 12 = 24)

$= \frac{9}{24} + \frac{14}{24}$

$= \frac{9+14}{24} = \frac{23}{24}$

Question 15.
What should be subtracted from $\frac{5}{9}$ to get $\frac{9}{5}$?

Solution:

Let the first number be x

Other number $= \frac{5}{9}$

Difference of two number $= \frac{9}{5}$

According to question,

$$\therefore \quad \frac{5}{9} - x = \frac{9}{5}$$

$$x = \frac{5}{9} - \frac{9}{5}$$

$$x = \frac{5\times5}{9\times5} - \frac{9\times9}{5\times9}$$

($\because$ L.C.M. of 9 and 5 = 45)

$$x = \frac{25}{45} - \frac{81}{45}$$

$$x = \frac{25-81}{45} = -\frac{56}{45}$$

EXERCISE 2 (D)

Question 1.

Evaluate:

(i) $\frac{5}{4} \times \frac{3}{7}$ (ii) $\frac{2}{3} \times -\frac{6}{7}$

(iii) $\left(\frac{-12}{5}\right) \times \left(\frac{10}{-3}\right)$ (iv) $\frac{-45}{39} \times \frac{-13}{15}$

(v) $3\frac{1}{8} \times \left(-2\frac{2}{5}\right)$ (vi) $2\frac{14}{25} \times \left(\frac{-5}{16}\right)$

(vii) $\left(\frac{-8}{9}\right) \times \left(\frac{-3}{16}\right)$ (viii) $\left(\frac{5}{-27}\right) \times \left(\frac{-9}{20}\right)$

Solution:

(i) $\frac{5}{4} \times \frac{3}{7} = \frac{5 \times 3}{4 \times 7} = \frac{15}{28}$

(ii) $\frac{2}{3} \times -\frac{6}{7} = \frac{2 \times -6}{3 \times 7} = \frac{2 \times -2}{7} = \frac{-4}{7}$

(iii) $\left(\frac{-12}{5}\right) \times \left(\frac{10}{-3}\right)$

$= \frac{(-12) \times 10}{5 \times (-3)} = 4 \times 2 = 8$

(iv) $\frac{-45}{39} \times \frac{-13}{15}$

$= \frac{(-45) \times (-13)}{39 \times 15} = \frac{(-3) \times (-1)}{3 \times 1}$

$= \frac{3}{3} = 1$

(v) $3\frac{1}{8} \times \left(-2\frac{2}{5}\right)$

$= \frac{3 \times 8 + 1}{8} \times \left(-\frac{2 \times 5 + 2}{5}\right)$

$= \frac{25}{8} \times \left(-\frac{12}{5}\right) = \frac{25 \times (-12)}{8 \times 5}$

$= \frac{5 \times (-3)}{2 \times 1} = \frac{-15}{2}$

(vi) $2\frac{14}{25} \times \left(\frac{-5}{16}\right)$

$= \frac{2 \times 25 + 14}{25} \times \left(\frac{-5}{16}\right)$

$= \frac{64}{25} \times \left(\frac{-5}{16}\right)$

$= \frac{64 \times (-5)}{25 \times 16} = \frac{4 \times (-1)}{5 \times 1} = -\frac{4}{5}$

(vii) $\left(\frac{-8}{9}\right) \times \left(\frac{-3}{16}\right)$

$= \frac{(-8) \times (-3)}{9 \times 16} = \frac{(-1) \times (-1)}{3 \times 2} = \frac{1}{6}$

(viii) $\left(\frac{5}{-27}\right) \times \left(\frac{-9}{20}\right)$

$= \frac{5 \times (-9)}{(-27) \times 20} = \frac{1 \times 1}{3 \times 4} = \frac{1}{12}$

Question 2.
Multiply:

(i) $\frac{3}{25}$ and $\frac{4}{5}$ (ii) $1\frac{1}{8}$ and $10\frac{2}{3}$

(iii) $6\frac{2}{3}$ and $\frac{-3}{8}$ (iv) $\frac{-13}{15}$ and $\frac{-25}{26}$

(v) $1\frac{1}{6}$ and 18 (vi) $2\frac{1}{14}$ and -7

(vii) $5\frac{1}{8}$ and -16 (viii) 35 and $\frac{-18}{25}$

(ix) $6\frac{2}{3}$ and $-\frac{3}{8}$ (x) $3\frac{3}{5}$ and -10

(xi) $\frac{27}{28}$ and -14 (xii) -24 and $\frac{5}{16}$

Solution:

(i) $\frac{3}{25}$ and $\frac{4}{5}$

$= \frac{3}{25} \times \frac{4}{5} = \frac{3 \times 4}{25 \times 5} = \frac{12}{125}$

(ii) $1\frac{1}{8}$ and $10\frac{2}{3}$

$= \frac{9}{8} \times \frac{32}{3} = \frac{9 \times 32}{8 \times 3} = 3 \times 4 = 12$

(iii) $6\frac{2}{3}$ and $\frac{-3}{8}$

$= \frac{20}{3} \times \frac{(-3)}{8} = \frac{20 \times (-3)}{3 \times 8}$

$= \frac{5 \times (-1)}{1 \times 2} = \frac{-5}{2}$

(iv) $\frac{-13}{15}$ and $\frac{-25}{26}$

$= \frac{-13 \times -25}{15 \times 26} = \frac{-1 \times -5}{3 \times 2} = \frac{5}{6}$

(v) $1\frac{1}{6}$ and 18

$= \frac{7}{6} \times 18 = 7 \times 3 = 21$

(vi) $2\frac{1}{14}$ and -7

$= \frac{2 \times 14 + 1}{14} \times (-7) = \frac{29}{14} \times (-7)$

$= \frac{29 \times (-1)}{2} = \frac{-29}{2}$

(vii) $5\frac{1}{8}$ and -16

$= \frac{41}{8} \times (-16) = 41 \times -2 = -82$

(*viii*) 35 and $\frac{-18}{25}$

$= 35 \times \frac{-18}{25} = \frac{35 \times (-18)}{25} = \frac{7 \times (-18)}{5}$

$= \frac{-126}{5} = -25\frac{1}{5}$

(*ix*) $6\frac{2}{3}$ and $-\frac{3}{8}$

$= \frac{20}{3} \times \frac{-3}{8} = \frac{20 \times (-3)}{3 \times 8}$

$= \frac{5 \times (-1)}{1 \times 2} = \frac{-5}{2} = -2\frac{1}{2}$

(*x*) $3\frac{3}{5}$ and -10

$= \frac{3 \times 5 + 3}{5} \times (-10)$

$= \frac{18}{5} \times (-10) = 18 \times (-2) = -36$

(*xi*) $\frac{27}{28}$ and -14

$= \frac{27}{28}$ and (-14)

$= \frac{27 \times (-1)}{2} = \frac{-27}{2} = -13\frac{1}{2}$

(*xii*) -24 and $\frac{5}{16}$

$= \frac{-24 \times 5}{16} = \frac{-3 \times 5}{2}$

$= \frac{-15}{2} = -7\frac{1}{2}$

Question 3.
Evaluate:

(i) $\left(-6\times\frac{5}{18}\right)-\left(-4\frac{2}{9}\right)$

(ii) $\left(\frac{7}{8}\times\frac{8}{7}\right)+\left(\frac{-5}{9}\right)\times\left(\frac{6}{-25}\right)$

(iii) $\left(\frac{11}{-9}\times\frac{21}{44}\right)+\left(\frac{-5}{9}\right)\times\left(\frac{63}{-100}\right)$

(iv) $\left(\frac{-5}{9}\times\frac{6}{-25}\right)+\left(\frac{24}{21}\times\frac{7}{8}\right)$

(v) $\left(\frac{-35}{39}\times\frac{-13}{7}\right)-\left(\frac{7}{90}\times\frac{-18}{14}\right)$

(vi) $\left(\frac{-4}{5}\times\frac{3}{2}\right)+\left(\frac{9}{-5}\times\frac{10}{3}\right)-\left(\frac{-3}{2}\times\frac{-1}{4}\right)$

Solution:

$$(i)\ \left(-6\times\frac{5}{18}\right)-\left(-4\frac{2}{9}\right)$$

$$=\left(-1\times\frac{5}{3}\right)-\left(\frac{-(4\times9+2)}{9}\right)$$

3	3, 9
3	1, 3
	1, 1

L.C.M. = 9

$$=\frac{-5}{3}-\left(\frac{-38}{9}\right)$$

$$=\frac{-5}{3}+\frac{38}{9}=\frac{-5\times3}{3\times3}+\frac{38\times1}{9\times1}$$

$$=\frac{-15+38}{9}\Rightarrow\frac{23}{9}=2\frac{5}{9}$$

$$(ii)\ \left(\frac{7}{8}\times\frac{8}{7}\right)+\left(\frac{-5}{9}\right)\times\left(\frac{6}{-25}\right)$$

$$=\left(\frac{7}{8}\times\frac{8}{7}\right)+\left(\frac{-5}{9}\times\frac{6}{(-25)}\right)$$

$$=\frac{1}{1}+\frac{1\times2}{3\times5}=\frac{1}{1}+\frac{2}{15}$$

$$=\frac{15+2}{15}=\frac{17}{15}=1\frac{2}{15}$$

(*iii*) $\left(\frac{11}{-9} \times \frac{21}{44}\right) + \left(\frac{-5}{9}\right) \times \left(\frac{63}{-100}\right)$

$= \left(\frac{11}{-9} \times \frac{21}{44}\right) + \left(\frac{5}{9} \times \frac{63}{100}\right)$

$= -\frac{1 \times 7}{3 \times 4} + \frac{1 \times 7}{1 \times 20} = -\frac{7}{12} + \frac{7}{20}$

$= -\frac{7 \times 5}{12 \times 5} + \frac{7 \times 3}{20 \times 3}$

($\because$ L.C.M. of 12 and 20 = 60)

$= -\frac{35}{60} + \frac{21}{60} = \frac{-35+21}{60} = \frac{-14}{60}$

(*iv*) $\left(\frac{-5}{9} \times \frac{6}{-25}\right) + \left(\frac{24}{21} \times \frac{7}{8}\right)$

$\left(\frac{5}{9} \times \frac{6}{25}\right) + \left(\frac{24}{21} \times \frac{7}{8}\right)$

$= \frac{2}{3 \times 5} + 1 = \frac{2}{15} + 1$

$= \frac{2+15}{15} = \frac{17}{15} = 1\frac{2}{15}$

(*v*) $\left(\frac{-35}{39} \times \frac{-13}{7}\right) - \left(\frac{7}{90} \times \frac{-18}{14}\right)$

$= \left(\frac{-35}{39} \times \frac{(-13)}{7}\right) - \left(\frac{7}{90} \times \frac{-18}{14}\right)$

$= \frac{(-5) \times (-1)}{3 \times 1} - \left(\frac{1 \times (-1)}{5 \times 2}\right)$

$= \frac{5}{3} - \left(\frac{-1}{10}\right)$

$= \frac{5 \times 10}{3 \times 10} + \frac{1}{10 \times 3}$

$= \frac{50+3}{30} = \frac{53}{30} = 1\frac{23}{30}$

$$(vi)\ \left(\frac{-4}{5}\times\frac{3}{2}\right)+\left(\frac{9}{-5}\times\frac{10}{3}\right)-\left(\frac{-3}{2}\times\frac{-1}{4}\right)$$

$$=\left(\frac{-2\times3}{5\times1}\right)+\left(\frac{3\times2}{-1\times1}\right)-\left(\frac{-3\times(-1)}{2\times4}\right)$$

$$=\left(\frac{-6}{5}\right)+\left(\frac{-6}{1}\right)-\left(\frac{3}{8}\right)$$

$$=\frac{-6\times8}{5\times8}-\frac{6\times40}{1\times40}-\frac{3\times5}{8\times5}$$

$$=\frac{-48-240-15}{40}$$

$$=\frac{-288-15}{40}=\frac{303}{40}=5\frac{3}{40}$$

Question 4.

Find the cost of 3 $\frac{1}{2}$ m cloth, if one metre cloth costs ₹325 $\frac{1}{2}$.

Solution:

Cost of 1 metre cloth = ₹$325\frac{1}{2}$

$$=\frac{2\times325+1}{2}=\frac{650+1}{2}=₹\frac{651}{2}$$

Now cost of $3\frac{1}{2}$ m $\left(\frac{2\times3+1}{2}=\frac{7}{2}\right)$m

$$=\frac{651}{2}\times\frac{7}{2}=\frac{651\times7}{2\times2}$$

$$=\frac{4557}{4}=₹1139\frac{1}{4}$$

Question 5.
A bus is moving with a speed of 65 $\frac{1}{2}$ km per hour. How much distance will it cover in 1 $\frac{1}{3}$ hours.

Solution:

Speed of bus per hour = $65\frac{1}{2}$

$= \frac{2\times65+1}{2} = \frac{130+1}{2} = \frac{131}{2}$ km

Time taken = $1\frac{1}{3}$ hour = $\frac{4}{3}$ hour

$= \frac{131}{2} \times \frac{4}{3} \Rightarrow \frac{131}{1} \times \frac{2}{3}$

Distance covered = Speed × Time

$= \frac{131}{2} \times \frac{4}{3}$

$= \frac{131\times2}{1\times3} = \frac{262}{3} = 87\frac{1}{3}$ km

Question 6.
Divide:

(i) $\frac{15}{28}$ by $\frac{3}{4}$ (ii) $\frac{-20}{9}$ by $\frac{-5}{9}$

(iii) $\frac{16}{-5}$ by $\frac{-8}{7}$ (iv) -7 by $\frac{-14}{5}$

(v) -14 by $\frac{7}{-2}$ (vi) $\frac{-22}{9}$ by $\frac{11}{18}$

(vii) 35 by $\frac{-7}{9}$ (viii) $\frac{21}{44}$ by $-\frac{11}{9}$

Solution:

(i) $\frac{15}{28}$ by $\frac{3}{4}$

$= \frac{15}{28} \div \frac{3}{4} \Rightarrow \frac{15}{28} \times \frac{4}{3}$

$= \frac{5}{7} \times \frac{1}{1} = \frac{5}{7}$

(ii) $\frac{-20}{9}$ by $\frac{-5}{9} = \frac{-20}{9} \div \frac{-5}{9}$

$\Rightarrow \frac{-20}{9} \times \frac{9}{-5} = \frac{-4}{-1} = \frac{4}{1} = 4$

(iii) $\frac{16}{-5}$ by $\frac{-8}{7}$

$= \frac{16}{-5} \div \frac{-8}{7} \Rightarrow \frac{16}{-5} \times \frac{7}{-8}$

$= \frac{2}{-5} \times \frac{7}{-1}$

$= \frac{2 \times 7}{-5 \times (-1)} = \frac{14}{5} = 2\frac{4}{5}$

(iv) -7 by $\frac{-14}{5}$

$= -7 \div \frac{-14}{5} \Rightarrow -7 \times \frac{5}{-14} \Rightarrow 1 \times \frac{5}{2}$

$= \frac{1 \times 5}{2} = \frac{5}{2} = 2\frac{1}{2}$

(v) -14 by $\frac{7}{-2}$

$$= -14 \div \frac{7}{-2} \Rightarrow -14 \times \frac{-2}{7}$$

$$= \frac{-2 \times (-2)}{1 \times 1} = 4$$

(*vi*) $\frac{-22}{9}$ by $\frac{11}{18}$

$$= \frac{-22}{9} \div \frac{11}{18} \Rightarrow \frac{-22}{9} \times \frac{18}{11}$$

$$= \frac{-2}{1} \times \frac{2}{1}$$

$$= \frac{-2 \times 2}{1 \times 1} = \frac{-4}{1} = -4$$

(*vii*) 35 by $\frac{-7}{9}$

$$= 35 \div \frac{-7}{9} \Rightarrow 35 \times \frac{9}{-7}$$

$$= 5 \times \frac{9}{-1}$$

$$= \frac{5 \times 9}{-1} = \frac{45}{-1} = -45$$

(*viii*) $\frac{21}{44}$ by $-\frac{11}{9}$

$$= \frac{21}{44} \div \left(-\frac{11}{9}\right) \Rightarrow \frac{21}{44} \times -\frac{9}{11}$$

$$= \frac{21 \times (-9)}{44 \times 11} = -\frac{189}{484}$$

Question 7.
Evaluate:

(i) $3\frac{5}{12} + 1\frac{2}{3}$ (ii) $3\frac{5}{12} - 1\frac{2}{3}$

(iii) $\left(3\frac{5}{12}+1\frac{2}{3}\right) \div \left(3\frac{5}{12}-1\frac{2}{3}\right)$

Solution:

(i) $3\frac{5}{12} + 1\frac{2}{3}$

$= \frac{12\times3+5}{12} + \frac{3\times1+2}{3}$

$= \frac{41}{12} + \frac{5}{3}$ ($\because$ L.C.M. of 12, 3 = 12)

$= \frac{41\times1}{12\times1} + \frac{5\times4}{3\times4} = \frac{41}{12} + \frac{20}{12}$

$= \frac{41+20}{12} = \frac{61}{12} = 5\frac{1}{12}$

(ii) $3\frac{5}{12} - 1\frac{2}{3}$

$= \frac{12\times3+5}{12} - \frac{3\times1+2}{3}$

$= \frac{41}{12} - \frac{5}{3}$ ($\because$ L.C.M. of 12, 3 = 12)

$= \frac{41\times1}{12\times1} - \frac{5\times4}{3\times4}$

$= \frac{41-20}{12} = \frac{21}{12} = \frac{7}{4} = 1\frac{3}{4}$

(iii) $\left(3\frac{5}{12}+1\frac{2}{3}\right) \div \left(3\frac{5}{12}-1\frac{2}{3}\right)$

$$= \left(\frac{12\times 3+5}{12}+\frac{3\times 1+2}{3}\right)$$

$$\div \left(\frac{12\times 3+5}{12}-\frac{3\times 1+2}{3}\right)$$

$$\left(\frac{41}{12}+\frac{5}{3}\right) \div \left(\frac{41}{12}-\frac{5}{3}\right)$$

$(\because$ L.C.M. of 12, 3 = 12)

$$= \left(\frac{41+20}{12}\right) \div \left(\frac{41-20}{12}\right)$$

$$= \frac{61}{12} \div \frac{21}{12} \Rightarrow \frac{61}{12} \times \frac{12}{21}$$

$$= \frac{61}{21} = 2\frac{19}{21}$$

Question 8.

The product of two numbers is 14. If one of the numbers is $\frac{-8}{7}$, find the other.

Solution:

$\because$ Product of two numbers = 14

and one of these two numbers = $\frac{-8}{7}$

The other number = $14 \div \frac{-8}{7}$

$$= 14 \times -\frac{7}{8} = -\frac{98}{8} = \frac{-49}{4}$$

Question 9.

The cost of 11 pens is ₹3 $\frac{2}{3}$. Find the cost of one pen.

Solution:

The cost of 11 pens = ₹$3\frac{2}{3}$

$= \frac{3\times3+2}{3} = ₹\frac{11}{3}$

The cost of one pen = $\frac{11}{3} \div 11$

$= \frac{11}{3} \times \frac{1}{11} = ₹\frac{1}{3}$

Question 10.

If 6 identical articles can be bought for ₹2 $\frac{6}{17}$. Find the cost of each article.

Solution:

Cost of 6 articles = ₹$2\frac{6}{17}$

$= \frac{2\times17+6}{17} = ₹\frac{40}{17}$

Cost of each article = $\frac{40}{17} \div 6$

$= \frac{40}{17} \times \frac{1}{6} = ₹\frac{20}{51}$

Question 11.

By what number should $\frac{-3}{8}$ be multiplied so that the product is $\frac{-9}{16}$?

Solution:

Number = $\frac{-3}{8} \div \left(\frac{-9}{16}\right)$

$= \frac{-3}{8} \times \frac{16}{-9} = \frac{2}{3} = 1\frac{1}{2}$

Question 12.
By what number should $\frac{-5}{7}$ be divided so -15 that the result is $\frac{-15}{28}$?

Solution:

$$\text{Number} = \frac{-15}{28} \div \frac{-5}{7}$$

$$= \frac{-15}{28} \times \frac{-7}{5} = \frac{3}{4}$$

Question 13.
Evaluate :

$$\left(\frac{32}{15}+\frac{8}{5}\right) \div \left(\frac{32}{15}-\frac{8}{5}\right).$$

Solution:

$$\left(\frac{32}{15}+\frac{8}{5}\right) \div \left(\frac{32}{15}-\frac{8}{5}\right)$$

$$\left(\frac{32\times1}{15\times1}+\frac{8\times3}{5\times3}\right) \div \left(\frac{32\times1}{15\times1}-\frac{8\times1}{5\times1}\right)$$

$$(\because \text{L.C.M. of } 15, 5 = 15)$$

$$= \left(\frac{32+24}{15}\right) \div \left(\frac{32-24}{15}\right)$$

$$= \frac{56}{15} \div \frac{8}{15} \Rightarrow \frac{56}{15} \times \frac{15}{8} = 7$$

Question 14.
Seven equal piece are made out of a rope 5 of 21 $\frac{5}{7}$ m. Find the length of each piece.

Solution:

Length of 7 pieces of rope $= 21\frac{5}{7}$ m

$$= \frac{21 \times 7 + 5}{7} = \frac{152}{7}$$

Length of each piece $= \frac{152}{7} \div 7$

$$= \frac{152}{7} \times \frac{1}{7} = \frac{152}{49} = 3\frac{5}{49} \text{ m}$$

EXERCISE 2 (E)

Question 1.
Evaluate:

(i) $\frac{-2}{3} + \frac{3}{4}$ (ii) $\frac{7}{-27} + \frac{11}{18}$

(iii) $\frac{-3}{8} + \frac{-5}{12}$ (iv) $\frac{9}{-16} + \frac{-5}{-12}$

(v) $\frac{-5}{9} + \frac{-7}{12} + \frac{11}{18}$ (vi) $\frac{7}{-26} + \frac{16}{39}$

(vii) $-\frac{2}{3} - \left(\frac{-5}{7}\right)$ (viii) $-\frac{5}{7} - \left(-\frac{3}{8}\right)$

(ix) $\frac{7}{26} + 2 + \frac{-11}{13}$ (x) $-1 + \frac{2}{-3} + \frac{5}{6}$

Solution:

(i) $\frac{-2}{3} + \frac{3}{4}$

$$\begin{array}{r|l} 3 & 3, 4 \\ \hline 4 & 1, 4 \\ \hline & 1, 1 \end{array}$$

L.C.M. of 3 and 4 = 3 × 4 = 12

$$\Rightarrow \frac{-2 \times 4}{3 \times 4} + \frac{3 \times 3}{4 \times 3}$$

($\because$ L.C.M. of 3 and 4 = 12)

$$\Rightarrow \frac{-8+9}{12} = \frac{1}{12}$$

(ii) $\frac{7}{-27} + \frac{11}{18}$

$$\begin{array}{r|l} 2 & 27, 18 \\ \hline 3 & 27, 9 \\ \hline 3 & 9, 3 \\ \hline 3 & 3, 1 \\ \hline & 1, 1 \end{array}$$

L.C.M. of 27 and 18 = 2 × 3 × 3 × 3 = 54

$$\Rightarrow \frac{7 \times 2}{-27 \times 2} + \frac{11 \times 3}{18 \times 3}$$

($\because$ L.C.M. of –27 and 18 = 54)

$$\Rightarrow \frac{-14+33}{54} = \frac{19}{54}$$

(*iii*) $\frac{-3}{8} + \frac{-5}{12}$

$$\begin{array}{r|l} 2 & 8, 12 \\ \hline 2 & 4, 6 \\ \hline 2 & 2, 3 \\ \hline 3 & 1, 3 \\ \hline & 1, 1 \end{array}$$

L.C.M. of 8 and 12 = 2 × 2 × 2 × 3 = 24

$$\Rightarrow \frac{-3\times3}{8\times3} + \frac{(-5\times2)}{12\times2}$$

($\because$ L.C.M. of 8, 12 = 24)

$$\Rightarrow \frac{-9-10}{24} = \frac{-19}{24}$$

(*iv*) $\frac{9}{-16} + \frac{-5}{-12}$ or $\frac{-5}{-12} = \frac{5}{12}$

$$\begin{array}{r|l} 2 & 16, 12 \\ \hline 2 & 8, 6 \\ \hline 2 & 4, 3 \\ \hline 2 & 2, 3 \\ \hline 3 & 1, 3 \\ \hline & 1, 1 \end{array}$$

L.C.M. of 16 and 12 = 2 × 2 × 2 × 2 × 3 = 48

$$\Rightarrow \frac{9\times 3}{-16\times 3} + \frac{5\times 4}{12\times 4}$$

$(\because$ L.C.M. of 16 and 12 = 48)

$$\Rightarrow \frac{-27+20}{48} = \frac{-7}{48}$$

(v) $\frac{-5}{9} + \frac{-7}{12} + \frac{11}{18}$

2	9, 12, 18
2	9, 6, 9
3	9, 3, 9
3	3, 1, 3
	1, 1, 1

L.C.M. of 9, 12 and 18 = 2 × 2 × 3 × 3 = 36

$$\Rightarrow \frac{-5\times 4}{9\times 4} - \frac{7\times 3}{12\times 3} + \frac{11\times 2}{18\times 2}$$

$(\because$ L.C.M. of 9, 12 and 18 = 36)

$$\Rightarrow \frac{-20-21+22}{36}$$

$$\Rightarrow \frac{-41+22}{36} = \frac{-19}{36}$$

(vi) $\frac{7}{-26} + \frac{16}{39}$

2	26, 39
3	13, 39
13	13, 13
	1, 1

L.C.M. of 26 and 39 = 2 × 3 × 13 = 78

$$\Rightarrow \frac{-7\times 3}{26\times 3} + \frac{16\times 2}{39\times 2}$$

(L.C.M. of –26 and 39 = 78)

$$\Rightarrow \frac{-21+32}{78} = \frac{11}{78}$$

(*vii*) $-\frac{2}{3} - \left(\frac{-5}{7}\right)$

$\Rightarrow -\frac{2}{3} + \frac{5}{7}$

$$\begin{array}{c|c} 3 & 3, 7 \\ \hline 7 & 1, 7 \\ \hline & 1, 1 \end{array}$$

L.C.M. of 3 and 7 = 3 × 7 = 21

$\Rightarrow \frac{-2 \times 7}{3 \times 7} + \frac{5 \times 3}{7 \times 3}$

($\because$ L.C.M. of 3 and 7 = 21)

$\Rightarrow \frac{-14 + 15}{21} = \frac{1}{21}$

(*viii*) $-\frac{5}{7} - \left(-\frac{3}{8}\right)$

$\Rightarrow -\frac{5}{7} + \frac{3}{8}$

$$\begin{array}{r|l} 2 & 7, 8 \\ \hline 2 & 7, 4 \\ \hline 2 & 7, 2 \\ \hline 7 & 7, 1 \\ \hline & 1 \end{array}$$

L.C.M. of 7 and 8 = 2 × 2 × 2 × 7 = 56

$\Rightarrow \frac{-5\times 8}{7\times 8} + \frac{3\times 7}{8\times 7}$

(∵ LCM of 7 and 8 = 56)

$\Rightarrow \frac{-40+21}{56} = \frac{-19}{56}$

(*ix*) $\frac{7}{26} + 2 + \frac{-11}{13}$

$\Rightarrow \frac{7}{26} + \frac{2}{1} + \frac{-11}{13}$

$$\begin{array}{r|l} 2 & 26, 13 \\ \hline 13 & 13, 13 \\ \hline & 1, 1 \end{array}$$

L.C.M. of 26 and 13 = 2 × 13 = 26

$\Rightarrow \frac{7\times 1}{26\times 1} + \frac{2\times 26}{1\times 26} - \frac{11\times 2}{13\times 2}$

(∵ L.C.M. of 26, 13 = 26)

$\Rightarrow \frac{7+52-22}{26}$

$$\Rightarrow \frac{59-22}{26} = \frac{37}{26}$$

(*x*) $-1 + \frac{2}{-3} + \frac{5}{6}$

$$\begin{array}{r|l} 2 & 3, 6 \\ \hline 3 & 3, 3 \\ \hline & 1, 1 \end{array}$$

L.C.M. of 3 and 6 = 2 × 3 = 6

$$\Rightarrow \frac{-1\times6}{1\times6} - \frac{2\times2}{3\times2} + \frac{5\times1}{6\times1}$$

($\because$ L.C.M. of 3 and 6 = 6)

$$= \frac{-6-4+5}{6}$$

$$= \frac{-10+5}{6} = \frac{-5}{6}$$

Question 2.
The sum of two rational numbers is $\frac{-3}{8}$. If one of them is $\frac{3}{16}$, find the other,

Solution:

Sum of two numbers = $\frac{-3}{8}$

One number = $\frac{3}{16}$

$\therefore$ Second number = $\frac{-3}{8} - \frac{3}{16}$

$$\begin{array}{r|l} 2 & 8, 16 \\ \hline 2 & 4, 8 \\ \hline 2 & 2, 4 \\ \hline 2 & 1, 2 \\ \hline & 1, 1 \end{array}$$

L.C.M. of 8 and 16 = $2 \times 2 \times 2 \times 2 = 16$

$$= \frac{-3 \times 2}{8 \times 2} - \frac{3 \times 1}{16 \times 1}$$

($\because$ L.C.M. of 8 and 16 = 16)

$$= \frac{-6-3}{16} = \frac{-9}{16}$$

$\therefore$ Second number = $\frac{-9}{16}$

Question 3.

The sum of two rational numbers is -5. If one of them is $\frac{-52}{25}$, find the other.

Solution:

Sum of two numbers = –5

One number = $\frac{-52}{25}$

Second number = $-5-\left(\frac{-52}{25}\right)$

$= \frac{-5 \times 25}{1 \times 25} + \frac{52 \times 1}{25 \times 1}$

$= \frac{-125+52}{25} = \frac{-77}{25}$

$\therefore$ Second number = $\frac{-77}{25}$

Question 4.
What rational number should be added to $-\frac{3}{16}$ to get $\frac{11}{24}$

Solution:

$$\text{Sum of two number} = \frac{11}{24}$$

$$\text{One number} = -\frac{3}{16}$$

$$\therefore \text{ The required number} = \frac{11}{24} - \left(\frac{-3}{16}\right)$$

$$\Rightarrow \frac{11}{24} + \frac{3}{16}$$

$$\begin{array}{r|l} 2 & 24, 16 \\ \hline 2 & 12, 8 \\ \hline 2 & 6, 4 \\ \hline 2 & 3, 2 \\ \hline 3 & 3, 1 \\ \hline & 1 \end{array}$$

$$\begin{aligned} \text{L.C.M. of 16 and 24} &= 2 \times 2 \times 2 \times 2 \times 3 \\ &= 48 \end{aligned}$$

$$(\because \text{L.C.M. of } 24, 16 = 48)$$

$$= \frac{11 \times 2}{24 \times 2} + \frac{3 \times 3}{16 \times 3}$$

$$= \frac{22 + 9}{48} = \frac{31}{48}$$

Question 5.
What rational number should be added to $-\frac{3}{5}$ to get 2?

Solution:

The required number $= 2 - \left(\frac{-3}{5}\right)$

$= 2 + \frac{3}{5}$

$= \frac{2 \times 5}{1 \times 5} + \frac{3 \times 1}{5 \times 1}$

$= \frac{10+3}{5} = \frac{13}{5} = 2\frac{3}{5}$

Question 6.
What rational number should be subtracted from $-\frac{5}{12}$ to get $\frac{5}{24}$

Solution:

The required number $= \frac{-5}{12} - \frac{5}{24}$

2	12, 24
2	6, 12
2	3, 6
3	3, 3
	1, 1

L.C.M. of 12 and 24 $= 2 \times 2 \times 2 \times 3 \times 3$
$= 72$

$\Rightarrow \frac{-5 \times 6}{12 \times 6} - \frac{5 \times 3}{24 \times 3}$

($\because$ L.C.M. of 12, 24 = 72)

$\Rightarrow \frac{-30-15}{72} = \frac{-45}{72}$ or $\frac{-5}{8}$

Question 7.
What rational number should be subtracted from $\frac{5}{8}$ to get $\frac{8}{5}$?

Solution:

The required number $= \frac{5}{8} - \frac{8}{5}$

$$\begin{array}{c|c} 2 & 8, 5 \\ \hline 2 & 4, 5 \\ \hline 2 & 2, 5 \\ \hline 5 & 1, 5 \\ \hline & 1, 1 \end{array}$$

$$\Rightarrow \frac{5\times5}{8\times5} - \frac{8\times8}{5\times8} \qquad \text{(L.C.M. of 8, 5 = 40)}$$

$$\Rightarrow \frac{25-64}{40} = \frac{-39}{40}$$

Question 8.
Evaluate:

(i) $\left(\frac{7}{8}\times\frac{24}{21}\right) + \left(\frac{-5}{9}\times\frac{6}{-25}\right)$

(ii) $\left(\frac{8}{15}\times\frac{-25}{16}\right) + \left(\frac{-18}{35}\times\frac{5}{6}\right)$

(iii) $\left(\frac{18}{33}\times\frac{-22}{27}\right) - \left(\frac{13}{25}\times\frac{-75}{26}\right)$

(iv) $\left(\frac{-13}{7}\times\frac{-35}{39}\right) - \left(\frac{-7}{45}\times\frac{9}{14}\right)$

Solution:

$$(i)\ \left(\frac{7}{8}\times\frac{24}{21}\right)+\left(\frac{-5}{9}\times\frac{6}{-25}\right)$$

$$\Rightarrow \frac{7\times 24}{8\times 21}+\frac{-5\times 6}{9\times(-25)}$$

$$\Rightarrow \frac{1\times 3}{1\times 3}+\frac{1\times 2}{3\times 5}$$

$$\Rightarrow \frac{3}{3}+\frac{2}{15}$$

$$\begin{array}{c|l} 3 & 3,\ 15 \\ \hline 5 & 1,\ 5 \\ \hline & 1,\ 1 \end{array}$$

($\because$ L.C.M. of 3 and 15 = 15)

$$\Rightarrow \frac{3\times 5}{3\times 5}+\frac{2\times 1}{15\times 1}$$

$$\Rightarrow \frac{15+2}{15}=\frac{17}{15}=1\frac{2}{15}$$

$$(ii)\ \left(\frac{8}{15}\times\frac{-25}{16}\right)+\left(\frac{-18}{35}\times\frac{5}{6}\right)$$

$$\Rightarrow \frac{8\times(-25)}{15\times 16}+\frac{-18\times 5}{35\times 6}$$

$$\Rightarrow \frac{1\times(-5)}{3\times 2}+\left(\frac{-3\times 1}{7\times 1}\right)$$

$$\Rightarrow \frac{-5}{6}-\frac{3}{7}$$

$$\begin{array}{c|l} 2 & 6,\ 7 \\ \hline 3 & 3,\ 7 \\ \hline 7 & 1,\ 7 \\ \hline & 1,\ 1 \end{array}$$

L.C.M. of 6 and 7 = 2 × 3 × 7 = 42

($\because$ L.C.M. of 6 and 7 = 42)

$$\Rightarrow \frac{-5\times 7}{6\times 7}-\frac{3\times 6}{7\times 6}$$

$$\Rightarrow \frac{-35-18}{42}=\frac{-53}{42}$$

(iii) $\left(\frac{18}{33} \times \frac{-22}{27}\right) - \left(\frac{13}{25} \times \frac{-75}{26}\right)$

$$\Rightarrow \frac{18 \times (-22)}{33 \times 27} - \frac{13 \times (-75)}{25 \times 26}$$

$$\Rightarrow \frac{2 \times (-2)}{3 \times 3} - \frac{1 \times (-3)}{1 \times 2}$$

$$\Rightarrow \frac{-4}{9} - \left(\frac{-3}{2}\right)$$

$$\Rightarrow \frac{-4}{9} + \frac{3}{2}$$

$$\begin{array}{r|l} 2 & 9, 2 \\ \hline 3 & 9, 1 \\ \hline 3 & 3, 1 \\ \hline & 1, 1 \end{array}$$

L.C.M. of 9 and 2 = 2 × 3 × 3 = 18

$$\Rightarrow \frac{-4 \times 2}{9 \times 2} + \frac{3 \times 9}{2 \times 9} \text{ } (\because \text{L.C.M. of 9 and 2} = 18)$$

$$\Rightarrow \frac{-8 + 27}{18} = \frac{19}{18} = 1\frac{1}{18}$$

(iv) $\left(\frac{-13}{7} \times \frac{-35}{39}\right) - \left(\frac{-7}{45} \times \frac{9}{14}\right)$

$$\Rightarrow \frac{-13 \times (-35)}{7 \times 39} + \frac{7 \times 9}{45 \times 14}$$

$$\Rightarrow \frac{-1 \times (-5)}{1 \times 3} + \frac{1 \times 1}{5 \times 2}$$

$$\Rightarrow \frac{5}{3} + \frac{1}{10}$$

$$\begin{array}{r|l} 2 & 3, 10 \\ \hline 3 & 3, 5 \\ \hline 5 & 1, 5 \\ \hline & 1, 1 \end{array}$$

L.C.M. of 3 and 10 = 2 × 3 × 5 = 10

$$\Rightarrow \frac{5\times10}{3\times10} + \frac{1\times3}{10\times3}$$

$(\because$ L.C.M. of 3 and 10 = 30)

$$\Rightarrow \frac{50+3}{30} = \frac{53}{30} = 1\frac{23}{30}$$

Question 9.
The product of two rational numbers is 24. If one of them is $-\frac{36}{11}$, find the other.

Solution:

Product of two numbers = 24

One number $= \frac{-36}{11}$

$\therefore$ Second number $= 24 \div \left(\frac{-36}{11}\right)$

$= 24 \times \left(\frac{-11}{36}\right)$

$= 2 \times \frac{(-11)}{3} = \frac{-22}{3}$

Question 10.
By what rational number should we multiply $\frac{20}{-9}$,so that the product may be $\frac{-5}{9}$?
Solution:

Required number $= \frac{-5}{9} \div \left(\frac{20}{-9}\right)$

$$\Rightarrow \frac{-5}{9} \times \left(\frac{-9}{20}\right) = \frac{1}{4}$$

$\therefore$ Required number $= \frac{1}{4}$

CHAPTER - 3
FRACTIONS

POINTS TO REMEMBER

1. **Fraction.** A rational number in form of — where a and b are integers is called a fraction.
   ‘a’ is called the numerator and Lb’ is called the denominator of the fraction.
2. **Classification of Fractions :**
   Decimal fraction : A fraction whose denominator is 10 or multiple of 10.
   Vulgur fraction : A fraction whose denominator is oilier than 10 or multiple of 10.
   Proper fraction : A fraction whose denominator is greater than its numerator.
   Improper fraction : A fraction whose denominator less than its numerator.
   Mixed fraction : A fraction which consists of an integer and a proper fraction.
   Note. If the numerator of a fraction is equal to its denominator, then the fraction is equal to unity i.e. 1.
3. **Equivalent Fractions**
   Fractions having the same value are called the equivalent fractions.
4. **Simple and Complex Fractions**
   A fraction whose numerator and denominator both are integers, is called a simple fraction.
   A fraction whose numerator or denominator or both are not integers, is called a complex fraction.
5. **Like and Unlike Fractions**
   Fractions having the same denominators are called like fractions.
   The fractions with different denominators are called unlike fractions.
6. **Converting unlike fractions into like fractions**
   Find the LCM of the denominators of all the give- fractions.
   For each given fraction, multiply its denominator by a suitable numbers so that the product obtained is equal to the LCM in (i).
   Multiply the numerator also by the same number.
7. **To insert a fraction between two given fractions .**
   Add the numerators as well as denominators of the given fractions. Then simplify if required.
8. **Addition and Subtraction of fractions**
9. For like fractions, add or subtract (as required) their numerators, keeping the denominator same.
   For unlike fractions, first change all the fractions into like fractions and then add or subtract as above given in (i).
10. **Multiplication**
    To multiply two or more fractions, multiply their numerators as well as their denominators.
11. **Division**
    To divide on fraction or integer by some other fractions or integer, multiply the first by the reciprocal of the second as given above in multiplication.

12. **Using 'BODMAS'**
The word 'BODMAS' is the abbreviation formed by taking the initial letters of six operations i.e. 'Bracket', ;OF, 'Division', 'Multiplication', 'Addition' and 'Subtraction'. So, according to the rule of 'BODMAS', working must be done in the order corresponding to the letters in the word 'BODMAS'.
13. **Brackets and their removal**
Brackets are four kinds i.e., bar bracket , circular brackets ( ), curly brackets { } and square brackets [ ] and these can be removed in this order i.e. firstly bar, then circular, then curly and lastly square brackets keeping in considerations of the sign given before them.

EXERCISE 3 (A)

Question 1.

Classify, each fraction given below, as decimal or vulgar fraction, proper or improper fraction and mixed fraction :

(i) $\frac{3}{5}$ (ii) $\frac{11}{10}$ (iii) $\frac{13}{20}$

(iv) $\frac{13}{7}$ (v) $3\frac{2}{9}$ (vi) $\frac{19}{10^3}$

(vii) $2\frac{7}{10}$ (viii) $\frac{23}{500}$

Solution:
(i) Vulgar and Proper
(ii) Decimal and Improper
(iii) Decimal and Proper
(iv) Vulgar and Improper
(v) Mixed
(vi) Decimal
(vii) Mixed and Decimal
(viii) Vulgar and Proper Ans.

Question 2.

Express the following improper fractions as mixed fractions :

(i) $\frac{18}{5}$ (ii) $\frac{7}{4}$ (iii) $\frac{25}{6}$

(iv) $\frac{38}{5}$ (v) $\frac{22}{5}$

Solution:

$$(i)\ \frac{18}{5} = 3\frac{3}{5} \qquad (ii)\ \frac{7}{4} = 1\frac{3}{4}$$

$$(iii)\ \frac{25}{6} = 4\frac{1}{6} \qquad (iv)\ \frac{38}{5} = 7\frac{3}{5}$$

$$(v)\ \frac{22}{5} = 4\frac{2}{5}\ \textbf{Ans.}$$

Question 3.

Express the following mixed fractions as improper fractions :

$(i)\ 2\frac{4}{9}$ $\quad (ii)\ 7\frac{5}{13}$ $\quad (iii)\ 3\frac{1}{4}$

$(iv)\ 2\frac{5}{48}$ $\quad (v)\ 12\frac{7}{11}$

Solution:

$$(i)\ 2\frac{4}{9} = \frac{2 \times 9 + 4}{9} = \frac{18+4}{9} = \frac{22}{9}$$

$$(ii)\ 7\frac{5}{13} = \frac{7 \times 13 + 5}{13} = \frac{91+5}{13} = \frac{96}{13}$$

$$(iii)\ 3\frac{1}{4} = \frac{3 \times 4 + 1}{4} = \frac{12+1}{4} = \frac{13}{4}$$

$$(iv)\ 2\frac{5}{48} = \frac{2 \times 48 + 5}{48} = \frac{96+5}{48} = \frac{101}{48}$$

$$(v)\ 12\frac{7}{11} = \frac{12 \times 11 + 7}{11} = \frac{132+7}{11} = \frac{139}{11}$$

Question 4.

Reduce the given fractions to lowest terms

$(i)\ \frac{8}{18}$ $\quad (ii)\ \frac{27}{36}$ $\quad (iii)\ \frac{18}{42}$

$(iv)\ \frac{35}{75}$ $\quad (v)\ \frac{18}{45}$

Solution:

(i) $\frac{8}{18} = \frac{8 \div 2}{18 \div 2}$

(Dividing by 2, the HCF of 8 and 18)

$= \frac{4}{9}$

(ii) $\frac{27}{36} = \frac{27 \div 9}{36 \div 9}$

(Dividing by 9, the HCF of 27 and 36)

$= \frac{3}{4}$

(iii) $\frac{18}{42} = \frac{18 \div 6}{42 \div 6}$

(Dividing by 6, the HCF of 18 and 42)

$= \frac{3}{7}$

(iv) $\frac{35}{75} = \frac{35 \div 5}{75 \div 5}$

(Dividing by 5, the HCF of 35 and 75)

$= \frac{7}{15}$

(v) $\frac{18}{45} = \frac{18 \div 9}{45 \div 9}$

(Dividing by 9, the HCF of 18 and 45)

$= \frac{2}{5}$ **Ans.**

Question 5.
State : true or false

(i) $\frac{30}{40}$ and $\frac{12}{16}$ are equivalent fractions.

(ii) $\frac{10}{25}$ and $\frac{25}{10}$ are equivalent fractions.

(iii) $\frac{35}{49}, \frac{20}{28}, \frac{45}{63}$ and $\frac{100}{140}$ are equivalent fractions.

Solution:

(i) True as $\frac{30}{40} = \frac{3}{4}$ and $\frac{12}{16} = \frac{3}{4}$

(ii) False as $\frac{10}{25} = \frac{2}{5}$ and $\frac{25}{10} = \frac{5}{2}$

(iii) True as $\frac{35}{49} = \frac{5}{7}, \frac{20}{28} = \frac{5}{7}, \frac{45}{63} = \frac{5}{7}$ and

$\frac{100}{140} = \frac{5}{7}$, all are equal **Ans.**

Question 6.
Distinguish each of the following fractions, given below, as a simple fraction or a complex fraction :

(i) $\frac{0}{8}$ (ii) $\frac{-3}{-8}$ (iii) $\frac{5}{-7}$

(iv) $\frac{3\frac{3}{5}}{18}$ (v) $\frac{-6}{2\frac{2}{5}}$ (vi) $\frac{3\frac{1}{3}}{7\frac{2}{7}}$

(vii) $\frac{-5\frac{2}{9}}{5}$ (viii) $\frac{-8}{0}$

Solution:

(i) $\frac{0}{8}$: It is a simple fraction

(ii) $\frac{-3}{-8}$: It is a simple fraction

(iii) $\frac{5}{-7}$: It is a simple fraction

(iv) $\frac{3\frac{3}{5}}{18}$: It is complex fraction

(v) $\frac{-6}{2\frac{2}{5}}$: It is complex fraction

(vi) $\frac{3\frac{1}{3}}{7\frac{2}{7}}$: It is complex fraction

(vii) $\frac{-5\frac{2}{9}}{5}$: It is complex fraction

(viii) $\frac{-8}{0}$: It neither complex nor simple as denominator is zero. **Ans.**

EXERCISE 3 (B)

Question 1.

For each pair, given below, state whether it forms like fractions or unlike fractions :

(i) $\frac{5}{8}$ and $\frac{7}{8}$ (ii) $\frac{8}{15}$ and $\frac{8}{21}$

(iii) $\frac{4}{9}$ and $\frac{9}{4}$

Solution:

(i) $\frac{5}{8}$ and $\frac{7}{8}$: These are like fractions.

(ii) $\frac{8}{15}$ and $\frac{8}{21}$: These are unlike fractions.

(iii) $\frac{4}{9}$ and $\frac{9}{4}$: These are unlike fractions.

Question 2.
Convert given fractions into fractions with equal denominators :

(i) $\frac{5}{6}$ and $\frac{7}{9}$

(ii) $\frac{2}{3}, \frac{5}{6}$ and $\frac{7}{12}$

(iii) $\frac{4}{5}, \frac{17}{20}, \frac{23}{40}$ and $\frac{11}{16}$

Solution:

(i) In $\frac{5}{6}$ and $\frac{7}{9}$: LCM of 6 and 9 = 18

$$\therefore \quad \frac{5}{6} = \frac{5 \times 3}{6 \times 3} = \frac{15}{18}$$

$$\frac{7}{9} = \frac{7 \times 2}{9 \times 2} = \frac{14}{18}$$

Hence, $\frac{15}{18}$ and $\frac{14}{18}$ are the required fractions.

(ii) In $\frac{2}{3}, \frac{5}{6}$ and $\frac{7}{12}$: LCM of 3, 6 and 12 = 12

$$\therefore \quad \frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

$$\frac{5}{6} = \frac{5 \times 2}{6 \times 2} = \frac{10}{12}$$

$$\frac{7}{12} = \frac{7}{12}$$

Hence, the required fractions are $\frac{8}{12}, \frac{10}{12}$ and $\frac{7}{12}$ **Ans.**

(*iii*) In $\frac{4}{5}, \frac{17}{20}, \frac{23}{40}$ and $\frac{11}{16}$

LCM of 5, 20, 40 and 16 = 80

$$\therefore \quad \frac{4}{5} = \frac{4 \times 16}{5 \times 16} = \frac{64}{80}$$

$$\frac{17}{20} = \frac{17 \times 4}{20 \times 4} = \frac{68}{80}$$

$$\frac{23}{40} = \frac{23 \times 2}{40 \times 2} = \frac{46}{80}$$

$$\text{and} \quad \frac{11}{16} = \frac{11 \times 5}{16 \times 5} = \frac{55}{80}$$

Hence the required fractions are

$\frac{64}{80}, \frac{68}{80}, \frac{46}{80}$ and $\frac{55}{80}$ **Ans.**

Question 3.

Convert given fractions into fractions with equal numerators :

(*i*) $\frac{8}{9}$ and $\frac{12}{17}$ (*ii*) $\frac{6}{13}, \frac{15}{23}$ and $\frac{12}{17}$

(*iii*) $\frac{15}{19}, \frac{25}{28}, \frac{9}{11}$ and $\frac{45}{47}$

Solution:

(*i*) In $\frac{8}{9}$ and $\frac{12}{17}$, LCM of 8 and 12 = 24

$$\therefore \quad \frac{8}{9} = \frac{8 \times 3}{9 \times 3} = \frac{24}{27}$$

$$\frac{12}{17} = \frac{12 \times 2}{17 \times 2} = \frac{24}{34}$$

Hence, the required fractions are $\frac{24}{27}$ and $\frac{24}{34}$

Ans.

(*ii*) In $\frac{6}{13}$, $\frac{15}{23}$ and $\frac{12}{17}$, LCM of 6, 15 and 12 = 60

$$\therefore \quad \frac{6}{13} = \frac{6 \times 10}{13 \times 10} = \frac{60}{130}$$

$$\frac{15}{23} = \frac{15 \times 4}{23 \times 4} = \frac{60}{92}$$

and $$\frac{12}{17} = \frac{12 \times 5}{17 \times 5} = \frac{60}{85}$$

Hence, required fractions are

$\frac{60}{130}, \frac{60}{92}$ and $\frac{60}{85}$ **Ans.**

(*iii*) In $\frac{15}{19}, \frac{25}{28}, \frac{9}{11}$ and $\frac{45}{47}$

LCM of 15, 25, 9 and 45 = 225

$$\therefore \quad \frac{15}{19} = \frac{15 \times 15}{19 \times 15} = \frac{225}{285}$$

$$\frac{25}{28} = \frac{25 \times 9}{28 \times 9} = \frac{225}{252}$$

$$\frac{9}{11} = \frac{9 \times 25}{11 \times 25} = \frac{225}{275}$$

$$\frac{45}{47} = \frac{45 \times 5}{47 \times 5} = \frac{225}{235}$$

Hence, required fractions are

$\frac{225}{285}, \frac{225}{252}, \frac{225}{275}$ and $\frac{225}{235}$ **Ans.**

Question 4.

Put the given fractions in ascending order by making denominators equal :

(*i*) $\frac{1}{3}, \frac{2}{5}, \frac{3}{4}$ and $\frac{1}{6}$

(*ii*) $\frac{5}{6}, \frac{7}{8}, \frac{11}{12}$ and $\frac{3}{10}$

(*iii*) $\frac{5}{7}, \frac{3}{8}, \frac{9}{14}$ and $\frac{20}{21}$

Solution:

(i) $\frac{1}{3}, \frac{2}{5}, \frac{3}{4}$ and $\frac{1}{6}$

LCM of denominators 3, 5, 4 and 6 = 60

$$\therefore \quad \frac{1}{3} = \frac{1 \times 20}{3 \times 20} = \frac{20}{60}$$

$$\frac{2}{5} = \frac{2 \times 12}{5 \times 12} = \frac{24}{60}$$

$$\frac{3}{4} = \frac{3 \times 15}{4 \times 15} = \frac{45}{60}$$

$$\frac{1}{6} = \frac{1 \times 10}{6 \times 10} = \frac{10}{60}$$

From above we see that

$$\frac{10}{60} < \frac{20}{60} < \frac{24}{60} < \frac{45}{60}$$

or $\frac{1}{6} < \frac{1}{3} < \frac{2}{5} < \frac{3}{4}$

Hence, $\frac{1}{6}, \frac{1}{3}, \frac{2}{5}, \frac{3}{4}$ are in ascending order

Ans.

(ii) $\frac{5}{6}, \frac{7}{8}, \frac{11}{12}$ and $\frac{3}{10}$

LCM of denominators 6, 8, 12 and 10 = 240

$$\therefore \quad \frac{5}{6} = \frac{5 \times 40}{6 \times 40} = \frac{200}{240}$$

$$\frac{7}{8} = \frac{7 \times 30}{8 \times 30} = \frac{210}{240}$$

$$\frac{11}{12} = \frac{11 \times 20}{12 \times 20} = \frac{220}{240}$$

$$\frac{3}{10} = \frac{3 \times 24}{10 \times 24} = \frac{72}{240}$$

It is clear from the above that

$$\frac{72}{240} < \frac{200}{240} < \frac{210}{240} < \frac{220}{240}$$

or $\frac{3}{10} < \frac{5}{6} < \frac{7}{8} < \frac{11}{12}$

Hence, $\frac{3}{10}, \frac{5}{6}, \frac{7}{8}$ and $\frac{11}{12}$ are in ascending order. **Ans.**

(iii) $\frac{5}{7}, \frac{3}{8}, \frac{9}{14}$ and $\frac{20}{21}$

LCM of denominators 7, 8, 14, 21 = 168

$$\therefore \quad \frac{5}{7} = \frac{5 \times 24}{7 \times 24} = \frac{120}{168}$$

$$\frac{3}{8} = \frac{3 \times 21}{8 \times 21} = \frac{63}{168}$$

$$\frac{9}{14} = \frac{9 \times 12}{14 \times 12} = \frac{108}{168}$$

$$\frac{20}{21} = \frac{20 \times 8}{21 \times 8} = \frac{160}{168}$$

It is clear from the above that

$$\frac{63}{168} < \frac{108}{168} < \frac{120}{168} < \frac{160}{168}$$

or $$\frac{3}{8} < \frac{9}{14} < \frac{5}{7} < \frac{20}{21}$$

Hence, $\frac{3}{8}, \frac{9}{14}, \frac{5}{7}$ and $\frac{20}{21}$ are in ascending order. **Ans.**

Question 5.

Arrange the given fractions in descending order by making numerators equal :

(i) $\frac{5}{6}, \frac{4}{15}, \frac{8}{9}$ and $\frac{i}{3}$

(ii) $\frac{3}{7}, \frac{4}{9}, \frac{5}{7}$ and $\frac{8}{11}$

(iii) $\frac{1}{10}, \frac{6}{11}, \frac{8}{11}$ and $\frac{3}{5}$

Solution:

(*i*) $\frac{5}{6}, \frac{4}{15}, \frac{8}{9}$ and $\frac{1}{3}$

LCM of numerators 5, 4, 8 and 1 = 40

$$\therefore \quad \frac{5}{6} = \frac{5 \times 8}{6 \times 8} = \frac{40}{48}$$

$$\frac{4}{15} = \frac{4 \times 10}{15 \times 10} = \frac{40}{150}$$

$$\frac{8}{9} = \frac{8 \times 5}{9 \times 5} = \frac{40}{45}$$

$$\frac{1}{3} = \frac{1 \times 40}{3 \times 40} = \frac{40}{120}$$

From above we see that

$$\frac{40}{45} > \frac{40}{48} > \frac{40}{120} > \frac{40}{150}$$

$$\therefore \quad \frac{8}{9} > \frac{5}{6} > \frac{1}{3} > \frac{4}{15}$$

Hence, $\frac{8}{9}, \frac{5}{6}, \frac{1}{3}, \frac{4}{15}$ are in descending order. **Ans.**

(*ii*) $\frac{3}{7}, \frac{4}{9}, \frac{5}{7}$ and $\frac{8}{11}$

LCM of numerators 3, 4, 5 and 8 = 120

$$\therefore \quad \frac{3}{7} = \frac{3 \times 40}{7 \times 40} = \frac{120}{280}$$

$$\frac{4}{9} = \frac{4 \times 30}{9 \times 30} = \frac{120}{270}$$

$$\frac{5}{7} = \frac{5 \times 24}{7 \times 24} = \frac{120}{168}$$

$$\frac{8}{11} = \frac{8 \times 15}{11 \times 15} = \frac{120}{165}$$

It is clear from the above that

$$\frac{120}{165} > \frac{120}{168} > \frac{120}{270} > \frac{120}{280}$$

or $$\frac{8}{11} > \frac{5}{7} > \frac{4}{9} > \frac{3}{7}$$

Hence, $\frac{8}{11}, \frac{5}{7}, \frac{4}{9}$ and $\frac{3}{7}$ are in descending order **Ans.**

(*iii*) $\frac{1}{10}, \frac{6}{11}, \frac{8}{11}$ and $\frac{3}{5}$

LCM of 1, 6, 8 and 3 = 24

$$\therefore \quad \frac{1}{10} = \frac{1 \times 24}{10 \times 24} = \frac{24}{240}$$

$$\frac{6}{11} = \frac{6 \times 4}{11 \times 4} = \frac{24}{44}$$

$$\frac{8}{11} = \frac{8 \times 3}{11 \times 3} = \frac{24}{33}$$

$$\frac{3}{5} = \frac{3 \times 8}{5 \times 8} = \frac{24}{40}$$

It is clear from the above that

$$\frac{24}{33} > \frac{24}{40} > \frac{24}{44} > \frac{24}{240}$$

or $$\frac{8}{11} > \frac{3}{5} > \frac{6}{11} > \frac{1}{10}$$

Hence, $\frac{8}{11}, \frac{3}{5}, \frac{6}{11}$ and $\frac{1}{10}$ are in descending order. **Ans.**

Question 6.
Find the greater fraction :

(i) $\frac{3}{5}$ and $\frac{11}{15}$ (ii) $\frac{4}{5}$ and $\frac{3}{10}$

(iii) $\frac{6}{7}$ and $\frac{5}{9}$ (iv) $\frac{3}{8}$ and $\frac{4}{9}$

(v) $\frac{-2}{7}$ and $\frac{-3}{10}$

Solution:

(i) In $\frac{3}{5}$ and $\frac{11}{15}$, LCM of 5 and 15 = 15

$\therefore \quad \frac{3}{5} = \frac{3 \times 3}{5 \times 3} = \frac{9}{15}$

$\frac{11}{15} = \frac{11}{15}$

It is clear from above that $\frac{11}{15} > \frac{9}{15}$

Hence $\frac{11}{15}$ is greater.

(ii) In $\frac{4}{5}$ and $\frac{3}{10}$, LCM of 5 and 10 = 10

$\therefore \quad \frac{4}{5} = \frac{4 \times 2}{5 \times 2} = \frac{8}{10}$

and $\quad \frac{3}{10} = \frac{3}{10}$

It is clear from the above that $\frac{8}{10} > \frac{3}{10}$

Hence $\frac{4}{5}$ is greater.

(iii) In $\frac{6}{7}$ and $\frac{5}{9}$, LCM of 7 and 9 = 63

$\therefore \quad \frac{6}{7} = \frac{6 \times 9}{7 \times 9} = \frac{54}{63}$

and $\quad \frac{5}{9} = \frac{5 \times 7}{9 \times 7} = \frac{35}{63}$

It is clear from the above that $\frac{54}{63} > \frac{35}{63}$

Hence $\frac{54}{63}$ or $\frac{6}{7}$ is greater.

(*iii*) In $\frac{6}{7}$ and $\frac{5}{9}$, LCM of 7 and 9 = 63

$\therefore \quad \frac{6}{7} = \frac{6 \times 9}{7 \times 9} = \frac{54}{63}$

and $\quad \frac{5}{9} = \frac{5 \times 7}{9 \times 7} = \frac{35}{63}$

It is clear from the above that $\frac{54}{63} > \frac{35}{63}$

Hence $\frac{54}{63}$ or $\frac{6}{7}$ is greater.

(*iv*) In $\frac{3}{8}$ and $\frac{4}{9}$, LCM of 8 and 9 = 72

$\therefore \quad \frac{3}{8} = \frac{3 \times 9}{8 \times 9} = \frac{27}{72}$

$\frac{4}{9} = \frac{4 \times 8}{9 \times 8} = \frac{32}{72}$

It is clear from the above that $\frac{32}{72} > \frac{27}{72}$

Hence $\frac{32}{72}$ or $\frac{4}{9}$ is greater

(*v*) In $\frac{-2}{7}$ and $\frac{-3}{10}$, LCM of 7 and 10 = 70

$\therefore \quad \frac{-2}{7} = \frac{-2 \times 10}{7 \times 10} = \frac{-20}{70}$

$\frac{-3}{10} = \frac{-3 \times 7}{10 \times 7} = \frac{-21}{70}$

It is clear from the above that $\frac{-20}{70} > \frac{-21}{70}$

Hence $\frac{-20}{70}$ or $\frac{-2}{7}$ is greater **Ans.**

Question 7.
Insert one fraction between :

(i) $\frac{3}{7}$ and $\frac{4}{9}$ (ii) 2 and $\frac{8}{3}$

(iii) $\frac{9}{17}$ and $\frac{6}{13}$

Solution:

(i) Fraction between $\frac{3}{7}$ and $\frac{4}{9}$

$$= \frac{3+4}{7+9} = \frac{7}{16}$$

(ii) Fraction between 2 and $\frac{8}{3}$

$$= \frac{2+8}{1+3} = \frac{10}{4} = \frac{5}{2} = 2\frac{1}{2}$$

(iii) Fraction between $\frac{9}{17}$ and $\frac{6}{13}$

$$= \frac{9+6}{17+13} = \frac{15}{30} = \frac{1}{2} \text{ Ans.}$$

Question 8.
Insert three fractions between

(i) $\frac{2}{5}$ and $\frac{4}{9}$ (ii) $\frac{1}{2}$ and $\frac{5}{7}$

(iii) $\frac{3}{8}$ and $\frac{6}{11}$ (iv) $\frac{11}{12}$ and $\frac{2}{3}$

(v) $\frac{4}{7}$ and $\frac{3}{4}$

Solution:

(*i*) Fraction between $\frac{2}{5}$ and $\frac{4}{9}$

$$= \frac{2+4}{5+9} = \frac{6}{14} = \frac{3}{7}$$

Fraction between $\frac{2}{5}$ and $\frac{3}{7}$

$$= \frac{2+3}{5+7} = \frac{5}{12}$$

and fraction between $\frac{3}{7}$ and $\frac{4}{9}$

$$= \frac{3+4}{7+9} = \frac{7}{16}$$

Hence, three fractions between $\frac{2}{5}$ and $\frac{4}{9}$ will be $\frac{5}{12}$, $\frac{3}{7}$ and $\frac{7}{16}$ **Ans.**

(*ii*) Fraction between $\frac{1}{2}$ and $\frac{5}{7}$

$$= \frac{1+5}{2+7} = \frac{6}{9} = \frac{2}{3}$$

Fraction between $\frac{1}{2}$ and $\frac{2}{3}$

$$= \frac{1+2}{2+3} = \frac{3}{5}$$

Fraction between $\frac{2}{3}$ and $\frac{5}{7}$

$$= \frac{2+5}{3+7} = \frac{7}{10}$$

Hence, three fractions between $\frac{1}{2}$ and $\frac{5}{7}$ will be $\frac{3}{5}$, $\frac{2}{3}$ and $\frac{7}{10}$ **Ans.**

(*iii*) Fraction between $\frac{3}{8}$ and $\frac{6}{11}$

$$= \frac{3+6}{8+11} = \frac{9}{19}$$

Fraction between $\frac{3}{8}$ and $\frac{9}{19}$

$$= \frac{3+9}{8+19} = \frac{12}{27} = \frac{4}{9}$$

Fraction between $\frac{9}{19}$ and $\frac{6}{11}$

$$= \frac{9+6}{19+11} = \frac{15}{30} = \frac{1}{2}$$

Hence, three fractions between $\frac{3}{8}$ and $\frac{6}{11}$ will be $\frac{4}{9}$, $\frac{9}{19}$ and $\frac{1}{2}$ **Ans.**

(*iv*) Fraction between $\frac{11}{12}$ and $\frac{2}{3}$

$$= \frac{11+2}{12+3} = \frac{13}{15}$$

Fraction between $\frac{11}{12}$ and $\frac{13}{15}$

$$= \frac{11+13}{12+15} = \frac{24}{27} = \frac{8}{9}$$

Fraction between $\frac{13}{15}$ and $\frac{2}{3}$

$$= \frac{13+2}{15+3} = \frac{15}{18} = \frac{5}{6}$$

Hence, three fractions between $\frac{11}{12}$ and $\frac{2}{3}$ will be $\frac{8}{9}$, $\frac{13}{15}$ and $\frac{5}{6}$ **Ans.**

(v) Fraction between $\frac{4}{7}$ and $\frac{3}{4}$

$$= \frac{4+3}{7+4} = \frac{7}{11}$$

Fraction between $\frac{4}{7}$ and $\frac{7}{11}$

$$= \frac{4+7}{7+11} = \frac{11}{18}$$

Fraction between $\frac{7}{11}$ and $\frac{3}{4}$

$$= \frac{7+3}{11+4} = \frac{10}{15} = \frac{2}{3}$$

Hence, three fractions between $\frac{4}{7}$ and $\frac{3}{4}$ will be $\frac{11}{18}, \frac{7}{11}, \frac{2}{3}$ Ans.

Question 9.

Insert two fractions between

(*i*) 1 and $\frac{3}{11}$ (*ii*) $\frac{5}{9}$ and $\frac{1}{4}$ (*iii*) $\frac{5}{6}$ and $1\frac{1}{5}$

Solution:

(*i*) Fraction between 1 and $\frac{3}{11}$

$= \frac{1+3}{1+11} = \frac{4}{12} = \frac{1}{3}$

Fraction between $\frac{1}{3}$ and $\frac{3}{11} = \frac{1+3}{3+11} = \frac{4}{14} = \frac{2}{7}$

Hence, two fractions between 1 and $\frac{3}{11}$

will be $\frac{1}{3}$ and $\frac{2}{7}$. **Ans.**

(*ii*) Fraction between $\frac{5}{9}$ and $\frac{1}{4} = \frac{5+1}{9+4} = \frac{6}{13}$

Fraction between $\frac{6}{13}$ and $\frac{1}{4} = \frac{6+1}{13+4} = \frac{7}{17}$

Hence, two fractions between $\frac{5}{9}$ and $\frac{1}{4}$ will

be $\frac{6}{13}$ and $\frac{7}{17}$ **Ans.**

(*iii*) Fraction between $\frac{5}{6}$ and $1\frac{1}{5}$ or $\frac{5}{6}$ and $\frac{6}{5}$

$= \frac{5+6}{6+5} = \frac{11}{11} = 1$

Fraction between 1 and $\frac{6}{5} = \frac{1+6}{1+5} = \frac{7}{6} = 1\frac{1}{6}$

Hence, two fractions between $\frac{5}{6}$ and $1\frac{1}{5}$

will be 1 and $1\frac{1}{6}$ **Ans.**

EXERCISE 3 (C)

Question 1.

Reduce to a single fraction :

(i) $\frac{1}{2}+\frac{2}{3}$ (ii) $\frac{3}{5}-\frac{1}{10}$

(iii) $\frac{2}{3}-\frac{1}{6}$ (iv) $1\frac{1}{3}+2\frac{1}{4}$

(v) $\frac{1}{4}+\frac{5}{6}-\frac{1}{12}$ (vi) $\frac{2}{3}-\frac{3}{5}+3-\frac{1}{5}$

(vii) $\frac{2}{3}-\frac{1}{5}+\frac{1}{10}$ (viii) $2\frac{1}{2}+2\frac{1}{3}-1\frac{1}{4}$

(ix) $2\frac{5}{8}-2\frac{1}{6}+4\frac{3}{4}$

Solution:

(i) $\frac{1}{2}+\frac{2}{3}=\frac{1\times 3}{2\times 3}+\frac{2\times 2}{3\times 2}$

(LCM of 2 and 3 = 6)

$=\frac{3}{6}+\frac{4}{6}=\frac{3+4}{6}=\frac{7}{6}=1\frac{1}{6}$ **Ans.**

(ii) $\frac{3}{5}-\frac{1}{10}=\frac{3\times 2}{5\times 2}-\frac{1}{10}=\frac{6}{10}-\frac{1}{10}$

(LCM of 5, 10 = 10)

$=\frac{6-1}{10}=\frac{5}{10}=\frac{1}{2}$ **Ans.**

(iii) $\frac{2}{3}-\frac{1}{6}=\frac{2\times 2}{3\times 2}-\frac{1}{6}=\frac{4}{6}-\frac{1}{6}$

(LCM of 3, 6 = 6)

$=\frac{4-1}{6}=\frac{3}{6}=\frac{1}{2}$ **Ans.**

(iv) $1\frac{1}{3}+2\frac{1}{4}=\frac{4}{3}+\frac{9}{4}$

$=\frac{4\times 4}{3\times 4}+\frac{9\times 3}{4\times 3}=\frac{16}{12}+\frac{27}{12}$

(LCM of 3, 4 = 12)

$=\frac{16+27}{12}=\frac{43}{12}=3\frac{7}{12}$ **Ans.**

(v) $\frac{1}{4}+\frac{5}{6}-\frac{1}{12}=\frac{1\times 3}{4\times 3}+\frac{5\times 2}{6\times 2}-\frac{1}{12}$

(LCM of 4, 6, 12 = 12)

$=\frac{3}{12}+\frac{10}{12}-\frac{1}{12}=\frac{3+10-1}{12}$

$$= \frac{13-1}{12} = \frac{12}{12} = 1 \textbf{ Ans.}$$

(*vi*) $\frac{2}{3} - \frac{3}{5} + 3 - \frac{1}{5}$

$$= \frac{2 \times 5}{3 \times 5} - \frac{3 \times 3}{5 \times 3} + \frac{3 \times 15}{15} - \frac{1 \times 3}{5 \times 3}$$

(LCM of 3 and 5 = 15)

$$= \frac{10}{15} - \frac{9}{15} + \frac{45}{15} - \frac{3}{15}$$

$$= \frac{10 - 9 + 45 - 3}{15} = \frac{55 - 12}{15}$$

$$= \frac{43}{15} = 2\frac{13}{15} \textbf{ Ans.}$$

(*vii*) $\frac{2}{3} - \frac{1}{5} + \frac{1}{10} = \frac{2 \times 10}{3 \times 10} - \frac{1 \times 6}{5 \times 6} + \frac{1 \times 3}{10 \times 3}$

(LCM of 3, 5, 10 = 30)

$$= \frac{20}{30} - \frac{6}{30} + \frac{3}{30} = \frac{20 - 6 + 3}{30}$$

$$= \frac{23 - 6}{30} = \frac{17}{30} \textbf{ Ans.}$$

(*viii*) $2\frac{1}{2} + 2\frac{1}{3} - 1\frac{1}{4} = \frac{5}{2} + \frac{7}{3} - \frac{5}{4}$

$$= \frac{5 \times 6}{2 \times 6} + \frac{7 \times 4}{3 \times 4} - \frac{5 \times 3}{4 \times 3}$$

(LCM of 2, 3, 4 = 12)

$$= \frac{30}{12} + \frac{28}{12} - \frac{15}{12}$$

$$= \frac{30 + 28 - 15}{12} = \frac{58 - 15}{12}$$

$$= \frac{43}{12} = 3\frac{7}{12} \textbf{ Ans.}$$

(ix) $2\frac{5}{8}-2\frac{1}{6}+4\frac{3}{4}=\frac{21}{8}-\frac{13}{6}+\frac{19}{4}$

$=\frac{21\times 3}{8\times 3}-\frac{13\times 4}{6\times 4}+\frac{19\times 6}{4\times 6}$

(LCM of 8, 6, 4 = 24)

$=\frac{63}{24}-\frac{52}{24}+\frac{114}{24}=\frac{63-52+114}{24}$

$=\frac{177-52}{24}=\frac{125}{24}=5\frac{5}{24}$ **Ans.**

Question 2.
Simplify :

(i) $\frac{3}{4}\times 6=\frac{3}{4}\times\frac{6}{1}=\frac{3\times 6}{4\times 1}=\frac{18}{4}$

$=\frac{18\div 2}{4\div 2}=\frac{9}{2}=4\frac{1}{2}$ **Ans.**

(ii) $\frac{2}{3}\times 15=\frac{2}{3}\times\frac{15}{1}=\frac{2\times 15}{3\times 1}=\frac{30}{3}=10$ **Ans.**

(iii) $\frac{3}{4}\times\frac{1}{2}=\frac{3\times 1}{4\times 2}=\frac{3}{8}$ **Ans.**

(iv) $\frac{9}{12}\times\frac{4}{7}=\frac{9\times 4}{12\times 7}=\frac{36}{84}$

$=\frac{36\div 12}{84\div 12}=\frac{3}{7}$ **Ans.**

(HCF of 36 and 84 = 12)

(v) $45\times 2\frac{1}{3}=\frac{45}{1}\times\frac{7}{3}=\frac{45\times 7}{1\times 3}$

$=\frac{315}{3}=105$ **Ans.**

Solution:

$$(vi)\ 36 \times 3\frac{1}{4} = \frac{36}{1} \times \frac{13}{4} = \frac{36 \times 13}{4 \times 1}$$

$$= \frac{468}{4} = 117 \textbf{ Ans.}$$

$$(vii)\ 2 \div \frac{1}{3} = \frac{2}{1} \times \frac{3}{1} = \frac{2 \times 3}{1 \times 1} = \frac{6}{1} = 6 \textbf{ Ans.}$$

$$(viii)\ 3 \div \frac{2}{5} = \frac{3}{1} \times \frac{5}{2} = \frac{3 \times 5}{1 \times 2} = \frac{15}{2} = 7\frac{1}{2} \textbf{ Ans.}$$

$$(ix)\ 1 \div \frac{3}{5} = 1 \times \frac{5}{3} = \frac{1 \times 5}{3} = \frac{5}{3} = 1\frac{2}{3} \textbf{ Ans.}$$

$$(x)\ \frac{1}{3} \div \frac{1}{4} = \frac{1}{3} \times \frac{4}{1} = \frac{1 \times 4}{3 \times 1} = \frac{4}{3} = 1\frac{1}{3} \textbf{ Ans.}$$

$$(xi)\ -\frac{5}{8} \div \frac{3}{4} = -\frac{5}{8} \times \frac{4}{3} = -\frac{5 \times 4}{8 \times 3} = -\frac{20}{24}$$

$$= -\frac{20 \div 4}{24 \div 4} = -\frac{5}{6} \textbf{ Ans.}$$

$$(xii)\ 3\frac{3}{7} \div 1\frac{1}{14} = \frac{24}{7} \times \frac{14}{15} = \frac{24 \times 14}{7 \times 15}$$

$$= \frac{336}{105} = \frac{336 \div 21}{105 \div 21}$$

(HCF of 336 and 105 = 21)

$$= \frac{16}{5} = 3\frac{1}{5} \textbf{ Ans.}$$

$$(xiii)\ 3\frac{3}{4} \times 1\frac{1}{5} \times \frac{20}{21} = \frac{15}{4} \times \frac{6}{5} \times \frac{20}{21}$$

$$= \frac{15 \times 6 \times 20}{4 \times 5 \times 21} = \frac{1800}{420} = \frac{1800 \div 60}{420 \div 60}$$

(HCF of 1800 and 420 = 60)

$$= \frac{30}{7} = 4\frac{2}{7} \textbf{ Ans.}$$

Question 3.
Subtract :

(i) 2 from $\frac{2}{3}$ (ii) $\frac{1}{8}$ from $\frac{5}{8}$

(iii) $-\frac{2}{5}$ from $\frac{2}{5}$ (iv) $-\frac{3}{7}$ from $\frac{3}{7}$

(v) 0 from $-\frac{4}{5}$ (vi) $\frac{2}{9}$ from $\frac{4}{5}$

(vii) $-\frac{4}{7}$ from $-\frac{6}{11}$

Solution:

(i) 2 from $\frac{2}{3} = \frac{2}{3} - \frac{2}{1}$

$= \frac{2}{3} - \frac{2\times 3}{3} = \frac{2}{3} - \frac{6}{3}$

$= \frac{2-6}{3} = -\frac{4}{3} = -1\frac{1}{3}$ **Ans.**

(ii) $\frac{1}{8}$ from $\frac{5}{8} = \frac{5}{8} - \frac{1}{8} = \frac{5-1}{8} = \frac{4}{8} = \frac{1}{2}$ **Ans.**

(iii) $-\frac{2}{5}$ from $\frac{2}{5} = \frac{2}{5} - \left(-\frac{2}{5}\right) = \frac{2}{5} + \frac{2}{5}$

$= \frac{2+2}{5} = \frac{4}{5}$ **Ans.**

(iv) $-\frac{3}{7}$ from $\frac{3}{7} = \frac{3}{7} - \left(-\frac{3}{7}\right) = \frac{3}{7} + \frac{3}{7}$

$= \frac{3+3}{7} = \frac{6}{7}$ **Ans.**

(v) 0 from $-\frac{4}{5} = -\frac{4}{5} - 0 = -\frac{4}{5}$ **Ans.**

(vi) $\frac{2}{9}$ from $\frac{4}{5} = \frac{4}{5} - \frac{2}{9} = \frac{4\times 9}{5\times 9} - \frac{2\times 5}{9\times 5}$

(LCM of 5 and 9 = 45)

$= \frac{36}{45} - \frac{10}{45} = \frac{36-10}{45} = \frac{26}{45}$ **Ans.**

(vii) $-\frac{4}{7}$ from $-\frac{6}{11} = -\frac{6}{11} - \left(-\frac{4}{7}\right) = -\frac{6}{11} + \frac{4}{7}$

$= \frac{-6\times 7}{11\times 7} + \frac{4\times 11}{7\times 11}$

(LCM of 7 and 11 = 77)

$= -\frac{42}{77} + \frac{44}{77} = \frac{-42+44}{77} = \frac{2}{77}$ **Ans.**

Question 4.
Find the value of

(i) $\frac{1}{2}$ of 10 kg (ii) $\frac{3}{5}$ of 1 hour

(iii) $\frac{4}{7}$ of $2\frac{1}{3}$ kg

(iv) $3\frac{1}{2}$ times of 2 metres

(v) $\frac{1}{2}$ of $2\frac{2}{3}$

(vi) $\frac{5}{11}$ of $\frac{4}{5}$ of 22 kg.

Solution:

(i) $\frac{1}{2}$ of 10 kg $= \left(\frac{1}{2} \times 10\right)$ kg $= 5$ kg **Ans.**

(ii) $\frac{3}{5}$ of 1 hour $= \left(\frac{3}{5} \times 60\right)$ minutes

$= 3 \times 12 = 36$ minutes **Ans.**

(iii) $\frac{4}{7}$ of $2\frac{1}{3}$ kg $= \left(\frac{4}{7} \times \frac{7}{3}\right)$ kg

$= \frac{4}{3}$ kg $= 1\frac{1}{3}$ kg **Ans.**

(iv) $3\frac{1}{2}$ times of 2 metres $= \left(\frac{7}{2} \times 2\right)$ metres

$= 7$ metres **Ans.**

(v) $\frac{1}{2}$ of $2\frac{2}{3} = \frac{1}{2} \times \frac{8}{3} = \frac{4}{3} = 1\frac{1}{3}$ **Ans.**

(vi) $\frac{5}{11}$ of $\frac{4}{5}$ of 22 kg $= \left(\frac{5}{11} \times \frac{4}{5} \times \frac{22}{1}\right)$ kg

$= (4 \times 2) = 8$ kg **Ans.**

Question 5.
Simplify and reduce to a simple fraction :

(i) $\dfrac{3}{3\frac{3}{4}}$ (ii) $\dfrac{\frac{3}{5}}{7}$

(iii) $\dfrac{3}{\frac{5}{7}}$ (iv) $\dfrac{2\frac{1}{5}}{1\frac{1}{10}}$

(v) $\frac{2}{5}$ of $\frac{6}{11} \times 1\frac{1}{4}$ (vi) $2\frac{1}{4} \div \frac{1}{7} \times \frac{1}{3}$

(vii) $\frac{1}{3} \times 4\frac{2}{3} \div 3\frac{1}{2} \times \frac{1}{2}$

(viii) $\frac{2}{3} \times 1\frac{1}{4} \div \frac{3}{7}$ of $2\frac{5}{8}$

(ix) $0 \div \frac{8}{11}$

Solution:

(x) $\frac{4}{5} \div \frac{7}{15}$ of $\frac{8}{9}$

(xi) $\frac{4}{5} \div \frac{7}{15} \times \frac{8}{9}$ (xii) $\frac{4}{5}$ of $\frac{7}{15} \div \frac{8}{9}$

(xiii) $\frac{1}{2}$ of $\frac{3}{4} \times \frac{1}{2} \div \frac{2}{3}$

(*i*) $\dfrac{3}{3\frac{3}{4}} = \dfrac{3}{\frac{15}{4}} = \dfrac{3 \times 4}{15} = \dfrac{4}{5}$ **Ans.**

(*ii*) $\dfrac{\frac{3}{5}}{7} = \dfrac{3}{5} \times \dfrac{1}{7} = \dfrac{3}{35}$ **Ans.**

(*iii*) $\dfrac{3}{\frac{5}{7}} = 3 \times \dfrac{7}{5} = \dfrac{21}{5} = 4\dfrac{1}{5}$

(*iv*) $\dfrac{2\frac{1}{5}}{1\frac{1}{10}} = \dfrac{\frac{11}{5}}{\frac{11}{10}} = \dfrac{11}{5} \times \dfrac{10}{11} = 2$ **Ans.**

(*v*) $\dfrac{2}{5}$ of $\dfrac{6}{11} \times 1\dfrac{1}{4}$

$= \dfrac{2}{5}$ of $\dfrac{6}{11} \times \dfrac{5}{4} = \dfrac{12}{55} \times \dfrac{5}{4}$

[Removing 'of']

$= \dfrac{3}{11}$ **Ans.**

(*vi*) $2\dfrac{1}{4} \div \dfrac{1}{7} \times \dfrac{1}{3}$

$= \dfrac{9}{4} \div \dfrac{1}{7} \times \dfrac{1}{3} = \dfrac{9}{4} \times \dfrac{7}{1} \times \dfrac{1}{3}$

[Removing (+)]

$= \dfrac{21}{4} = 5\dfrac{1}{4}$ **Ans.**

(*vii*) $\dfrac{1}{3} \times 4\dfrac{2}{3} \div 3\dfrac{1}{2} \times \dfrac{1}{2}$

$$= \frac{1}{3} \times \frac{14}{3} \div \frac{7}{2} \times \frac{1}{2} = \frac{1}{3} \times \frac{14}{3} \times \frac{2}{7} \times \frac{1}{2}$$

[Solving '÷']

$$= \frac{2}{9}$$

(*viii*) $\frac{2}{3} \times 1\frac{1}{4} \div \frac{3}{7} \text{ of } 2\frac{5}{8}$

$$= \frac{2}{3} \times \frac{5}{4} \div \frac{3}{7} \text{ of } \frac{21}{8}$$

$$= \frac{2}{3} \times \frac{5}{4} \div \frac{9}{8}$$ [Solving 'of']

$$= \frac{2}{3} \times \frac{5}{4} \times \frac{8}{9}$$ [Solving '÷']

$$= \frac{20}{27}$$

(*ix*) $0 \div \frac{8}{11} = 0 \times \frac{11}{8} = 0$

(*x*) $\frac{4}{5} \div \frac{7}{15} \text{ of } \frac{8}{9}$

Using BODMAS, we get

$$= \frac{4}{5} \div \frac{56}{135}$$

$$= \frac{4}{5} \times \frac{135}{56} = \frac{27}{14} = 1\frac{13}{14}$$

(xi) $\frac{4}{5} \div \frac{7}{15} \times \frac{8}{9}$

$= \frac{4}{5} \times \frac{15}{7} \times \frac{8}{9}$

$= \frac{32}{21} = 1\frac{11}{21}$

(xii) $\frac{4}{5}$ of $\frac{7}{15} \div \frac{8}{9}$

Using BODMAS,

$= \frac{28}{75} \div \frac{8}{9} = \frac{28}{75} \times \frac{9}{8}$

$= \frac{7 \times 3}{25 \times 2} = \frac{21}{50}$

(xiii) $\frac{1}{2}$ of $\frac{3}{4} \times \frac{1}{2} \div \frac{2}{3}$

Using BODMAS

$= \frac{3}{8} \times \frac{1}{2} \div \frac{2}{3} = \frac{3}{8} \times \frac{1}{2} \times \frac{3}{2} = \frac{9}{32}$

Question 6.

A bought $3\frac{3}{4}$ kg of wheat and $2\frac{1}{2}$ kg of rice. Find the total weight of wheat and rice bought.

Solution:

Weight of wheat $= 3\frac{3}{4}$ kg $= \frac{15}{4}$ kg

Weight of rice $= 2\frac{1}{2}$ kg $= \frac{5}{2}$ kg

$\therefore$ Total weight of wheat and rice

$= \frac{15}{4} + \frac{5}{2}$

$= \frac{15 \times 1}{4 \times 1} + \frac{5 \times 2}{2 \times 2}$ ($\because$ L.C.M. of 4 and 2 = 4)

$= \frac{15+10}{4} = \frac{25}{4}$ kg $= 6\frac{1}{4}$ kg

Question 7.

Which is greater, $\frac{3}{5}$ or $\frac{7}{10}$ and by how much?

Solution:

Taking the cross multiplication, we get

$3 \times 10 = 30$ and $7 \times 5 = 35$

Since, 3×10 (*i.e.*, 30) is smaller than 7×5 (*i.e.*, 35)

$$\therefore \frac{3}{5} < \frac{7}{10}$$

Difference between $\frac{7}{10}$ and $\frac{3}{5}$

$$\Rightarrow \frac{7}{10} - \frac{3}{5} \qquad (\because \text{L.C.M. of 10 and 5} = 10)$$

$$\Rightarrow \frac{7 \times 1}{10 \times 1} - \frac{3 \times 2}{5 \times 2}$$

$$\Rightarrow \frac{7-6}{10} = \frac{1}{10}$$

$$\therefore \frac{7}{10} \text{ is greater than } \frac{3}{5} \text{ by } \frac{1}{10}$$

Question 8.
What number should be added to 8 $\frac{2}{3}$ to 12 $\frac{5}{6}$

Solution:

For finding the required fraction, we have to subtract $8\frac{2}{3}$ from $12\frac{5}{6}$

$\therefore$ Required number $= 12\frac{5}{6} - 8\frac{2}{3}$

$$= \frac{77}{6} - \frac{26}{3}$$

$$= \frac{77\times1}{6\times1} - \frac{26\times2}{3\times2}$$

($\because$ L.C.M. of 3 and 6 = 6)

$$= \frac{77-52}{6} = \frac{25}{6} = 4\frac{1}{6}$$

Question 9.
What should be subtracted from 8$\frac{3}{4}$ to get 2 $\frac{2}{3}$

Solution:

The required number $= 8\frac{3}{4} - 2\frac{2}{3}$

$$\Rightarrow \frac{35}{4} - \frac{8}{3}$$

$$\Rightarrow \frac{35\times3}{4\times3} - \frac{8\times4}{3\times4} \quad (\because \text{L.C.M. of 4 and 3} = 12)$$

$$\Rightarrow \frac{105-32}{12} = \frac{73}{12} = 6\frac{1}{12}$$

Question 10.
A field is 16 $\frac{1}{2}$ m long and 12 $\frac{2}{5}$ m wide. Find the perimeter of the field.

Solution:

Length of field = $16\frac{1}{2}$ m

Breadth of field = $12\frac{2}{5}$ m

$\therefore$ Perimeter of field = $2(l + b)$

$= 2 \times \left(16\frac{1}{2} + 12\frac{2}{5}\right)$

$= 2 \times \left(\frac{33}{2} + \frac{62}{5}\right)$

$= 2 \times \left(\frac{33 \times 5}{2 \times 5} + \frac{62 \times 2}{5 \times 2}\right)$

($\because$ L.C.M. of 2 and 5 = 10)

$= 2 \times \left(\frac{165 + 124}{10}\right) = 2 \times \frac{289}{10}$

$= \frac{289}{5}$ m $= 57\frac{4}{5}$ m

Question 11.

Sugar costs ₹37 $\frac{1}{2}$per kg. Find the cost of 8$\frac{3}{4}$ kg sugar.

Solution:

Cost of 1 kg sugar = ₹$37\frac{1}{2}$

$\therefore$ Cost of $8\frac{3}{4}$ kg sugar

$= 37\frac{1}{2} \times 8\frac{3}{4}$

$= \frac{75}{2} \times \frac{35}{4}$

$=$ ₹$\frac{2625}{8}$ = ₹$328\frac{1}{8}$

Question 12.
A motor cycle runs $31\frac{1}{4}$ km consuming 1 litre of petrol. How much distance will it run consuming $1\frac{3}{5}$ liter of petrol?

Solution:

Distance covered in 1 litre petrol

$$= 31\frac{1}{4}\text{ km} = \frac{125}{4}\text{ km}$$

$\therefore$ Distance covered in $1\frac{3}{5}$ litre of petrol

$$= \frac{125}{4} \times \frac{8}{5}$$

$$= \frac{1000}{20} = 50\text{ km}$$

Question 13.
A rectangular park has length = $23\frac{2}{3}$ m and breadth = $16\frac{2}{3}$ m. Find the area of the

Solution:

Length of rectangular park

$$= 23\frac{2}{5}\text{ m} = \frac{117}{5}\text{ m}$$

Breadth of rectangular park

$$= 16\frac{2}{3}\text{ m} = \frac{50}{3}\text{ m}$$

$\therefore$ Area of the park $= l \times b$

$$= \frac{117}{5} \times \frac{50}{3}$$

$$= 39 \times 10 = 390\text{ m}^2$$

Question 14.
Each of 40 identical boxes weighs $4\frac{4}{5}$ kg Find the total weight of all the boxes.

Solution:

Weight of one box $= 4\frac{4}{5}$ kg $= \frac{24}{5}$ kg

Weight of 40 boxes $= 40 \times \frac{24}{5}$

$= 8 \times 24 = 192$ kg

Question 15.
Out of 24 kg of wheat, $\frac{5}{6}$ th of wheat is consumed. Find, how much wheat is still left?

Solution:

Total wheat available = 24 kg

Wheat consumed $= \frac{5}{6}$ th of 24 kg

$= \frac{5}{6} \times 24 = 20$ kg

$\therefore$ Remaining wheat = 24 – 20 kg = 4 kg

Question 16.
A rod of length $2\frac{2}{5}$ metre is divided into five equal parts. Find the length of each part so obtained.

Solution:

Total length of rod $= 2\frac{2}{5}$ m

Length of rod to be divided into 5 equal parts.

$\therefore$ Length of each part of rod $= 2\frac{2}{5} \div 5$

$= \frac{12}{5} \times \frac{1}{5} = \frac{12}{25}$ metre

Question 17.
IfA = $3\frac{3}{8}$ and B = $6\frac{5}{8}$ find :
(i) A+B
(ii) B A
Solution:

$$A = 3\frac{3}{8} = \frac{27}{8}$$

$$B = 6\frac{5}{8} = \frac{53}{8}$$

(*i*) A ÷ B

$$\Rightarrow \frac{27}{8} \div \frac{53}{8}$$

$$\Rightarrow \frac{27}{8} \times \frac{8}{53} = \frac{27}{53}$$

(*ii*) B ÷ A

$$\Rightarrow \frac{53}{8} \div \frac{27}{8} \Rightarrow \frac{53}{8} \times \frac{8}{27}$$

$$\Rightarrow \frac{53}{27} = 1\frac{26}{27}$$

Question 18.

Cost of 3 $\frac{5}{7}$ litres of oil is ₹83 $\frac{1}{2}$. Find the cost of one litre oil.

Solution:

Cost of $3\frac{5}{7}$ litres of oil = ₹$83\frac{1}{2}$

∴ Cost of 1 litre oil = ₹$83\frac{1}{2} \div 3\frac{5}{7}$

$$= ₹\frac{167}{2} \div \frac{26}{7}$$

$$= ₹\frac{167}{2} \times \frac{7}{26}$$

$$= ₹\frac{1169}{52} = ₹22\frac{25}{52}$$

Question 19.
The product of two numbers is 20 $\frac{5}{7}$. If one of these numbers is 6 $\frac{2}{3}$, find the other.
Solution:

$$\text{The product of two numbers} = 20\frac{5}{7} = \frac{145}{7}$$

$$\text{One number} = 6\frac{2}{3} = \frac{20}{3}$$

$$\therefore \text{ Second number} = \frac{145}{7} \div \frac{20}{3}$$

$$= \frac{145}{7} \times \frac{3}{20} = \frac{87}{28} = 3\frac{3}{28}$$

Question 20.
By what number should 5 $\frac{5}{6}$ be multiplied 1 to get 3$\frac{1}{3}$?

Solution:

$$\text{Required number} = 3\frac{1}{3} \div 5\frac{5}{6}$$

$$= \frac{10}{3} \div \frac{35}{6} \Rightarrow \frac{10}{3} \times \frac{6}{35} = \frac{4}{7}$$

$$\therefore \text{ Required number} = \frac{4}{7}$$

EXERCISE 3(D)

Question 1.

Simplify

Solution:

$$6+\left\{\frac{4}{3}+\left(\frac{3}{4}-\frac{1}{3}\right)\right\}$$

$$6+\left\{\frac{4}{3}+\left(\frac{3}{4}-\frac{1}{3}\right)\right\}$$

$$=6+\left\{\frac{4}{3}+\frac{3}{4}-\frac{1}{3}\right\}=\frac{6}{1}+\frac{4}{3}+\frac{3}{4}-\frac{1}{3}$$

$$=\frac{72+16+9-4}{12}\qquad\text{(LCM of 3, 4 = 12)}$$

$$=\frac{97-4}{12}=\frac{93}{12}=\frac{31}{4}=7\frac{3}{4}$$

Question 2.

$$8-\left\{\frac{3}{2}+\left(\frac{3}{5}-\frac{1}{2}\right)\right\}$$

Solution:

$$8-\left\{\frac{3}{2}+\left(\frac{3}{5}-\frac{1}{2}\right)\right\}$$

$$=8-\left\{\frac{3}{2}+\frac{3}{5}-\frac{1}{2}\right\}=\frac{8}{1}-\frac{3}{2}-\frac{3}{5}+\frac{1}{2}$$

$$=\frac{80-15-6+5}{10}=\frac{85-21}{10}=\frac{64}{10}$$

$$=\frac{32}{5}=6\frac{2}{5}$$

Question 3.

$$\frac{1}{4}\left(\frac{1}{4}+\frac{1}{3}\right)-\frac{2}{5}$$

Solution:

$$=\frac{1}{4}\left(\frac{3+4}{12}\right)-\frac{2}{5}=\frac{1}{4}\times\frac{7}{12}-\frac{2}{5}$$

$$=\frac{7}{48}-\frac{2}{5}=\frac{35-96}{240}=\frac{-61}{240} \textbf{ Ans.}$$

Question 4.

$$2\frac{3}{4}-\left[3\frac{1}{8}\div\left\{5-\left(4\frac{2}{3}-\frac{11}{12}\right)\right\}\right]$$

$$=\frac{11}{4}-\left[\frac{25}{8}\div\left\{5-\left(\frac{14}{3}-\frac{11}{12}\right)\right\}\right]$$

$$=\frac{11}{4}-\left[\frac{25}{8}\div\left\{5-\left(\frac{56-11}{12}\right)\right\}\right]$$

$$=\frac{11}{4}-\left[\frac{25}{8}\div\left\{5-\frac{45}{12}\right\}\right]$$

$$=\frac{11}{4}-\left[\frac{25}{8}\div\left\{\frac{60-45}{12}\right\}\right]$$

$$=\frac{11}{4}-\left[\frac{25}{8}\div\frac{15}{12}\right]=\frac{11}{4}-\left[\frac{25}{8}\times\frac{12}{15}\right]$$

$$=\frac{11}{4}-\frac{5}{2}=\frac{11-10}{4}=\frac{1}{4} \textbf{ Ans.}$$

Solution:

$$2\frac{3}{4}-\left[3\frac{1}{8}\div\left\{5-\left(4\frac{2}{3}-\frac{11}{12}\right)\right\}\right]$$

Question 5.

$$12\frac{1}{2}-\left[8\frac{1}{2}+\left\{9-(5-\overline{3-2})\right\}\right]$$

$$=\frac{25}{2}-\left[\frac{17}{2}+\{9-(5-1)\}\right]$$

$$=\frac{25}{2}-\left[\frac{17}{2}+\{9-4\}\right]=\frac{25}{2}-\left[\frac{17}{2}+5\right]$$

$$=\frac{25}{2}-\frac{17}{2}-\frac{5}{1}=\frac{25-17-10}{2}$$

$$=\frac{25-27}{2}=-\frac{2}{2}=-1 \textbf{ Ans.}$$

Solution:

$$12\frac{1}{2}-\left[8\frac{1}{2}+\left\{9-(5-\overline{3-2})\right\}\right]$$

Question 6.

$$1\frac{1}{5}\div\left\{2\frac{1}{3}-(5+\overline{2-3})\right\}-3\frac{1}{2}$$

Solution:

$$1\frac{1}{5}\div\left\{2\frac{1}{3}-(5+\overline{2-3})\right\}-3\frac{1}{2}$$

$$=\frac{6}{5}\div\left\{\frac{7}{3}-(5-1)\right\}-\frac{7}{2}$$

$$=\frac{6}{5}\div\left\{\frac{7}{3}-4\right\}-\frac{7}{2}=\frac{6}{5}\div\left\{\frac{7-12}{3}\right\}-\frac{7}{2}$$

$$=\frac{6}{5}\div\frac{-5}{3}-\frac{7}{2}=\frac{6}{5}\times\frac{3}{-5}-\frac{7}{2}$$

$$=-\frac{18}{25}-\frac{7}{2}=\frac{-36-175}{50}=\frac{-211}{50}$$

$$=-4\frac{11}{50} \textbf{ Ans.}$$

Question 7.

$$\left(\frac{1}{2}+\frac{2}{3}\right) \div \left(\frac{3}{4}-\frac{2}{9}\right)$$

Solution:

$$\left(\frac{1}{2}+\frac{2}{3}\right) + \left(\frac{3}{4}-\frac{2}{9}\right)$$

$$= \frac{3+4}{6} + \frac{27-8}{36} \qquad \text{(Using BODMAS)}$$

$$= \frac{7}{6} \div \frac{19}{36}$$

$$= \frac{7}{6} \times \frac{36}{19} = \frac{42}{19} = 2\frac{4}{19}$$

Question 8.

$$\frac{6}{5} \text{ of } \left(3\frac{1}{3}-2\frac{1}{2}\right) + \left(2\frac{5}{21}-2\right)$$

Solution:

$$\frac{6}{5} \text{ of } \left(3\frac{1}{3} - 2\frac{1}{2}\right) \div \left(2\frac{5}{21} - 2\right)$$

$$= \frac{6}{5} \text{ of } \left(\frac{10}{3} - \frac{5}{2}\right) \div \left(\frac{47}{21} - \frac{2}{1}\right)$$

(Using BODMAS)

$$= \frac{6}{5} \text{ of } \left(\frac{20-15}{6}\right) \div \left(\frac{47-42}{21}\right)$$

$$= \frac{6}{5} \text{ of } \frac{5}{6} \div \frac{5}{21}$$

$$= 1 \div \frac{5}{21}$$

$$= 1 \times \frac{21}{5} = \frac{21}{5} = 4\frac{1}{5}$$

Question 9.

$$10\frac{1}{8} \text{ of } \frac{4}{5} \div \frac{35}{36} \text{ of } \frac{20}{49}$$

Solution:

$$10\frac{1}{8} \text{ of } \frac{4}{5} \div \frac{35}{36} \text{ of } \frac{20}{49}$$

Using BODMAS

$$\frac{81}{8} \text{ of } \frac{4}{5} \div \frac{35}{36} \text{ of } \frac{20}{49}$$

$$= \frac{81}{10} \div \frac{25}{63}$$

$$= \frac{81}{10} \times \frac{63}{25} = \frac{5103}{250}$$

$$= 20\frac{103}{250}$$

Question 10.

$$5\frac{3}{4} - \frac{3}{7} \times 15\frac{3}{4} + 2\frac{2}{35} \div 1\frac{11}{25}$$

Solution:

$$\frac{23}{4} - \frac{3}{7} \times \frac{63}{4} + \frac{72}{35} \div \frac{36}{25}$$

$$= \frac{23}{4} - \frac{3}{7} \times \frac{63}{4} + \frac{72}{35} \times \frac{25}{36}$$

$$= \frac{23}{4} - \frac{27}{4} + \frac{10}{7}$$

$$= \frac{161 - 189 + 40}{28} = \frac{201 - 189}{28} = \frac{12}{28} = \frac{3}{7}$$

Question 11.

$$\frac{3}{4} \text{ of } 7\frac{3}{7} - 5\frac{3}{5} + 3\frac{4}{15}$$

Solution:

$$\frac{3}{4} \text{ of } 7\frac{3}{7} - 5\frac{3}{5} \div 3\frac{4}{15}$$

Using BODMAS

$$\frac{3}{4} \text{ of } \frac{52}{7} - \frac{28}{5} + \frac{49}{15}$$

$$= \frac{39}{7} - \frac{28}{5} \div \frac{49}{15}$$

$$= \frac{39}{7} - \frac{28}{5} \times \frac{15}{49} = \frac{39}{7} - \frac{12}{7}$$

$$= \frac{39 - 12}{7} = \frac{27}{7} = 3\frac{6}{7}$$

EXERCISE 3 (E)

Question 1.

A line AB is of length 6 cm. Another line CD is of length 15 cm. What fraction is :

(i) The length of AB to that of CD ?

(ii) $\frac{1}{2}$ the length of AB to that of $\frac{1}{3}$ of CD ?

(iii) $\frac{1}{5}$ of CD to that of AB ?

Solution:

Length of line AB = 6 cm

and length of line CD = 15 cm

(*i*) Length of AB to length of

$$CD = \frac{6}{15} = \frac{2}{5}$$

(*ii*) $\frac{1}{2}$ of AB $= \frac{1}{2} \times 6 = 3$ cm

$\frac{1}{3}$ of CD $= \frac{1}{3} \times 15 = 5$ cm

$\therefore \frac{1}{2}$ of AB to $\frac{1}{3}$ of CD $= \frac{3}{5}$

(*iii*) $\frac{1}{5}$ of CD $= \frac{1}{5} \times 15 = 3$ cm.

$\therefore \frac{1}{5}$ of CD to that of AB $= \frac{3}{6} = \frac{1}{2}$ **Ans.**

Question 2.
Subtract $\frac{2}{7} - \frac{5}{21}$ from the sum of $\frac{3}{4}$, $\frac{5}{7}$ and $\frac{7}{12}$

Solution:

$$\left(\frac{3}{4}+\frac{5}{7}+\frac{7}{12}\right)-\left(\frac{2}{7}-\frac{5}{21}\right)$$

$$\left(\frac{63+60+49}{84}\right)-\left(\frac{6-5}{21}\right)$$

$$\frac{172}{84}-\frac{1}{21}=\frac{172-4}{84}=\frac{168}{84}=2 \textbf{ Ans.}$$

Question 3.
From a sack of potatoes weighing 120 kg, a merchant sells portions weighing 6 kg, $5\frac{1}{4}$ kg, $9\frac{1}{2}$ kg and $9\frac{3}{4}$ kg respectively.
(i) How many kg did he sell ?
(ii) How many kg are still left in the sack ?

Solution:

Total quantity of potatoes = 120 kg

(*i*) Quantity of potatoes he sold

$$= 6 \text{ kg} + 5\frac{1}{4}\text{kg} + 9\frac{1}{2}\text{kg} + 9\frac{3}{4}\text{kg}$$

$$= \left(6+\frac{21}{4}+\frac{19}{2}+\frac{39}{4}\right)\text{kg}$$

$$= \frac{24+21+38+39}{4}\text{kg}$$

$$= \frac{122}{4} = \frac{61}{2}\text{kg} = 30\frac{1}{2}\text{kg.}$$

(*ii*) Quantity of potatoes left

$$= 120 \text{ kg} - 30\frac{1}{2}\text{kg} = \left(\frac{120}{1}-\frac{61}{2}\right)\text{kg}$$

$$= \frac{240-61}{2} = \frac{179}{2}\text{kg} = 89\frac{1}{2}\text{kg}$$

Question 4.

If a boy works for six consecutive days for 8 hours, 7$\frac{1}{2}$ hours, 8$\frac{1}{4}$ hours, 6 $\frac{1}{4}$4 3hours, 6$\frac{3}{4}$ hours and 7 hours respectively. How much money will he earn at the rate of Rs. 36 per hour ?

Solution:

No. of hours, a boy worked in 6 days

$$= 8 \text{ hrs} + 7\frac{1}{2} \text{ hrs} + 8\frac{1}{4} \text{ hrs} + 6\frac{1}{4} \text{ hrs} + 6\frac{3}{4} \text{ hrs} + 7 \text{ hrs}$$

$$= \left(8 + \frac{15}{2} + \frac{33}{4} + \frac{25}{4} + \frac{27}{4} + 7\right) \text{hours}$$

$$= \frac{32 + 30 + 33 + 25 + 27 + 28}{4} \text{hours}$$

(LCM of 2, 4 = 4)

$$= \frac{175}{4} \text{ hours} = 43\frac{3}{4} \text{ hours}$$

Earning per hour = Rs. 36

$$\therefore \text{ Total earnings} = \text{Rs. } \frac{175}{4} \times 36$$

$$= \text{Rs. } 175 \times 9 = \text{Rs. } 1575$$ **Ans.**

Question 5.

A student bought $4\frac{1}{3}$ m of yellow ribbon, $6\frac{1}{6}$ m of red ribbon and $3\frac{2}{9}$ m of blue ribbon for decorating a room. How many metres of ribbon did he buy ?

Solution:

Length of yellow ribbon $= 4\frac{1}{3}\text{ m} = \frac{13}{3}\text{ m}$

Length of red ribbon $= 6\frac{1}{6}\text{ m} = \frac{37}{6}\text{ m}$

Length of blue ribbon $= 3\frac{2}{9}\text{ m} = \frac{29}{9}\text{ m}$

Total length of ribbon $= \frac{13}{3} + \frac{37}{6} + \frac{29}{9}$

$= \frac{78 + 111 + 58}{18}$ (LCM of 3, 6, 9 = 18)

$= \frac{247}{18} = 13\frac{13}{18}$ metres **Ans.**

Question 6.

In a business, Ram and Deepak invest $\frac{3}{5}$ and $\frac{2}{5}$ of the total investment. IfRs. 40,000 is the total investment, calculate the amount invested by each ?

Solution:

Total investment = Rs. 40,000

Ram's investment $= \frac{3}{5}$ of Rs. 40,000

$= \text{Rs. } \frac{3}{5} \times 40.000$

$= \text{Rs. } 3 \times 8000 = \text{Rs. } 24,000$

Deepak's investment $= \frac{2}{5}$ of Rs. 40000

$= \text{Rs. } \frac{2}{5} \times 40000$

$= \text{Rs. } 2 \times 8000 = \text{Rs. } 16,000$ **Ans.**

Question 7.
Geeta had 30 problems for home work. She worked out $\frac{2}{5}$ of them. How many problems were still left to be worked out by her ?

Solution:

No. of problems of Geeta = 30

No. of problems worked out = $\frac{2}{3}$ of 30

$= \frac{2}{3} \times 30 = 20$

No. of problems left out = 30 – 20

= 10 **Ans.**

Question 8.
A picture was marked at Rs. 90. It was sold at $\frac{3}{4}$ of its marked price. What was the sale price ?

Solution:

Marked price = Rs. 90

Sale price = $\frac{3}{4}$ of Rs. 90 = $\frac{3}{4} \times 90$

= Rs. $\frac{270}{4}$ = Rs. $67\frac{1}{2}$ = Rs. 67·50 **Ans.**

Question 9.
Mani had sent fifteen parcels of oranges. What was the total weight of the parcels, if each weighed $10\frac{1}{2}$ kg ?

Solution:

Total no. of parcels = 15

Weight of each parcel = $10\frac{1}{2}$ kg = $\frac{21}{2}$ kg

Total weight = 15 of $\frac{21}{2}$ kg = $\frac{21}{2} \times 15$ kg

$= \frac{315}{2} = 157\frac{1}{2}$ kg = 157·5 kg **Ans.**

Question 10.

A rope is 25$\frac{1}{2}$ m long. How many pieces , 1 $\frac{1}{2}$ each of length can be cut out from it?

Solution:

Total length of the rope $= 25\frac{1}{2}\text{ m} = \frac{51}{2}\text{ m}$

Length of each piece $= 1\frac{1}{2}\text{ m} = \frac{3}{2}\text{ m}$

$\therefore$ No. of pieces $= \frac{51}{2} \div \frac{3}{2} = \frac{51}{2} \times \frac{2}{3}$

$= 17$ pieces **Ans.**

Question 11.

The heights of two vertical poles, above the earth's surface, are 14 $\frac{1}{4}$ m and 22 $\frac{1}{3}$ respectively. How much higher is the second pole as compared with the height of the first pole ?

Solution:

Height of one pole above earth's surface

$= 14\frac{1}{4}\text{ m}$

and height of second pole $= 22\frac{1}{3}$

$\therefore$ Second pole is higher than the first pole

$= 22\frac{1}{3} - 14\frac{1}{4} = \frac{67}{3} - \frac{57}{4}$

$= \frac{268-171}{12} = \frac{97}{12}\text{ m} = 8\frac{1}{12}\text{ m}$

Question 12.
Vijay weighed $65\frac{1}{2}$ kg. He gained $1\frac{2}{5}$ kg during the first week, $1\frac{1}{4}$ kg during the second week, but lost $\frac{5}{16}$ kg during the 16 third week. What was his weight after the third week ?

Solution:

In the beginning, weight of Vijay

$= 65\frac{1}{2}$ kg.

Gained in first week $= 1\frac{2}{5}$ kg.

Gained in the second week $= 1\frac{1}{4}$ kg

Lost in the third week $= \frac{5}{16}$ kg

$\therefore$ Weight of Vijay after third week

$$= \left(65\frac{1}{2} + 1\frac{2}{5} + 1\frac{1}{4} - \frac{5}{16}\right) \text{ kg}$$

$$= \left(\frac{131}{2} + \frac{7}{5} + \frac{5}{4} - \frac{5}{16}\right) \text{ kg}$$

$$= \frac{5240 + 112 + 100 - 25}{80}$$

{LCM of 2, 5, 4 and 16 = 80}

$$= \frac{5452 - 25}{80} = \frac{5427}{80} \text{ kg} = 67\frac{67}{80} \text{ kg } \textbf{Ans.}$$

Question 13.

A man spends $\frac{2}{5}$ of his salary on food and $\frac{3}{10}$ on house rent, electricity, etc. What fraction of his salary is still left with him ?

Solution:

Let salary of man = Re. 1

Amount spent on food $= \frac{2}{5}$ of Re. 1 = Re. $\frac{2}{5}$

and amount spent house rent $= \frac{3}{10}$ of Re. 1

= Re. $\frac{3}{10}$

Total amount spent = Re. $\frac{2}{5} + \frac{3}{10} = \frac{4+3}{10} = \frac{7}{10}$

$\therefore$ Amount left with him $= 1 - \frac{7}{10} = \frac{10-7}{10} = \frac{3}{10}$

Question 14.

A man spends $\frac{2}{5}$ of his salary on food and $\frac{3}{10}$ of the remaining on house rent, electricity, etc. What fraction of his salary is still left with him ?

Solution:

Let total amount of salary = Re. 1

Amount spent on food $= \frac{2}{5}$ of Re. 1

$= \text{Rs. } \frac{2}{5}$

Remaining amount $= 1 - \frac{2}{5}$

$= \frac{5-2}{5} = \text{Rs. } \frac{3}{5}$

Amount spent on house rent etc.

$= \frac{3}{10} \text{ of } \frac{3}{5} = \text{Rs. } \frac{9}{50}$

Remaining amount left

$= \frac{3}{5} - \frac{9}{50}$

$= \text{Rs } \frac{30-9}{50} = \text{Rs. } \frac{21}{50}$

$\therefore$ Fraction of amount left $= \frac{21}{50}$

Question 15.

Shyam bought a refrigerator for Rs. 5000. He paid $\frac{1}{10}$ of the price in cash and the rest in 12 equal monthly instalments. How much had he to pay each month ?

Solution:

Total amount of the refrigerator = Rs. 5000

Amount paid in cash = $\frac{1}{10}$ of Rs. 5000

$= \frac{1}{10} \times 5000 =$ Rs. 500

Balance amount = Rs. 5000 – Rs. 500

= Rs. 4500

No. of equally instalments = 12

$\therefore$ Amount of each instalment

= Rs. 4500 ÷ 12 = Rs. 4500 $\times \frac{1}{12}$

= Rs. 375 **Ans.**

Question 16.

A lamp post has half of its length in mud, and $\frac{1}{3}$ of its length in water.
(i) What fraction of its length is above the water ?
(ii) If $3\frac{1}{3}$ m of the lamp post is above the water, find the whole length of the lamp post.

Solution:

(i) Let length of the post = 1 m

then length of post in mud = $\frac{1}{2}$ m

and length of post in water = $\frac{1}{3}$ m

$\therefore$ Length of post above the water

$= 1 - \left(\frac{1}{2} + \frac{1}{3}\right) = 1 - \left(\frac{3+2}{6}\right)$

$= 1 - \frac{5}{6} = \frac{6-5}{6} = \frac{1}{6}$ m

(ii) But length of post above water

$= 3\frac{1}{3}$ m $= \frac{10}{3}$ m

$\therefore \frac{1}{6}$th of total length $= \frac{10}{3}$ m

$\therefore$ Total length $= \frac{10}{3} \times \frac{6}{1} = 20$ m **Ans.**

Question 17.
I spent $\frac{3}{5}$ of my savings and still have Rs. 2,000 left. What were my savings ?

Solution:

Let my saving = 1, Part spent = $\frac{3}{5}$ of savings

$\therefore$ part left $= 1 - \frac{3}{5} = \frac{5-3}{5} = \frac{2}{5}$ of savings

But he left = Rs. 2000

$\therefore \frac{2}{5}$ of savings = Rs. 2000

$\therefore$ Total savings = Rs. $2000 \times \frac{5}{2}$

= Rs. 5000 **Ans.**

Question 18.
In a school, $\frac{4}{5}$ of the children are boys. If the number of girls is 200, find the number of boys.

Solution:

No. of boys = $\frac{4}{5}$ of the total children

$\therefore$ No. of girls = $\left(1 - \frac{4}{5}\right)$ of total children

$= \frac{5-4}{5} = \frac{1}{5}$ of total children.

But no. of girls = 200

$\therefore \frac{1}{5}$ of total children = 200

Hence total number of children = $200 \times \frac{5}{1}$

= 1000

$\therefore$ No. of boys = $\frac{4}{5}$ of 1000 = $\frac{4}{5} \times 1000$

= 800 **Ans.**

Question 19.
If $\frac{4}{5}$ of an estate is worth Rs. 42,000, find the worth of whole estate. Also, find the value of $\frac{3}{7}$ of it.
Solution:

$\frac{4}{5}$ of an estate = Rs. 42000

$\therefore$ Total value of estate = Rs. $42000 \times \frac{5}{4}$

= Rs. 10500 × 5 = Rs. 52500 **Ans.**

and value of $\frac{3}{7}$ of it = $\frac{3}{7}$ of its value

$= \frac{3}{7}$ of ₹52500 = ₹$\frac{3}{7} \times 52500$

= ₹3 × 7500 = ₹22500

Question 20.
After going $\frac{3}{4}$ of my journey, I find that I have covered 16 km. How much Journey is still left ?

Solution:

$\frac{3}{4}$ of journey = 16 km.

$\therefore$ Total Journey $= 16 \text{ km} \times \frac{4}{3} = \frac{64}{3}$ km

$\therefore$ Journey Left $= \frac{64}{3} - \frac{16}{1}$

$= \frac{64 - 48}{3} = \frac{16}{3}$ km $= 5\frac{1}{3}$

Question 21.
When Krishna travelled 25 km, he found that $\frac{3}{5}$ of his journey was still left. What was the length of the whole journey.

$\frac{3}{5}$ of the total journey was left

$\therefore$ Journey travelled by him $= 1 - \frac{3}{5}$

$= \frac{5-3}{5} = \frac{2}{5}$

$\therefore \frac{2}{5}$ of total journey = 25 km

$\therefore$ Total journey = 25 km $\times \frac{5}{2} = \frac{125}{2}$ km

$= 62\frac{1}{2}$ km

Solution:

Question 22.
From a piece of land, one-third is bought by Rajesh and one-third of remaining is bought by Manoj. If 600 m² land is still left unsold, find the total area of the piece of land.

Solution:
Let the piece of land = 1 m

Land bought by Rajesh $= 1 \times \frac{1}{3} = \frac{1}{3}$ m

Remaining land $= 1 - \frac{1}{3} = \frac{3-1}{3} = \frac{2}{3}$ m

Now, Land bought by Manoj $= \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$ m

Land unsold $= \frac{2}{3} - \frac{2}{9}$

$= \frac{6-2}{9} = \frac{4}{9}$ m

Land of $\frac{4}{9}$ m remain unsold = 600 m^2

$\therefore$ Total area of the land $= 600 \times \frac{9}{4}$

$= 150 \times 9\ m^2 = 1350\ m^2$

Question 23.

A boy spent $\frac{3}{5}$ of his money on buying 1 cloth and $\frac{1}{4}$ of the remaining on buying shoes. If initially he has ?2,400; how much did he spend on shoes?

Solution:

Money in hand = ₹2400

Money spent on buying clothes

$$= \frac{3}{5} \text{ of } ₹2400$$

$$= \frac{3}{5} \times 2400 = 3 \times 480 = ₹1440$$

Remaining money = ₹2400 – ₹1440 = ₹960

Now, boy spent $\frac{1}{4}$ of the remaining money on buying shoes.

∴ Money spent on buying shoes

$$= ₹960 \times \frac{1}{4} = ₹240$$

Question 24.

A boy spent $\frac{3}{5}$ of his money on buying cloth and $\frac{1}{4}$ of his money on buying shoes.

Solution:

Money in hand = ₹2400

Money spent on buying clothes

$$= \frac{3}{5} \text{ of } ₹2400$$

$$= \frac{3}{5} \times 2400 = ₹1440$$

Remaining money = ₹2400 – ₹1440 = ₹960

Now, boy spent $\frac{1}{4}$ of the remaining money on buying shoes.

Money spent on buying shoes = ₹960 × $\frac{1}{4}$

= ₹240

CHAPTER - 4
DECIMAL FRACTIONS

POINTS TO REMEMBER

1. **Decimal fraction (or a decimal number)**
   A decimal fraction is a fraction whose denominator is 10 or a higher power of 10. In order to express a given decimal fraction in shorter form, the denominator is not written but its absence is shown by a dot which is called a decimal point inserted in a proper place.
   Note :
   (i) When there is no number is the left of the decimal point, generally, a zero is written.
   (ii) Generally, a decimal fraction has two parts, the first part which is on the right ox the decimal point is called decimal part and the part on the left side of the decimal point, is called integeral part.
   (iii) The decimal part is always less than 1.
2. **Reading Decimal Numbers**
   The integral part is read according to its value and the decimal part is read by naming each digit, in order, separately
3. **Converting a decimal number into a vulgar fraction :**
   Remove the decimal point from the decimal number and write in its denominator with as many zeros as the number of digits are in the decimal parts to the right of 1.
   In the decimal number, the name of each place is given as under is the place value chart:

Thousands	**Hundreds**	**Tens**	**Units**	**•**	**Tenths**	**Hundredths**	**Thousandth**
				Decimal point			**and so on**

4. **Converting a given fraction in to a decimal fraction :**
   (a) When the denominator in given fraction is 10. 100, 1000 etc., then count from extreme right to left, mark decimal point after as many digits of the numerator as there are zeroes in the denominator.
   (b) When the denominator of the given number is other then 10 or higher power of 10, then divide in an ordinary way and mark the decimal point in the quotient just after the division of unit digit is completed. After this, any number of zeroes can be borrowed to.complete the division.

Note : The number of figures that follow the decimal part is called the number of decimal places.

5. **Addition and Subtraction of decimal numbers.**
   (a) Addition : Write the given decimal numbers in such a way, that the decimal points of all the numbers fall in the same vertical line. Digits with the same place value are placed one below the other units are written below units, tens below tens and so on.
   Addition is started from the right side, as done in the usual addition (empty places may be filled up by zeroes). In the result (total), the decimal point is placed under decimal points of the numbers added.
   (b) Subtraction : In subtraction also, the numbers are written in such a way that their decimals are in ihe same vertical line. Digits with the same place value are placed one below the other (empty places may be filled by zeroes).
   Subtraction is started from the right side, as in the case of normal subtraction. In the result, decimal point is placed just under the other decimal points.
6. **Multiplication of decimal numbers :**
   (1) Multiplication by 10,100, 1000 etc. shift the decimal point, in the multiplicand, to the right by as many digits as there are zeroes in the multiplier.
   (2) Multiplication by a whole number : Multiply in an ordinary way, without considering the decimal point. In the product, the decimal point should be fixed by counting as many digits from the right as there are decimal places in the multiplicand.
   Thus, (i) 0.3 x 6 = 1.8 (ii) 0.26 x 18 = 4.68 and so on.
   (3) Multiplication of a decimal number by another decimal number :
   Multiply the two numbers in a normal way, ignoring their decimals. In the product, decimal point is fixed counting from right, the digits equal to the sum of decimal places in the multiplicand and the multiplier.
   Thus, 32.5 x ().()7 = 2.275
   Since, the multiplicand (32.5) has one decimal place and multiplier (0.07) has two decimal places, their product will have 1+2 = 3 decimal places.
7. **Division of decimal numbers :**
   (1) Division by 10, 100, 1000 etc : Shift the decimal points to the left as many digits as there are
   zeroes in the divisor. .
   (2) Division by a whole number : Divide in the normal manner, ignoring the decimal, and mark tire decimal point; in the quotient, while just crossing over the decimal point in the dividend.
8. **Recurring Decimals :**
   On performing a division, sometimes we find that the same remainder is left, no matter how long we continue the division process. For this reason, the same digit appeares again and again in the quotient. This fact is shown by puting a dot as a bar over the repeating digits.
9. **Rounding off of decimal numbers :**
   (i) If the answer required is correct to two decimal places, we retain digits upto three decimal places.
   (ii) If the digit in the third decimal place is five or more than five, then the digit in

the second decimal place is increased by one and, if the digit in the third decimal place is less than five, then the digit in the second decimal place is not altered.
(iii)The third digit which was retained is now omitted.
Thus, for getting 8.4813 correct to two decimal places.
Write the given number upto three decimal places i.e. 8.481.
Since, the digit in the third decimal place is 1 which is less than 5.
∴The digit in the second decimal place is not altered.
And, so 8.4813 = 8.48, correct to two decimal places.
In the same way, to get 3.946824 correct to nearest thousandths i.e., correct to three decimal places, first write it as 3.9468.
Then according to the rule, the digit in the third place changes from 6 to 7.
3.9468 = 3.947, correct to three decimal places.

EXERCISE 4 (A)

Question 1.

Convert the following into fractions in their lowest terms :
(i) 3.75
(ii) 0.5
(iii) 2.04
(iv) 0.65
(v) 2.405
(vi) 0.085
(vii) 8.025

Answer:

(i) $3\cdot75 = \frac{375}{100} = \frac{375 \div 25}{100 \div 25}$

(HCF of 375 and 100 = 25)

$= \frac{15}{4}$ **Ans.**

(ii) $0\cdot5 = \frac{5}{10} = \frac{1}{2}$ **Ans.**

(iii) $2\cdot04 = \frac{204}{100} = \frac{204 \div 4}{100 \div 4}$

(HCF of 204 and 100 = 4)

$= \frac{51}{25}$ **Ans.**

(iv) $0\cdot65 = \frac{65}{100} = \frac{65 \div 5}{100 \div 5} = \frac{13}{20}$ **Ans.**

(v) $2\cdot405 = \frac{2405}{1000} = \frac{2405 \div 5}{1000 \div 5} = \frac{481}{200}$ **Ans.**

(vi) $0\cdot085 = \frac{85}{1000} = \frac{85 \div 5}{1000 \div 5} = \frac{17}{200}$ **Ans.**

(vii) $8\cdot025 = \frac{8025}{1000} = \frac{8025 \div 25}{1000 \div 25}$

Question 2.
Convert into decimal fractions

(i) $2\frac{4}{5}$

(ii) $\frac{79}{100}$

(iii) $\frac{37}{10,000}$

(iv) $\frac{7543}{10^4}$

(v) $\frac{3}{4}$

(vi) $9\frac{3}{5}$

(vii) $8\frac{5}{8}$

(viii) $5\frac{7}{8}$

Answer:

(i) (i) $2\frac{4}{5} = \frac{14}{5}$

$= \frac{14}{5} \times \frac{2}{2} = \frac{28}{10} = 2.8$

(ii) $\frac{79}{100} = 0{\cdot}79$ **Ans.**

(iii) $\frac{37}{10,000} = 0{\cdot}0037$ **Ans.**

(iv) $\frac{7543}{10^4} = \frac{7543}{10000}$

$= 0{\cdot}7543$ **Ans.**

(v) $\frac{3}{4} = 0{\cdot}75$ **Ans.**

$$\begin{array}{r|l} & 0{\cdot}75 \\ 4 & 3{\cdot}00 \\ & 28 \\ \hline & 20 \\ & 20 \\ \hline & \times \end{array}$$

(vi) $9\frac{3}{5} = \frac{48}{5} = 9 \cdot 6$

$$\begin{array}{r|l} & 9\cdot6 \\ 5 & 48\cdot0 \\ & 45 \\ \hline & 30 \\ & 30 \\ \hline & \times \end{array}$$

(vii) $8\frac{5}{8} = 8 \cdot 625$

$$\begin{array}{r|l} & 0\cdot625 \\ 8 & 5\cdot000 \\ & 48 \\ \hline & 20 \\ & 16 \\ \hline & 40 \\ & 40 \\ \hline & \times \end{array}$$

(viii) $5\frac{7}{8} = 5 \cdot 875$

$$\begin{array}{r|l} & 8\cdot75 \\ 8 & 7\cdot000 \\ & 64 \\ \hline & 60 \\ & 56 \\ \hline & 40 \\ & 40 \\ \hline & \times \end{array}$$

Question 3.
Write the number of decimal places in :
(i) 0.4762
(ii) 7.00349
(iii) 8235.403
(iv) 35.4
(v) 2.608
(vi) 0.000879

Answer:
(i) In 0.4762, there are four places.
(ii) In 7.00349, there are five places.
(iii) In 8235.403, there are three places.
(iv) In 35.4, there is one place.
(v) In 2.608, there are three places.
(vi) In 0.000879, there are six places.

Question 4.
Write the following decimals as word statements :
(i) 0.4,0.9,0.1
(ii) 1.9, 4.4, 7.5
(iii) 0.02, 0.56, 13.06
(iv) 0.005,0.207, 111.519
(v) 0.8, 0.08, 0.008, 0.0008
(vi)256.1, 10.22, 0.634

Answer:
(i) 0.4 = zero point four, 0.9 = zero point nine, 0.1 = zero point one.
(ii) 1 .9 = one point nine, 4.4 = four point four, 7.5 = seven point five.
(iii) 0.02 = zero point zero two, 0.56 = zero point five six, 13.06 = thirteen point zero six.
(iv) 0.005 = zero point zero zero five, 0.207 = zero point two zero seven, 111.519 = one hundred eleven point five one nine.
(v) 0.8 = zero point eight, 0.08 = zero point zero eight, 0.008 = zero point zero zero eight, 0.0008 = zero point zero zero zero eight
(vi) 256.1 = Two hundred fifty six point one, 10.22 = Ten point two two, 0.634 = zero point six three four.

Question 5.
Convert the given fractions into like fractions:
(i) 0.5,3.62,43.987 and 232.0037
(ii) 215.78, 33.0006, 530.3 and 0.03569

Answer:
(i) 0.5, 3.62, 43.987 and 232.0037
In these decimals, the greatest places of decimal is 4
∴0.5 = 0.5000
3.62 = 3.6200
43.987 = 43.9870
232.0037 = 232.0037

(ii) 215.78, 33.0006, 530.3 and 0.03569
In these decimals, the greatest places of decimal is 5
∴215.78 = 215.78000
33.0006 = 33.00060
530.3 = 530.30000
0.03569 = 0.03569

EXERCISE 4 (B)

Question 1.
Add:
(i) 0.5 and 0.37 (ii) 3.8 and 8.7
(iii) 0.02, 0.008 and 0.309
(iv) 0. 4136, 0. 3195 and 0.52
(v) 9.25, 3.4 and 6.666
(vi) 3.007, 0.587 and 18.341
(vii) 0.2, 0.02 and 2.0002
(viii) 6. 08, 60.8, 0.608 and 0.0608
(ix) 29.03, 0.0003, 0.3 and 7.2
(x) 3.4, 2.025, 9.36 and 3.6221

Answer:

(i) $0{\cdot}5 + 0{\cdot}37 = 0{\cdot}87$ **Ans.**

$$\begin{array}{r} 0{\cdot}5 \\ +\ 0{\cdot}37 \\ \hline 0{\cdot}87 \\ \hline \end{array}$$

(ii) $3{\cdot}8 + 8{\cdot}7 = 12{\cdot}5$ **Ans.**

$$\begin{array}{r} 3{\cdot}8 \\ +\ 8{\cdot}7 \\ \hline 12{\cdot}5 \\ \hline \end{array}$$

(iii) $0{\cdot}02 + 0{\cdot}008 + 0{\cdot}309 = 0{\cdot}337$ **Ans.**

$$\begin{array}{r} 0{\cdot}02 \\ +\ 0{\cdot}008 \\ +\ 0{\cdot}309 \\ \hline 0{\cdot}337 \\ \hline \end{array}$$

(iv) $0{\cdot}4136 + 0{\cdot}3195 + 0{\cdot}52 = 1{\cdot}2531$ **Ans.**

$$\begin{array}{r} 0{\cdot}4136 \\ +\ 0{\cdot}3195 \\ +\ 0{\cdot}52 \\ \hline 1{\cdot}2531 \\ \hline \end{array}$$

(v) $9{\cdot}25 + 3{\cdot}4 + 6{\cdot}666 = 19{\cdot}316$ **Ans.**

$$\begin{array}{r} 9{\cdot}25 \\ +\ 3{\cdot}4 \\ +\ 6{\cdot}666 \\ \hline 19{\cdot}316 \\ \hline \end{array}$$

(vi) $3{\cdot}007 + 0{\cdot}587 + 18{\cdot}341 = 21{\cdot}935$ **Ans.**

$$\begin{array}{r} 3{\cdot}007 \\ +\ 0{\cdot}587 \\ +\ 18{\cdot}341 \\ \hline 21{\cdot}935 \\ \hline \end{array}$$

(vii) $0{\cdot}2 + 0{\cdot}02 + 2{\cdot}0002 = 2{\cdot}2202$ **Ans.**

$$\begin{array}{r} 0{\cdot}2 \\ +\ 0{\cdot}02 \\ +\ 2{\cdot}0002 \\ \hline 2{\cdot}2202 \\ \hline \end{array}$$

(viii) $6{\cdot}08 + 60{\cdot}8 + 0{\cdot}608 + 0{\cdot}0608 = 67{\cdot}5488$

Ans.

$$\begin{array}{r} 6{\cdot}08 \\ +\ 60{\cdot}8 \\ +\ 0{\cdot}608 \\ +\ 0{\cdot}0608 \\ \hline 67{\cdot}5488 \\ \hline \end{array}$$

(ix) $29{\cdot}03 + 0{\cdot}0003 + 0{\cdot}3 + 7{\cdot}2 = 36{\cdot}5303$ **Ans.**

$$\begin{array}{r} 29{\cdot}03 \\ +\ 0{\cdot}0003 \\ +\ 0{\cdot}3 \\ +\ 7{\cdot}2 \\ \hline 36{\cdot}5303 \\ \hline \end{array}$$

(x) $3{\cdot}4 + 2{\cdot}025 + 9{\cdot}36 + 3{\cdot}6221 = 18{\cdot}4071$ **Ans.**

$$\begin{array}{r} 3{\cdot}4 \\ +\ 2{\cdot}025 \\ +\ 9{\cdot}36 \\ +\ 3{\cdot}6221 \\ \hline 18{\cdot}4071 \\ \hline \end{array}$$

Question 2.
Subtract the first! number from the second :
(i) 5.4, 9.8
(ii) 0.16, 4.3
(iii) 0.82, 8.6
(v) 2.237, 9.425
(vi) 41 .03, 59.46
(vii) 3.92. 26.86
(viii) 4.73, 8.5
(ix) 12.63, 36.2
(x) 0.845, 3.71

Answer:

(*i*) $9 \cdot 8 - 5 \cdot 4 = 4 \cdot 4$ **Ans.**

$$\begin{array}{r} 9 \cdot 8 \\ -\ 5 \cdot 4 \\ \hline 4 \cdot 4 \\ \hline \end{array}$$

(*ii*) $4 \cdot 30 - 0 \cdot 16 = 4 \cdot 14$ **Ans.**

$$\begin{array}{r} 4 \cdot 30 \\ -\ 0 \cdot 16 \\ \hline 4 \cdot 14 \\ \hline \end{array}$$

(*iii*) $8 \cdot 60 - 0 \cdot 82 = 7 \cdot 78$ **Ans.**

$$\begin{array}{r} 8 \cdot 60 \\ -\ 0 \cdot 82 \\ \hline 7 \cdot 78 \\ \hline \end{array}$$

(*iv*) $8 \cdot 43 - 0 \cdot 07 = 8 \cdot 36$ **Ans.**

$$\begin{array}{r} 8 \cdot 43 \\ -\ 0 \cdot 07 \\ \hline 8 \cdot 36 \\ \hline \end{array}$$

(*v*) $9 \cdot 425 - 2 \cdot 237 = 7 \cdot 188$ **Ans.**

$$\begin{array}{r} 9 \cdot 425 \\ -\ 2 \cdot 237 \\ \hline 7 \cdot 188 \\ \hline \end{array}$$

(vi) $59 \cdot 46 - 41 \cdot 03 = 18 \cdot 43$ **Ans.**

$$\begin{array}{r} 59 \cdot 46 \\ -\ 41 \cdot 03 \\ \hline 18 \cdot 43 \\ \hline \end{array}$$

(vii) $26 \cdot 86 - 3 \cdot 92 = 22 \cdot 94$ **Ans.**

$$\begin{array}{r} 26 \cdot 86 \\ -\ \ 3 \cdot 92 \\ \hline 22 \cdot 94 \\ \hline \end{array}$$

(viii) $8 \cdot 50 - 4 \cdot 73 = 3 \cdot 77$ **Ans.**

$$\begin{array}{r} 8 \cdot 50 \\ -\ 4 \cdot 73 \\ \hline 3 \cdot 77 \\ \hline \end{array}$$

(ix) $36 \cdot 20 - 12 \cdot 63 = 23 \cdot 57$ **Ans.**

$$\begin{array}{r} 36 \cdot 20 \\ -\ 12 \cdot 63 \\ \hline 23 \cdot 57 \\ \hline \end{array}$$

(x) $3 \cdot 710 - 0 \cdot 845 = 2 \cdot 865$ **Ans.**

$$\begin{array}{r} 3 \cdot 710 \\ -\ 0 \cdot 845 \\ \hline 2 \cdot 865 \\ \hline \end{array}$$

Question 3.
Simplify :
(i) 28.796 -13.42 – 2.555
(ii) 93.354 – 62.82 – 13.045
(iii) 36 – 18.59 – 3.2
(iv) 86 + 16.95 – 3.0042
(v) 32.8 – 13 – 10.725 +3.517
(vi) 4000 – 30.51 – 753.101 – 69.43
(vii) 0.1835 + 163.2005 – 25.9 – 100
(viii) 38.00 – 30 + 200.200 – 0.230
(ix) 555.555 + 55.555 – 5.55 – 0.555

Answer:

(i) $28.796 - 13.42 - 2.555$

$= 28.796 - (13.42 + 2.555)$

$= 28.796 - 15.975 = 12.821$ **Ans.**

$$\begin{array}{r} 28.796 \\ -\,15.975 \\ \hline 12.821 \\ \hline \end{array} \qquad \begin{array}{r} 13.420 \\ +\;\;2.555 \\ \hline 15.975 \\ \hline \end{array}$$

(ii) $93.354 - 62.82 - 13.045$

$= 93.354 - (62.82 + 13.045)$

$= 93.354 - 75.865 = 17.489$ **Ans.**

$$\begin{array}{r} 93.354 \\ -\,75.865 \\ \hline 17.489 \\ \hline \end{array} \qquad \begin{array}{r} 62.820 \\ +\,13.045 \\ \hline 75.865 \\ \hline \end{array}$$

(iii) $36 - 18.59 - 3.2 = 36 - (18.59 + 3.2)$

$= 36 - (21.79) = 14.21$ **Ans.**

$$\begin{array}{r} 36.00 \\ -\,21.79 \\ \hline 14.21 \\ \hline \end{array} \qquad \begin{array}{r} 18.59 \\ +\;\;3.20 \\ \hline 21.79 \\ \hline \end{array}$$

(iv) $86 + 16.95 - 3.0042 = 102.95 - 3.0042$

$= 99.9458$ **Ans.**

$$\begin{array}{r} 102.9500 \\ -\;\;\;3.0042 \\ \hline 99.9458 \\ \hline \end{array} \qquad \begin{array}{r} 86.00 \\ +\;16.95 \\ \hline 102.95 \\ \hline \end{array}$$

(v) $32.8 - 13 - 10.725 + 3.517$

$= (32.8 + 3.517) - (13 + 10.725)$

$= 36.317 - 23.725 = 12.592$ **Ans.**

$$\begin{array}{r} 13.000 \\ +\,10.725 \\ \hline 23.725 \\ \hline \end{array} \qquad \begin{array}{r} 32.8 \\ +\;\;3.517 \\ \hline 36.317 \\ \hline \end{array} \qquad \begin{array}{r} 36.317 \\ -\,23.725 \\ \hline 12.592 \\ \hline \end{array}$$

(vi) $4000 - 30.51 - 753.101 - 69.43$

$= 4000 - (30.51 + 753.101 + 69.43)$

$= 4000 - 853.041 = 3146.959$ **Ans.**

$$\begin{array}{r} 30.510 \\ +.753.101 \\ +\;\;69.430 \\ \hline 853.041 \\ \hline \end{array} \qquad \begin{array}{r} 4000.000 \\ -\;\;\;853.041 \\ \hline 3146.959 \\ \hline \end{array}$$

(*vii*) $0{\cdot}1835 + 163{\cdot}2005 - 25{\cdot}9 - 100$

$= (0{\cdot}1835 + 163{\cdot}2005) - (25{\cdot}9 + 100)$

$= 163{\cdot}3840 - 125{\cdot}9 = 37{\cdot}484$ **Ans.**

$$\begin{array}{r} 25{\cdot}9 \\ +\ 100.0 \\ \hline 125{\cdot}9 \\ \hline \end{array}$$

$$\begin{array}{r} 0{\cdot}1835 \\ +\ 163{\cdot}2005 \\ \hline 163{\cdot}3840 \\ \hline \end{array} \qquad \begin{array}{r} 163{\cdot}3840 \\ -\ \ 125{\cdot}9000 \\ \hline 37{\cdot}4840 \\ \hline \end{array}$$

(*viii*) $38{\cdot}00 - 30 + 200{\cdot}200 - 0{\cdot}230$

$= (38{\cdot}00 + 200{\cdot}200) - (30 + 0{\cdot}230)$

$= 238{\cdot}200 - 30{\cdot}230$

$= 207{\cdot}970 = 207{\cdot}97$ **Ans.**

$$\begin{array}{r} 238{\cdot}200 \\ -\ \ 30{\cdot}230 \\ \hline 207{\cdot}970 \\ \hline \end{array}$$

(*ix*) $555{\cdot}555 + 55{\cdot}555 - 5{\cdot}55 - 0{\cdot}555$

$= (555{\cdot}555 + 55{\cdot}555) - (5{\cdot}55 + 0{\cdot}555)$

$= 611{\cdot}110 - 6{\cdot}105 = 605{\cdot}005$ **Ans.**

$$\begin{array}{r} 555{\cdot}555 \\ +\ \ 55{\cdot}555 \\ \hline 611{\cdot}110 \\ \hline \end{array} \qquad \begin{array}{r} 611{\cdot}110 \\ -\ \ \ 6{\cdot}105 \\ \hline 605{\cdot}005 \\ \hline \end{array}$$

Question 4.
Find the difference between 6.85 and 0.685.
Answer:
Difference between $6{\cdot}85$ and $0{\cdot}685$

$= 6{\cdot}85 - 0{\cdot}685 = 6{\cdot}165$ **Ans.**

$$\begin{array}{r} 6{\cdot}850 \\ -\ 0{\cdot}685 \\ \hline 6{\cdot}165 \\ \hline \end{array}$$

Question 5.
Take out the sum of 19.38 and 56.025 then subtract it from 200. 111.
Answer:
Sum of $19 \cdot 38 + 56 \cdot 025 = 75 \cdot 405$

$$\begin{array}{r} 19 \cdot 38 \\ + 56 \cdot 025 \\ \hline 75 \cdot 405 \\ \hline \end{array}$$

Difference of $200 \cdot 111$ and $75 \cdot 405$
$= 200 \cdot 111 - 75 \cdot 405 = 124 \cdot 706$ **Ans.**

$$\begin{array}{r} 200 \cdot 111 \\ - \quad 75 \cdot 405 \\ \hline 124 \cdot 706 \\ \hline \end{array}$$

Question 6.
Add 13.95 and 1.003 ; and from the result, subtract the sum of 2.794 and 6.2.
Answer:
Sum of $13 \cdot 95$ and $1 \cdot 003$
$= 13 \cdot 95 + 1 \cdot 003 = 14 \cdot 953$

$$\begin{array}{r} 13 \cdot 95 \\ + \quad 1 \cdot 003 \\ \hline 14 \cdot 953 \\ \hline \end{array}$$

Sum of $2 \cdot 794$ and $6 \cdot 2$
$= 2 \cdot 794 + 6 \cdot 2 = 8 \cdot 994$

$$\begin{array}{r} 2 \cdot 794 \\ + 6 \cdot 200 \\ \hline 8 \cdot 994 \\ \hline \end{array}$$

Difference of $14 \cdot 953$ and $8 \cdot 994$
$= 14 \cdot 953 - 8 \cdot 994 = 5 \cdot 959$ **Ans.**

$$\begin{array}{r} 14 \cdot 953 \\ - \quad 8 \cdot 994 \\ \hline 5 \cdot 959 \\ \hline \end{array}$$

Question 7.
What should be added to 39.587 to give 80.375 ?

Answer:
Sum = 80·375

Given number = 39·587

∴ The number which is to be added

$= 80{\cdot}375 - 39{\cdot}587 = 40{\cdot}788$ **Ans.**

$$\begin{array}{r} 80{\cdot}375 \\ -\ 39{\cdot}587 \\ \hline 40{\cdot}788 \\ \hline \end{array}$$

Question 8.
What should be subtracted from 100 to give 19.29?
Answer:
Sum = 100

The number = 19·29

∴ The number which is to be subtracted

$= 100 - 19{\cdot}29 = 80{\cdot}71$ **Ans.**

$$\begin{array}{r} 100{\cdot}00 \\ -\ 19{\cdot}29 \\ \hline 80{\cdot}71 \\ \hline \end{array}$$

Question 9.
What is the excess of 584.29 over 213.95 ?
Answer:
Total = 584·29

Given number = 213·95

Required difference = 584·29 – 213·95

$= 370{\cdot}34$ **Ans.**

$$\begin{array}{r} 584{\cdot}29 \\ -\ 213{\cdot}95 \\ \hline 370{\cdot}34 \\ \hline \end{array}$$

Question 10.
Evaluate:
(i) (5.4 – 0.8) + (2.97 -1.462)
(ii) (6.25 + 0.36) -(17.2 – 8.97)
(iii) 9.004 + (3 -2.462)
(iv) 879.4 – (87.94 – 8 .794)

Answer:

(i) $(5\cdot4 - 0\cdot8) + (2\cdot97 - 1\cdot462)$
$= 4\cdot6 + 1\cdot508 = 6\cdot108$

$$\begin{array}{r} -1\cdot4 \\ \hline 1\cdot5 \\ \hline \end{array}$$

(ii) $(6\cdot25 + 0\cdot36) - (17\cdot2 - 8\cdot97)$
$= 6\cdot61 - 8\cdot23 = -1\cdot62$

$$\begin{array}{r} 6\cdot25 \\ +0\cdot36 \\ \hline 6\cdot61 \\ \hline \end{array} \qquad \begin{array}{r} 17\cdot20 \\ -\ \ 8\cdot97 \\ \hline 8\cdot23 \\ \hline \end{array} \qquad \begin{array}{r} 8\cdot23 \\ -\ \ 6\cdot61 \\ \hline -1\cdot62 \\ \hline \end{array}$$

(iii) $9\cdot004 + (3 - 2\cdot462)$
$= 9\cdot004 + 0\cdot538 = 9\cdot542$

$$\begin{array}{r} 3\cdot000 \\ -2\cdot462 \\ \hline 0\cdot538 \\ \hline \end{array}$$

(iv) $879\cdot4 - (87\cdot94 - 8\cdot794)$
$= 879\cdot4 - 79\cdot146 = 800\cdot254$

$$\begin{array}{r} 87\cdot940 \\ -\ \ 8\cdot794 \\ \hline 79\cdot146 \\ \hline \end{array} \qquad \begin{array}{r} 879\cdot400 \\ -\ \ 79\cdot146 \\ \hline 800\cdot254 \\ \hline \end{array}$$

Question 11.
What is the excess of 75 over 48.29?
Answer:

Excess of 75 over 48.29

$$\begin{array}{r} 75.00 \\ -48.29 \\ \hline 26.71 \\ \hline \end{array}$$

$\therefore$ Excess of 75 over 48.29 is 26.71

Question 12.
If A = 237.98 and B = 83.47.
Find :
(i) A – B
(ii) B – A.
Answer:

(i) A – B

A = 237.98

B = 83.47

$$\begin{array}{r} 237.98 \\ -83.47 \\ \hline 154.51 \\ \hline \end{array}$$

$\Rightarrow$ A – B = 154.51

(ii) B – A

= 83.47 – 237.98 = –154.51

Question 13.
The cost of one kg of sugar increases from ?28.47 to T32.65. Find the increase in cost.

Answer:

Initial cost of sugar = ₹28.47

Increase cost of sugar = ₹32.65

$\therefore$ Increase of sugar in cost = ₹5.18

$$\begin{array}{r} 32.65 \\ -28.47 \\ \hline 5.18 \\ \hline \end{array}$$

EXERCISE 4 (C)

Question 1.
Multiply:
(i) 0.87 by 10
(ii) 2.948 by 100
(iii) 6.4 by 1000
(iv) 5.8 by 4
(v) 16.32 by 28
(vi) 5. 037 by 8
(vi) 4.6 by 2.1
(viii) 0.568 by 6.4

Answer:

(*i*) $0{\cdot}87 \times 10 = 8{\cdot}7$

(*ii*) $2{\cdot}948 \times 100 = 294{\cdot}8$

(*iii*) $6{\cdot}4 \times 1000 = 6400$

(*iv*) $5{\cdot}8 \times 4 = 23{\cdot}2$

(*v*) $16{\cdot}32 \times 28 = 456{\cdot}96$

$$\begin{array}{r} 16{\cdot}32 \\ \times \quad 28 \\ \hline 130{\cdot}56 \\ 326{\cdot}40 \\ \hline 456{\cdot}96 \\ \hline \end{array}$$

(*vi*) $5{\cdot}037 \times 8 = 40{\cdot}296$

(*vii*) $4{\cdot}6 \times 2{\cdot}1 = 9{\cdot}66$

$$\begin{array}{r} 4{\cdot}6 \\ \times 2{\cdot}1 \\ \hline 46 \\ 92\times \\ \hline 9{\cdot}66 \\ \hline \end{array}$$

(*viii*) $0{\cdot}568 \times 6{\cdot}4 = 3{\cdot}6352$

$$\begin{array}{r} 0{\cdot}568 \\ \times \quad 6{\cdot}\ 4 \\ \hline 2272 \\ 34080 \\ \hline 3{\cdot}6352 \\ \hline \end{array}$$

Question 2.
Multiply each number by 10, 100, 1000 :
(i) 0.5
(ii) 0.112
(iii) 4.8
(iv) 0.0359
(v) 16.27
(vi) 234.8

Answer:
(i) 0.5 x 10 = 5,0.5 x 100 = 50,
0.5 x 1000 = 500
(ii) 0.112 x 10= 1.12,0.112 x 100
= 11.2, 0.112 x 1000= 112
(iii) 4.8 x 10 = 48, 4.8 x 100 = 480,
4.8 x 1000 = 4800
(iv) 0.0359 x 10 = 0.359,0.0359 x 100 = 3.59, 0.0359 x 1000 = 35-9
(v) 16.27 x 10 = 162.7, 16.27 x 100 = 1627, 16.27 x 1000= 16270
(vi) 234.8 x 10 = 2348, 234.8 x 100 = 23480, 234.8 x 1000 = 234800

Question 3.
Evaluate:
(i) 5.897 x 2.3
(ii) 0.894 x 87
(iii) 0.01 x 0.001
(iv) 0.84 x 2.2 x 4
(v) 4.75 x 0.08 x 3
(vi) 2.4 x 3.5 x 4.8
(vii) 0.8 x 1.2 x 0.25
(viii) 0.3 x 0.03 x 0.003
(ix) 12.003 x (0.2)5

Answer:

(*i*) $5{\cdot}897 \times 2{\cdot}3 = 13{\cdot}5631$ **Ans.**

$$\begin{array}{r} 5{\cdot}897 \\ \times\ 2{\cdot}3 \\ \hline 17691 \\ 11794\times \\ \hline 13{\cdot}5631 \\ \hline \end{array}$$

(*ii*) $0{\cdot}894 \times 87 = 77{\cdot}778$ **Ans.**

$$\begin{array}{r} {\cdot}894 \\ \times 87 \\ \hline 6258 \\ 7152\times \\ \hline 77{\cdot}778 \\ \hline \end{array}$$

(*iii*) $0{\cdot}01 \times 0{\cdot}001 = 0{\cdot}00001$ **Ans.**

(*iv*) $0{\cdot}84 \times 2{\cdot}2 \times 4$

$= 0{\cdot}84 \times 8{\cdot}8$

$= 7{\cdot}392$ **Ans.**

$$\begin{array}{r} 84 \\ \times 88 \\ \hline 672 \\ 672\times \\ \hline 7392 \\ \hline \end{array}$$

(*v*) $4{\cdot}75 \times 0{\cdot}08 \times 3 = 4{\cdot}75 \times 0{\cdot}08 \times 3$

$= 4{\cdot}75 \times 0{\cdot}24$

$= 1{\cdot}1400 = 1{\cdot}14$ **Ans.**

$$\begin{array}{r} 4.75 \\ 0.24 \\ \hline 1900 \\ 950\times \\ \hline 1.1400 \\ \hline \end{array}$$

(*vi*) $2{\cdot}4 \times 3{\cdot}5 \times 4{\cdot}8 = 8{\cdot}40 \times 4{\cdot}8$

$= 8{\cdot}4 \times 4{\cdot}8$

$= 40{\cdot}32$ **Ans.**

$$\begin{array}{r} 24 \\ \times 35 \\ \hline 120 \\ 72\times \\ \hline 840 \\ \hline \end{array} \qquad \begin{array}{r} 8{\cdot}4 \\ \times 4\,8 \\ \hline 672 \\ 336\times \\ \hline 4032 \\ \hline \end{array}$$

(*vii*) $0{\cdot}8 \times 1{\cdot}2 \times 0{\cdot}25 = 0{\cdot}96 \times 0{\cdot}25$

$= 0{\cdot}2400$

$= 0{\cdot}24$ **Ans.**

$$\begin{array}{r} 96 \\ \times 25 \\ \hline 480 \\ 192\times \\ \hline 2400 \\ \hline \end{array}$$

(*viii*) $0{\cdot}3 \times 0{\cdot}03 \times 0{\cdot}003$

$= 0{\cdot}009 \times 0{\cdot}003$

$= 0{\cdot}000027$ **Ans.**

(*ix*) $12{\cdot}003 \times (0{\cdot}2)^5$

$= 12{\cdot}003 \times 0{\cdot}2 \times 0{\cdot}2 \times 0{\cdot}2 \times 0{\cdot}2 \times 0{\cdot}2$

$= 12{\cdot}003 \times 0{\cdot}00032 = 0{\cdot}00384096$ **Ans.**

$$\begin{array}{r} 12003 \\ \times \quad 32 \\ \hline 24006 \\ 36009\times \\ \hline 384096 \\ \hline \end{array}$$

Question 4.
Divide :
(i) 54.9 by 10
(ii) 7.8 by 100
(iii) 324.76 by 1000
(iv) 12.8 by 4
(v) 27.918 by 9
(vi) 4.672 by 8
(vii) 4.32 by 1.2
(viii) 7.644 by 1.4
(ix) 4.8432 by 0.08

Answer:

(*i*) $54{\cdot}9 \div 10 = 5{\cdot}49$ **Ans.**

(*ii*) $7{\cdot}8 \div 100 = 0{\cdot}078$ **Ans.**

(*iii*) $324{\cdot}76 \div 1000 = 0{\cdot}32476$ **Ans.**

(*iv*) $12{\cdot}8 \div 4 = 3{\cdot}2$ **Ans.**

(*v*) $27{\cdot}918 \div 9 = 3{\cdot}102$ **Ans.**

(*vi*) $4{\cdot}672 \div 8 = 0{\cdot}584$ **Ans.**

$$\begin{array}{r|l} & 0{\cdot}584 \\ 8 & 4{\cdot}672 \\ & -40 \\ \hline & 67 \\ & -64 \\ \hline & 32 \\ & -32 \\ \hline & \times \end{array}$$

(*vii*) $4{\cdot}32 \div 1{\cdot}2 = 4{\cdot}32 \div 1{\cdot}20$
$= 432 \div 120 = 3{\cdot}6$ **Ans.**

$$\begin{array}{r|l} & 3{\cdot}6 \\ 120 & 432{\cdot}0 \\ & -360 \\ \hline & 720 \\ & -720 \\ \hline & \times \end{array}$$

(*viii*) $7{\cdot}644 \div 1{\cdot}4 = 7{\cdot}644 \div 1{\cdot}400$
$= 7644 \div 1400 = 5{\cdot}46$ **Ans.**

$$\begin{array}{r|l} & 5{\cdot}46 \\ 1400 & 7644{\cdot}00 \\ & -7000 \\ \hline & 6440 \\ & -5600 \\ \hline & 8400 \\ & -8400 \\ \hline & \times \end{array}$$

(ix) $4 \cdot 8432 \div 0 \cdot 08 = 4 \cdot 8432 \div 0 \cdot 0800$

$= 48432 \div 800 = 60 \cdot 54$ **Ans.**

$$
\begin{array}{r|l}
 & 60 \cdot 54 \\
800 & 48432 \cdot 00 \\
 & -4800 \\
\hline
 & 4320 \\
 & -4000 \\
\hline
 & 3200 \\
 & -3200 \\
\hline
 & \times
\end{array}
$$

Question 5.
Divide each of the given numbers by 10, 100, 1000 and 10000
(i) 2.1
(ii) 8.64
(iii) 5-01
(iv) 0.0906
(v) 0.125
(vi) 111.11
(vii) 0.848 x 3
(viii)4.906 x $(0.2)^2$
(ix) $(1.2)^2$ x$(0.9)^2$

Answer:

(i) 2.1 ÷ 10 = 0.21, 2.1 ÷ 100 = 0.021,
2.1 ÷ 1000 = 0.0021
and 2.1 ÷10000 = 0.00021

(ii) 8.64 ÷ 10 = 0-864, 8.64 ÷ 100 = 0-0864,
8.64 ÷ 1000 = 0-00864
and 8.64 ÷ 10000 = 0.000864

(iii)5.01 ÷ 10 = 0.501,
5.01 ÷ 100 = 0.0501,
5.01 ÷1000 = 0.00501,
5.01 ÷ 10000 = 0.000501

(iv) $0.0906 \div 10 = 0.00906$,
$0.0906 \div 100 = 0.000906$,
$0.0906 \div 1000 = 0.0090906$,
$0.0906 \div 10000 = 0.00000906$

(v) 0.125
Now $0.125 + 10 = 0.0125$,
$0.125 \div 100 = 0\text{-}00125$,
$0.125 \div 1000 = 0.000125$,
$0.125 \div 10000 = 0.0000125$

(vi) $111.11 \div 10 = 11.111$,
$111.11 \div 100 = 1.1111$,
$111.11 \div 1000 = 0.11111$,
$111.11 \div 10000 = 0.011111$

(vii) $0.848 \times 3 = 2.544$,
Now $2.544 \div 10 = 0.2544$,
$2.544 \div 100 = 0\text{-}02544$,
$2\text{-}544 \div 1000 = 0\text{-}002544$,
$2\text{-}544 \div 10000 = 0\text{-}0002544$

(viii) $4.906 \times (0.2)^2 = 4.906 \times 0.2 \times 0.2$
$= 4.906 \times 0.04 = 0.19624$
Now $0.19624 + 10 = 0.019624$,
$0.19624 + 100 = 0.0019624$,
$0.19624 + 1000 = 0.00019624$,
$0.19624 + 10000 = 0.000019624$

(ix) $(1.2)^2 \times (0.9)^2 = 1.2 \times 1.2 \times 0.9 \times 0.9 = 1.44 \times 0.81 = 1.1664$
Now $1 .1664 + 10 = 0.11664$,
$1.1664 \div 100 = 0.011664$,
$1.1664 \div 1000 = 0.0011664$,
$1.1664 \div 10000 = 0.00011664$

Question 6.
Evaluate :
(i) 9.75 + 5
(ii) 4.4064 + 4
(iii) 27.69 + 30
(iv) 19.25 + 25
(v) 20.64+ 16
(vi) 3.204 + 9
(vii) 0.125 + 25

(viii) 0.14616 + 72
(ix) 0.6227+ 1300
(x) 257.894+ 0-169
(xi) 6.3 + $(0.3)^2$

Answer:

(*i*) $9{\cdot}75 \div 5 = 1{\cdot}95$ **Ans.**

$$\begin{array}{r|l} & \;\;1{\cdot}95 \\ \hline 5 & \;\;9{\cdot}75 \\ & -5 \\ \hline & \;\;\;47 \\ & \;-45 \\ \hline & \;\;\;\;25 \\ & \;\;-25 \\ \hline & \;\;\;\;\times \\ \hline \end{array}$$

(*ii*) $4{\cdot}4064 \div 4 = 1{\cdot}1016$ **Ans.**

(*iii*) $27{\cdot}69 \div 30 = 0{\cdot}923$ **Ans.**

$$\begin{array}{r|l} & \;\;0{\cdot}923 \\ \hline 30 & \;\;27{\cdot}690 \\ & -270 \\ \hline & \;\;\;\;69 \\ & \;\;-60 \\ \hline & \;\;\;\;\;90 \\ & \;\;\;-90 \\ \hline & \;\;\;\;\;\times \\ \hline \end{array}$$

(*iv*) $19{\cdot}25 \div 25 = 0{\cdot}77$ **Ans.**

$$\begin{array}{r@{\,}l}
 & 0{\cdot}77 \\
25\,\big) & \overline{19{\cdot}25} \\
 & -175 \\ \cline{2-2}
 & 175 \\
 & -175 \\ \cline{2-2}
 & \times \\ \cline{2-2}
\end{array}$$

(*v*) $20{\cdot}64 \div 16 = 1{\cdot}29$ **Ans.**

$$\begin{array}{r@{\,}l}
 & 1{\cdot}29 \\
16\,\big) & \overline{20{\cdot}64} \\
 & -16 \\ \cline{2-2}
 & 46 \\
 & -32 \\ \cline{2-2}
 & 144 \\
 & -144 \\ \cline{2-2}
 & \times \\ \cline{2-2}
\end{array}$$

(*vi*) $3{\cdot}204 \div 9 = 0{\cdot}356$ **Ans.**

$$\begin{array}{r@{\,}l}
 & 0{\cdot}356 \\
9\,\big) & \overline{3{\cdot}204} \\
 & -27 \\ \cline{2-2}
 & 50 \\
 & -45 \\ \cline{2-2}
 & 54 \\
 & -54 \\ \cline{2-2}
 & \times \\ \cline{2-2}
\end{array}$$

(*vii*) $0{\cdot}125 \div 25 = 0{\cdot}005$ **Ans.**

$$\begin{array}{r@{\,}l}
 & 0{\cdot}005 \\
25\,\big) & \overline{0{\cdot}125} \\
 & -125 \\ \cline{2-2}
 & \times \\ \cline{2-2}
\end{array}$$

(*viii*) $0 \cdot 14616 \div 72 = 0 \cdot 00203$ **Ans.**

$$\begin{array}{r|l} & 0\cdot00203 \\ \hline 72 & 0\cdot14616 \\ & -144 \\ \hline & \quad 216 \\ & \quad -216 \\ \hline & \quad \times \\ \hline \end{array}$$

(*ix*) $0 \cdot 6227 \div 1300 = 0 \cdot 000479$

$$\begin{array}{r|l} & 0\cdot000479 \\ \hline 1300 & 0\cdot62270 \\ & -5200 \\ \hline & 10270 \\ & -9100 \\ \hline & \ 11700 \\ & -11700 \\ \hline & \quad \times \\ \hline \end{array}$$

(*x*) $257 \cdot 894 \div 0 \cdot 169$

$= 257894 \div 169 = 1526$

$$\begin{array}{r|l} & 1526 \\ \hline 169 & 257894 \\ & -169 \\ \hline & \ 888 \\ & -845 \\ \hline & \quad 439 \\ & \ -338 \\ \hline & \quad 1014 \\ & \ -1014 \\ \hline & \quad \times \\ \hline \end{array}$$

(*xi*) $6 \cdot 3 \div (0 \cdot 3)^2 = 6 \cdot 3 \div (0 \cdot 3 \times 0 \cdot 3)$

$= 6 \cdot 3 \div (0 \cdot 09) = 630 \div 09$

$= 630 \div 9 = 70$

Question 7.
Evaluate:
(i) 4.3 x 0.52 x 0.3
(ii) 3.2 x 2.5 x 0.7
(iii) 0.8 x 1.5 x 0.6
(iv) 0.3 x 0.3 x 0.3
(v) 1.2 x 1.2 x 0.4
(vi) 0.4 x 0.04 x 0.004
(vii) 0.5 x 0.6 x 0.7
(Viii) 0.5 x 0.06 x 0.007

Answer:

(i) $4.3 \times 0.52 \times 0.3$

$$\begin{array}{r} 0.52 \\ \times 4.3 \\ \hline 156 \\ 208\times \\ \hline 2.236 \\ \times 0.3 \\ \hline 6708 \\ 0000\times \\ \hline 0.6708 \\ \hline \end{array}$$

(Sum of decimal places = 1 + 2 + 1 = 4)

$\therefore$ 4.3 × 0.52 × 0.3 = 0.6708

(*ii*) 3.2 × 2.5 × 0.7

$$\begin{array}{r} 3.2 \\ \times 2.5 \\ \hline 160 \\ 64\times \\ \hline 8.00 \\ \times 0.7 \\ \hline 5600 \\ 000\times \\ \hline 5.600 \\ \hline \end{array}$$

(Sum of decimal places = 1 + 1 + 1 = 3)

$\therefore$ 3.2 × 2.5 × 0.7 = 5.600 or 5.6

(*iii*) 0.8 × 1.5 × 0.6

$$\begin{array}{r} 1.5 \\ \times 0.8 \\ \hline 120 \\ 00\times \\ \hline 1.20 \\ \times 0.6 \\ \hline 720 \\ 000\times \\ \hline 0.720 \\ \hline \end{array}$$

(Sum of decimal places = 1 + 1 + 1 = 3)

$\therefore$ 0.8 × 1.5 × 0.6 = 0.720 or 0.72

(*iv*) 0.3 × 0.3 × 0.3

$$\begin{array}{r} 0.3 \\ \times 0.3 \\ \hline 09 \\ 00\times \\ \hline 0.09 \\ \times 0.3 \\ \hline 0.027 \\ \hline \end{array}$$

(Sum of decimal places = 1 + 1 + 1 = 3)

$\therefore$ 0.3 × 0.3 × 0.3 = 0.027

(*v*) 1.2 × 1.2 × 0.4

$$\begin{array}{r} 1.2 \\ \times 1.2 \\ \hline .24 \\ 12\times \\ \hline 1.44 \\ \times 0.4 \\ \hline 576 \\ 000\times \\ \hline 0.576 \\ \hline \end{array}$$

(Sum of decimal places = 1 + 1 + 1 = 3)

$\therefore$ 1.2 × 1.2 × 0.4 = 0.576

(*vi*) 0.4 × 0.04 × 0.004

$$\begin{array}{r} 0.004 \\ \times 0.04 \\ \hline 0016 \\ 0000\times \\ 0000\times\times \\ \hline 0.00016 \\ \times 0.4 \\ \hline 0.000064 \\ \hline \end{array}$$

(Sum of decimal places = 1 + 2 + 3 = 6)

$\therefore$ 0.4 × 0.04 × 0.004 = 0.000064

(*vii*) 0.5 × 0.6 × 0.7

$$\begin{array}{r} 0.5 \\ \times 0.6 \\ \hline .30 \\ 00\times \\ \hline 0.30 \\ \times 0.7 \\ \hline 210 \\ 000\times \\ \hline 0.210 \\ \hline \end{array}$$

(Sum of decimal places = 1 + 1 + 1 = 3)

$\therefore$ 0.5 × 0.6 × 0.7 = 0.210 or 0.21

(*viii*) 0.5 × 0.06 × 0.007

$$\begin{array}{r} 0.007 \\ \times 0.06 \\ \hline 0.00042 \\ \times 0.5 \\ \hline 0.00021 \\ \hline \end{array}$$

(Sum of decimal places = 1 + 2 + 3 = 5)

$\therefore$ 0.5 × 0.06 × 0.007 = 0.00021

Question 8.
Evaluate:
(i) $(0.9)^2$
(ii) $(0.6)^2$ x 0.5
(iii) 0.3 x $(0.5)^2$
(iv) $(0.4)^3$
(v) $(0.2)^3$ x 5
(vi) $(0.2)^3$ x 0.05

Answer:

(i) $(0.9)^2$
⇒ 0.9 x 0.9 = 0.81
(Sum of decimal places 1 + 1=2)

(ii) $(0.6)^2$ x 0.5
⇒ 0.6 x 0.6 x 0.5
⇒ 0.36 x 0.5 = 0.180 or 0.18
(Sum of decimal places = 1 + 1 + 1 = 3)

(iii) 0.3 x $(0.5)^2$
⇒ 0.3 x 0.5 x 0.5
⇒ 0.3 x 0.25 = 0.075
(Sum of decimal places 1 + 1 + 1 = 3)
(iv) $(0.4)^3$
⇒ 0.4 x 0.4 x 0.4

⇒ 0.16 x 0.4 = 0.064
(Sum of decimal places 1 + 1 + 1 = 3)

(v) $(0.2)^3$ x 5
⇒ 0.2 x 0.2 x 0.2 x 5
⇒ 0.08 x 5 = 0.40 or 0.4
(Sum of decimal places 1 + 1 + 1 = 3)

(vi) $(0.2)^3$ x 0.05
⇒ 0.2 x 0.2 x 0.2 x 0.05
⇒ 0.008 x 0.05 = 0.00040
(Sum of decimal places = 5)

Question 9.
Find the cost of 36.75 kg wheat at the rate of ₹12.80 per kg.
Answer:

Total weight of wheat = 36.75 kg
Cost of 1 kg of wheat = ₹12.80
∴ Cost of 36.75 kg of wheat
= 36.75 × 12.80 = ₹470.40

$$\begin{array}{r} 36.75 \\ \times 12.80 \\ \hline 470.40 \\ \hline \end{array}$$

Question 10.
The cost of a pen is ₹56.15. Find the cost of 16 such pens.
Answer:

Cost of one pen = ₹56.15
∴ Cost of 16 pens
= ₹56.15 × 16 = ₹898.40

$$\begin{array}{r} 56.15 \\ \times 16 \\ \hline 898.40 \\ \hline \end{array}$$

Question 11.
Evaluate:
(i) 0.0072 ÷ 0.06
(ii) 0.621 ÷ 0.3
(iii) 0.0532 ÷ 0.005
(iv) 0.01162 ÷ 0.14
(v) (7.5 x 40.4) ÷ 25
(vi) 2.1 ÷ (0.1 x 0.1)

Answer:

(i) $0.0072 \div 0.06$

$$= \frac{0.0072 \times 100}{0.06 \times 100}$$

$$= \frac{0.72}{6} = 0.12$$

(ii) $0.621 \div 0.3$

$$= \frac{0.621 \times 10}{0.3 \times 10}$$

$$= \frac{6.21}{3} = 2.07$$

(iii) $0.0532 \div 0.005$

$$= \frac{0.0532 \times 1000}{0.005 \times 1000} = \frac{53.2}{5} = 10.64$$

(iv) $0.01162 \div 0.14$

$$= \frac{0.01162 \times 100}{0.14 \times 100}$$

$$= \frac{1.162}{14} = 0.083$$

(v) $(7.5 \times 40.4) \div 25$

$$= \frac{303}{25} = 12.12$$

(vi) $2.1 \div (0.1 \times 0.1)$

$$= \frac{2.1 \times 100}{0.01 \times 100} = \frac{210}{1} = 210$$

Question 12.
Fifteen indentical articles weigh 31.50 kg. Find the weight of each article.
Answer:
Weight of 15 articles = 31.50 kg
∴ Weight of one article
= 31.50- 15 = 2.1 kg

Question 13.
The product of two numbers is 211.2. If one of these two numbers is 16.5, find the other number.
Answer:

The product of two numbers = 211.2

One number = 16.5

∴ Second number = 211.2 ÷ 16.5

$$= \frac{211.2 \times 10}{16.5 \times 10}$$

$$= \frac{2112}{165} = 12.8$$

Question 14.
One dozen identical articles cost ₹45.96. Find the cost of each article.
Answer:
∴ Weight of one dozen articles = ₹45.96
One dozen = 12
∴ Cost of one article = 45.96 + 12 = ₹3.83

EXERCISE 4 (D)

Question 1.
Find whether the given division forms a terminating decimal or a non-terminating decimal:
(i) 3 ÷ 8
(ii)8 ÷ 3
(iii) 6÷ 5
(iv) 5 ÷ 6
(v) 12.5 ÷ 4
(vi) 23 ÷ 0.7
(vii) 42 ÷ 9
(viii) 0.56÷ 0.11

Answer:

(*i*) $3 \div 8 = 0{\cdot}375$

Hence it is terminating decimal.

$$
\begin{array}{r|l}
 & 0{\cdot}375 \\
\hline
8 & 3{\cdot}00 \\
 & 24 \\
\hline
 & 60 \\
 & 56 \\
\hline
 & 40 \\
 & 40 \\
\hline
 & \times \\
\hline
\end{array}
$$

(*ii*) $8 \div 3 = 2{\cdot}666......$

Hence it is non-terminating decimal.

$$
\begin{array}{r|l}
 & 2{\cdot}666...... \\
\hline
3 & 8{\cdot}000 \\
 & 6 \\
\hline
 & 20 \\
 & 18 \\
\hline
 & 20 \\
 & 18 \\
\hline
 & 20 \\
 & 18 \\
\hline
 & 2 \\
\hline
\end{array}
$$

(*iii*) $6 \div 5 = 1{\cdot}2$

Hence it is terminating decimal.

$$\begin{array}{r|l} & 1{\cdot}2 \\ \hline 5 & 6{\cdot}0 \\ & 5 \\ \hline & 10 \\ & 10 \\ \hline & \times \\ \hline \end{array}$$

(*iv*) $5 \div 6 = 0{\cdot}8333......$

Hence it is non-terminating decimal.

$$\begin{array}{r|l} & 0{\cdot}8333...... \\ \hline 6 & 5{\cdot}0000 \\ & 48 \\ \hline & 20 \\ & 18 \\ \hline & 20 \\ & 18 \\ \hline & 20 \\ & 18 \\ \hline & 2 \\ \hline \end{array}$$

(*v*) $12{\cdot}5 \div 4 = 3{\cdot}125$

Hence it is terminating decimal.

$$\begin{array}{r|l} & 3{\cdot}125 \\ \hline 4 & 12{\cdot}500 \\ & 12 \\ \hline & 5 \\ & 4 \\ \hline & 10 \\ & 8 \\ \hline & 20 \\ & 20 \\ \hline & \times \\ \hline \end{array}$$

(*vi*) $23 \div 0{\cdot}7 = 230 \div 7 = 32{\cdot}8571428......$

Hence it is non-terminating decimal. **Ans.**

$$
\begin{array}{r|l}
 & 32{\cdot}8571428...... \\
7 & 230{\cdot}000000 \\
 & \underline{21} \\
 & 20 \\
 & \underline{14} \\
 & 60 \\
 & \underline{56} \\
 & 40 \\
 & \underline{35} \\
 & 50 \\
 & \underline{49} \\
 & 10 \\
 & \underline{7} \\
 & 30 \\
 & \underline{28} \\
 & 20 \\
 & \underline{14} \\
 & 60 \\
 & \underline{56} \\
 & \underline{4}
\end{array}
$$

(*vii*) $42 \div 9 = 4{\cdot}666......$

Hence it is non-terminating decimal. **Ans.**

$$
\begin{array}{r|l}
 & 4{\cdot}666...... \\
9 & 42{\cdot}000 \\
 & \underline{36} \\
 & 60 \\
 & \underline{54} \\
 & 60 \\
 & \underline{54} \\
 & 60 \\
 & \underline{54} \\
 & \underline{6}
\end{array}
$$

(*viii*) $0{\cdot}56 \div 0{\cdot}11 = 56 \div 11 = 5{\cdot}0909......$

Hence it is non-terminating decimal. **Ans.**

$$\begin{array}{r|l} & 5{\cdot}0909...... \\ 11 & 56{\cdot}0000 \\ & \underline{55} \\ & \quad 100 \\ & \quad \underline{99} \\ & \qquad 100 \\ & \qquad \underline{99} \\ & \qquad\quad 1 \end{array}$$

Question 2.

Express as recurring decimals :

(*i*) $1\frac{1}{3}$ (*ii*) $\frac{10}{11}$

(*iii*) $\frac{5}{6}$ (*iv*) $\frac{2}{13}$

(*v*) $\frac{1}{9}$ (*vi*) $\frac{17}{90}$

(*vii*) $\frac{5}{18}$ (*viii*) $\frac{7}{12}$

Answer:

(i) $1\frac{1}{3} = \frac{4}{3} = 1 \cdot 333......$

$= 1 \cdot \bar{3}$ **Ans.**

$$\begin{array}{r} 1\cdot333...... \\ 3\,\overline{)\,4\cdot000} \\ \underline{3} \\ 10 \\ \underline{9} \\ 10 \\ \underline{9} \\ 10 \\ \underline{9} \\ \underline{1} \end{array}$$

(ii) $\frac{10}{11} = 0 \cdot 909090......$

$= 0 \cdot \overline{90}$ **Ans.**

$$\begin{array}{r} 0\cdot909090...... \\ 11\,\overline{)\,10\cdot0000000} \\ \underline{99} \\ 100 \\ \underline{99} \\ 100 \\ \underline{99} \\ 100 \\ \underline{99} \\ \underline{1} \end{array}$$

(*iii*) $\frac{5}{6} = 0.8333.....$

$6\overline{)5.0000}(0.8333$

48

20

18

20

18

20

18

2

$= 0.8\overline{3}$

(*iv*) $\frac{2}{13} = 0{\cdot}153846153846... = 0{\cdot}\overline{153846}$

$0{\cdot}153846153846......$

$13\overline{)2{\cdot}000000000000}$

13

70

65

50

39

110

104

60

52

80

78

20

13

70

65

50

39

110

104

60

52

80

78

2

(*v*) $\frac{1}{9} = 0{\cdot}1111......$

$= 0{\cdot}\overline{1}$ **Ans.**

$$
\begin{array}{r|l}
 & 0{\cdot}1111...... \\
9 & 1{\cdot}0000 \\
 & 9 \\
\hline
 & 10 \\
 & 9 \\
\hline
 & 10 \\
 & 9 \\
\hline
 & 10 \\
 & 9 \\
\hline
 & 1
\end{array}
$$

(*vi*) $\frac{17}{90} = 0{\cdot}1888......$

$= 0{\cdot}1\overline{8}$ **Ans.**

$$
\begin{array}{r|l}
 & 0{\cdot}1888...... \\
90 & 17{\cdot}0000 \\
 & 90 \\
\hline
 & 800 \\
 & 720 \\
\hline
 & 800 \\
 & 720 \\
\hline
 & 800 \\
 & 720 \\
\hline
 & 80
\end{array}
$$

(vii) $\frac{5}{18}$

$= 0.2777.....$

$$
\begin{array}{l}
18\overline{)5.000}(0.2777 \\
\underline{\;36\;\;\;} \\
\;\;140 \\
\;\;\underline{126\;} \\
\;\;\;\;140 \\
\;\;\;\;\underline{126} \\
\;\;\;\;\;\underline{14}
\end{array}
$$

$= 0.2\overline{7}$

(viii) $\frac{7}{12}$

$= 0.58333.....$

$$
\begin{array}{l}
12\overline{)7.0000}(0.58333 \\
\underline{\;60\;\;\;} \\
\;100 \\
\;\underline{\;96\;} \\
\;\;\;40 \\
\;\;\underline{\;36\;} \\
\;\;\;\;40 \\
\;\;\;\underline{\;36\;} \\
\;\;\;\;\;40 \\
\;\;\;\;\underline{\;36} \\
\;\;\;\;\;\underline{\;4\;}
\end{array}
$$

$= 0.58\overline{3}$

Question 3.
Convert into vulgar fraction :
(i) 0.$\bar{3}$
(ii) 0.$\bar{8}$
(iii) 4.$\bar{4}$
(iv) 23.$\bar{7}$

Answer:

$$(i)\ 0.\bar{3} = \frac{3}{9}$$

$$= \frac{3-0}{9} = \frac{3}{9} = \frac{1}{3}$$

$$(ii)\ 0.\bar{8} = \frac{8}{9}$$

$$= \frac{8-0}{9} = \frac{8}{9}$$

$$(iii)\ 4.\bar{4} = \frac{44}{9}$$

$$= \frac{44-4}{9} = \frac{40}{9}$$

$$= 4\frac{4}{9}$$

$$(iv)\ 23.\bar{7} = \frac{237}{9}$$

$$= \frac{237-23}{9} = \frac{214}{9}$$

$$= 23\frac{7}{9}$$

Question 4.
Convert into vulgar fraction :
(i) $0.\overline{35}$
(ii) $2.\overline{23}$
(iii) $1.\overline{28}$
(iv) $5.\overline{234}$

Answer:

$$(i)\ 0\overline{35} = \frac{35}{99}$$

$$= \frac{35-0}{99} = \frac{35}{99}$$

$$(ii)\ 2.\overline{23} = 2 + 0.\overline{23}$$

$$= 2 + \frac{23-0}{99}$$

$$= 2 + \frac{23}{99} = 2\frac{23}{99}$$

$$(iii)\ 1.\overline{28} = 1 + 0.\overline{28}$$

$$= 1 + \frac{28-0}{99}$$

$$= 1 + \frac{28}{99} = 1\frac{28}{99}$$

$$(iv)\ 5.\overline{234} = 5 + 0.\overline{234}$$

$$= 5 + \frac{234-0}{999} = 5\frac{234}{999}$$

Question 5.
Convert into vulgar fraction :
(i) 0 $0.3\overline{7}$
(ii) $0.2\overline{45}$
(iii) $0.68\overline{5}$
(iv) $0.4\overline{42}$

Answer:

$$(i)\ 0.3\overline{7} = \frac{37-3}{90}$$

$$= \frac{34}{90} = \frac{17}{45}$$

$$(ii)\ 0.2\overline{45} = \frac{245-2}{990}$$

$$= \frac{243}{990} = \frac{81}{330}$$

$$= \frac{27}{110}$$

$$(iii)\ 0.68\overline{5} = \frac{685-68}{900}$$

$$= \frac{617}{900}$$

$$(iv)\ 0.4\overline{42} = \frac{442-4}{990}$$

$$= \frac{438}{990} = \frac{219}{495}$$

EXERCISE 4 (E)

Question 1.
Round off:
(i) 0 .07, 0.112, 3.59, 9.489 to the nearest tenths.
(ii) 0.627, 100.479, 0 065 and 0.024 to the nearest hundredths.
(iii) 4.83,0.86,451 .943 and 9.08 to the nearest whole number.

Answer:
(i) 0.07 = 0.1,
0.112 = 0.1
3 . 59 = 3.6, 9.489 = 9.5

(ii) 0.627 = 0.63,
100.479 = 100.48
0.065 = 0.07,
0.024 = 0.02
(iii) 4.83 = 5,
0.86= 1,
451.943 = 452
9.08 = 9

Question 2.
Simplify, and write your answers correct to the nearest hundredths :
(i) 18 .35 x 1.2
(ii) 62.89 x 0.02

Answer:

(*i*) $18 \cdot 35 \times 1 \cdot 2 = 22 \cdot 02$

$$\begin{array}{r} 18\cdot35 \\ \times 1\cdot2 \\ \hline 36\cdot70 \\ 1835\times \\ \hline 22\cdot020 \\ \hline \end{array}$$

(*ii*) $62 \cdot 89 \times 0 \cdot 02 = 1 \cdot 2578 = 1 \cdot 26$

$$\begin{array}{r} 62\cdot89 \\ \times 0\cdot02 \\ \hline 1\cdot2578 \\ \hline \end{array}$$

Question 3.
Write the number of significant figures (digits) in:
(i) 35.06
(ii) 0.35
(iii) 7.0068
(iv) 19 .0
(v) 0.0062
(vi) 0 4.2 x 0.6
(vii) 0.08 x 25
(viii) 3.6 ÷ 0.12 .

Answer:
(i) 35.06 : In this significant figures i.e. digits are 4
(ii) In 0.35, significant figures are 2
(iii) In 7.0068, significant figures are 5
(iv) In 19.0, significant figures are 3
(v) In 0.0062, significant figures are 2
(vi) In 4.2 x 0.6 = 2.52, significant figure are 3
(vii) In 008 x 25 = 2.00 = 2 significant figure is 1
(viii) In 3.6 ÷0 .12 or 360 ÷ 12 = 30, significant figure are 2.

Question 4.
Write :
(i) 35.869,0 008426,4.952 and 382.7, correct lo three significant figures.
(ii) 60.974. 2.8753, 0.001789 and 400.04, correct to four significant figures.
(iii) 14.29462, 19.2, 46356.82 and 69, correct to five significant figures.

Answer:
(i) Correct to three significant figures are
35.869 → 35.9
0.008426 →0.00843
4. 952→ 4.95
382.7 →383
(ii) Correct to four significant figures
60.974 →60.97
2. 8753 → 2.875
0.001789 → 0.001789
400.04 → 400.0
(iii) Correct to five significant figures
14.29462→ 14.295
19.2 → 19.200
46356.82 →46357
69 → 69.000

EXERCISE 4 (F)

Question 1.
The weight of an object is 3 .06 kg. Find the total weight of 48 similar objects.

Answer:

Weight of one object = 3·06 kg.

$\therefore$ Weight of 48 objects = 3·06 × 48

= 146·88 kg. **Ans.**

$$\begin{array}{r} 3{\cdot}06 \\ \times\ 48 \\ \hline 2448 \\ 1224\times \\ \hline 14688 \\ \hline \end{array}$$

Question 2.
Find die cost of 17.5 m cloth at the rate of Rs. 112.50 per metre.

Answer:

Cost of 1 metre cloth = Rs. 112·5(

$\therefore$ Cost of 17·5 m cloth

= Rs. 112·50 × 17·5

= Rs. 1968·750

= 1968·75 **Ans.**

$$\begin{array}{r} 112{\cdot}50 \\ 17{\cdot}5 \\ \hline 56250 \\ 78750\times \\ 11250\times\times \\ \hline 1968750 \\ \hline \end{array}$$

Question 3.
One kilogramme of oil costs Rs. 73.40. Find the cost of 9.75 kilogramme of the oil.

Answer:

Cost of 1 kg oil = Rs. 73·40

∴ Cost of 9·75 kg oil = Rs. 73·40 × 9·75

= Rs. 715·6500

= Rs. 715·65 **Ans.**

$$\begin{array}{r} 73{\cdot}40 \\ \times 9{\cdot}75 \\ \hline 36700 \\ 51380\times \\ 66060\times\times \\ \hline 7156500 \\ \hline \end{array}$$

Question 4.
Total weight of 8 identical objects is 51.2 kg. Find the weight of each object.
Answer:
Weight of 8 objects = 51-2 kg
∴ Weight of 1 object = 51 -2 + 8 kg = 6-4 kg Ans.

Question 5.
18.5 m of cloth costs Rs. 666. Find the cost of 3.8 m cloth.

Answer:
Cost of 18.5 m cloth = Rs. 666
Cost of 1 m cloth = Rs. 666 ÷18.5 and cost of 3.8 m cloth
= Rs. (666 ÷18.5) x 3-8 = Rs. (6660 ÷ 185) x 3.8 = Rs. 36 x 3.8 = Rs. 136.80

$$\begin{array}{r|l} & 36 \\ \hline 185 & 6660 \\ & 555 \\ \hline & 1110 \\ & 1110 \\ \hline & \times \\ \hline \end{array}$$

$$\begin{array}{r} 3{\cdot}8 \\ \times 36 \\ \hline 228 \\ 114\times \\ \hline 1368 \\ \hline \end{array}$$

Question 6.
Find die value of:
(i) 0.5 of Rs. 7.60 + 1.62 of Rs. 30
(ii) 2.3 of 7.3 kg + 0.9 of 0.48 kg
(iii) 6.25 of 8.4 – 4.7 of 3.24
(iv) 0.98 of 235 – 0 .09 of 3.2

Answer:

(i) 0·5 of Rs. 7·60 + 1·62 of Rs. 30
= Rs. 3·80 + Rs. 48·60
= Rs. 52·40 **Ans.**

$$\begin{array}{r} 7\cdot60 \\ \times 0\cdot5 \\ \hline 3\cdot800 \\ \hline \end{array} \qquad \begin{array}{r} 1\cdot62 \\ \times 30 \\ \hline 48\cdot60 \\ \hline \end{array}$$

(ii) 2·3 of 7·3 kg + 0·9 of 0·48 kg
= 16·79 kg + 0·432 kg = 17·222 kg **Ans.**

$$\begin{array}{r} 7\cdot3 \\ \times 2\cdot3 \\ \hline 219 \\ 146\times \\ \hline 1679 \\ \hline \end{array} \qquad \begin{array}{r} 0\cdot48 \\ \times 0\cdot9 \\ \hline 0\cdot432 \\ \hline \end{array} \qquad \begin{array}{r} 16\cdot790 \\ +0\cdot432 \\ \hline 17\cdot222 \\ \hline \end{array}$$

(iii) 6·25 of 8·4 – 4·7 of 3·24
= 52·500 – 15·228 = 37·272 **Ans.**

$$\begin{array}{r} 6\cdot25 \\ \times 8\cdot4 \\ \hline 2500 \\ 5000\times \\ \hline 52500 \\ \hline \end{array} \qquad \begin{array}{r} 3\cdot24 \\ \times 4\cdot7 \\ \hline 2268 \\ 1296\times \\ \hline 15228 \\ \hline \end{array}$$

(iv) 0·98 of 235 – 0·09 of 3·2 = 230·30 – 0·288
= 230·012 **Ans.**

$$\begin{array}{r} 230\cdot300 \\ -0\cdot288 \\ \hline 230\cdot012 \end{array} \qquad \begin{array}{r} 3\cdot20 \\ \cdot09 \\ \hline \cdot2880 \end{array} \qquad \begin{array}{r} 235 \\ \times \cdot98 \\ \hline 1880 \\ 2115\times \\ \hline 23030 \end{array}$$

Question 7.
Evaluate:
(i) 5.6 – 1 .5 of 3.4
(ii) 4.8 ÷ 0.04 of 5
(iii) 0.72 of 80 + 0.2
(iv) 0.72 ÷ 80 of 0.2
(v) 6.45 + (3.9 – 1.75)
(vi) 0.12 of(0.104 – 0.02)+ 0.36 x 0.5

Answer:

(i) $5{\cdot}6 - 1{\cdot}5 \text{ of } 3{\cdot}4 = 5{\cdot}6 - 5{\cdot}1$

$$\begin{array}{r} 3{\cdot}4 \\ \times\ 1{\cdot}5 \\ \hline 170 \\ 34\times \\ \hline 5{\cdot}1\,0 \end{array}$$

$= 5{\cdot}6 - 5{\cdot}1 = 0{\cdot}5$ **Ans.**

(ii) $4{\cdot}8 \div 0{\cdot}04 \text{ of } 5 = 4{\cdot}8 \div 0{\cdot}20$

$= 4{\cdot}8 \div 0{\cdot}2 = 48 \div 2 = 24$ **Ans.**

(iii) $0{\cdot}72 \text{ of } 80 \div 0{\cdot}2 = 57{\cdot}60 \div 0{\cdot}2$

$= 57{\cdot}6 \div 0{\cdot}2 = 576 \div 2 = 288$ **Ans.**

(iv) $0{\cdot}72 \div 80 \text{ of } 0{\cdot}2 = 0{\cdot}72 \div 16{\cdot}0$

$= 0{\cdot}72 \div 16 = 72 \div 1600$

$$\begin{array}{r} 0{\cdot}045 \\ 1600\overline{)72,000}(\cdot 045 \\ -6400 \\ \hline 8000 \\ -8000 \\ \hline \times \\ \hline \end{array}$$

$= 0{\cdot}045$ **Ans.**

(v) $6{\cdot}45 \div (3{\cdot}9 - 1{\cdot}75) = 6{\cdot}45 \div (3{\cdot}90 - 1{\cdot}75)$

$= 6{\cdot}45 \div 2{\cdot}15 = 645 \div 215 = 3$ **Ans.**

$$\begin{array}{r} 3 \\ 215\overline{)\ 645}(\\ -645 \\ \hline \times \\ \hline \end{array}$$

(vi) $0{\cdot}12 \text{ of } (0{\cdot}104 - 0{\cdot}02) + 0{\cdot}36 \times 0{\cdot}5$

$= 0{\cdot}12 \text{ of } 0{\cdot}084 + 0{\cdot}36 \times 0{\cdot}5$

$= 0{\cdot}01008 + 0{\cdot}180 = 0{\cdot}19008$ **Ans.**

$$\begin{array}{r} 0{\cdot}104 \\ -\ 0{\cdot}020 \\ \hline 0{\cdot}084 \\ \hline \end{array} \qquad \begin{array}{r} \cdot 084 \\ \times \cdot 12 \\ \hline 0{\cdot}01008 \\ \hline \end{array} \qquad \begin{array}{r} 0{\cdot}01008 \\ +\ 0{\cdot}180 \\ \hline 0{\cdot}19008 \\ \hline \end{array}$$

CHAPTER - 5
EXPONENTS

EXERCISE 5 (A)

Question 1.
Find the value of:
(i) 6^2
(ii) 7^3
(iii) 4^4
(iv) 5^5
(v) 8^3
(vi) 7^5

Solution:
(i) $6^2 = 6 \times 6 = 36$
(ii) $7^3 = 7 \times 7 \times 7 = 343$
(iii) $4^4 = 4 \times 4 \times 4 \times 4 = 256$
(iv) $5^5 = 5 \times 5 \times 5 \times 5 \times 5 = 3125$
(v) $8^3 = 8 \times 8 \times 8 = 512$
(vi) $7^5 = 7 \times 7 \times 7 \times 7 \times 7 = 16807$

Question 2.
Evaluate:
(i) $2^3 \times 4^2$
(ii) $2^3 \times 5^2$
(iii) $3^3 \times 5^2$
(iv) $2^2 \times 3^3$
(v) $3^2 \times 5^2$
(vi) $5^3 \times 2^4$
(vii) $3^2 \times 4^2$
(ix) $(5 \times 4)^2$

Solution:
(i) $2^3 \times 4^2$
$= 2 \times 2 \times 2 \times 4 \times 4$
$= 8 \times 16$
$= 128$
(ii) $2^3 \times 5^2$
$= 2 \times 2 \times 2 \times 5 \times 5$
$= 8 \times 25$
$= 200$
(iii) $3^3 \times 5^2$
$=3 \times 3 \times 3 \times 5 \times 5$
$= 27 \times 25$

= 675
(iv) $2^2 \times 3^3$
= 2 x 2 x 3 x 3 x 3
= 4 x 27
= 108
(v) $3^2 \times 5^3$
=3 x3 x 5 x 5 x 5
= 9 x 125
= 1125
(vi) $5^3 \times 2^4$
= 5 x 5 x 5 x 2 x 2 x 2 x 2
= 125 x 16
= 2000
(vii) $3^2 \times 4^2$
=3 x 3 x 4 x 4
= 9 x 16
=144
(viii) $(4 \times 3)^3$
=4 x 4 x 4 x 3 x 3 x 3
= 64 x 27
= 1728
(ix) $(5 \times 4)^2$
=5 x 5 x 4 x 4
= 25 x 16
= 400

Question 3.
Evaluate:

(i) $\left(\frac{3}{4}\right)^4$ (ii) $\left(-\frac{5}{6}\right)^5$

(iii) $\left(\frac{-3}{-5}\right)^3$

Solution:

(i) $\left(\frac{3}{4}\right)^4 = \left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right)$

$= \frac{3\times3\times3\times3}{4\times4\times4\times4} = \frac{81}{256}$

(ii) $\left(-\frac{5}{6}\right)^5$

$= \left(\frac{-5}{6}\right)\times\left(\frac{-5}{6}\right)\times\left(\frac{-5}{6}\right)\times\left(\frac{-5}{6}\right)\times\left(\frac{-5}{6}\right)$

$= \frac{(-5)\times(-5)\times(-5)\times(-5)\times(-5)}{6\times6\times6\times6\times6}$

$= -\frac{3125}{776}$

(iii) $\left(\frac{-3}{-5}\right)^3 = \left(\frac{-3}{-5}\right) \times \left(\frac{-3}{-5}\right) \times \left(\frac{-3}{-5}\right)$

$= \frac{(-3)\times(-3)\times(-3)}{(-5)\times(-5)\times(-5)}$

$= \frac{27}{125}$

Question 4.
Evaluate :

(i) $\left(\frac{2}{3}\right)^3 \times \left(\frac{3}{4}\right)^2$ (ii) $\left(-\frac{3}{4}\right)^3 \times \left(\frac{2}{3}\right)^4$

(iii) $\left(\frac{3}{5}\right)^2 \times \left(-\frac{2}{3}\right)^3$

Solution:

(i) $\left(\frac{2}{3}\right)^3 \times \left(\frac{3}{4}\right)^2$

$= \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right)$

$= \frac{8}{27} \times \frac{9}{16} = \frac{1}{6}$

(ii) $\left(-\frac{3}{4}\right)^3 \times \left(\frac{2}{3}\right)^4$

$= \left(\frac{-3}{4}\right) \times \left(\frac{-3}{4}\right) \times \left(\frac{-3}{4}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right)$

$\times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right)$

$= \frac{-27}{64} \times \frac{16}{81} = -\frac{1}{2}$

(iii) $\left(\frac{3}{5}\right)^2 \times \left(-\frac{2}{3}\right)^3$

$= \left(\frac{3}{5}\right) \times \left(\frac{3}{5}\right) \times \left(\frac{-2}{3}\right) \times \left(\frac{-2}{3}\right) \times \left(\frac{-2}{3}\right)$

$= \frac{9}{25} \times \left(\frac{-8}{27}\right)$

$= -\frac{8}{75}$

Question 5.
Which is greater :
(i) 2^3 or 3^2
(ii) 2^5 or 5^2
(iii) 4^3 or 3^4
(iv) 5^4 or 4^5

Solution:
(i) 2^3 or 3^3
Since, $2^3 = 2 \times 2 \times 2 = 8$
and, $3^2 = 3 \times 3 = 9$
$\because 9$ is greater than 8 $\Rightarrow 3^2 > 2^3$
(ii) 2^5 or 5^2
Since, $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$
and, $5^2 = 5 \times 5 = 25$
$\because 32$ is greater than 25 $\Rightarrow 2^{35} > 5^{32}$
(iii) 4^3 or 3^4
Since, $4^3 = 4 \times 4 \times 4 = 64$
and, $3^4 = 3 \times 3 \times 3 \times 3 = 81$
$\because$ 81 is greater than 64 $\Rightarrow 3^4 > 4^3$
(iv) 5^4 or 4^5
Since, $5^4 = 5 \times 5 \times 5 \times 5 = 625$
and, $4^5 = 4 \times 4 \times 4 \times 4 \times 4 = 1024$
$\because$ 1024 is greater than 625 $\Rightarrow 4^5 > 5^4$

Question 6.
Express each of the following in exponential form :
(i) 512
(ii) 1250
(iii) 1458
(iv) 3600
(v) 1350
(vi) 1176

Solution:

(i) 512

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^9$

(ii) 1250

2	1250
5	625
5	125
5	25
5	5
	1

$= 2 \times 5 \times 5 \times 5 \times 5 = 2 \times 5^4$

(iii) 1458

2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

$= 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 2 \times 3^6$

(iv) 3600

$$\begin{array}{r|l} 2 & 3600 \\ \hline 2 & 1800 \\ \hline 2 & 900 \\ \hline 2 & 450 \\ \hline 3 & 225 \\ \hline 3 & 75 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$

$= 2^4 \times 3^2 \times 5^2$

(v) 1350

$$\begin{array}{r|l} 2 & 1350 \\ \hline 3 & 675 \\ \hline 3 & 225 \\ \hline 3 & 75 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$= 2 \times 3 \times 3 \times 3 \times 5 \times 5$

$= 2 \times 3^3 \times 5^2$

(vi) 1176

$$\begin{array}{r|l} 2 & 1176 \\ \hline 2 & 588 \\ \hline 2 & 294 \\ \hline 3 & 147 \\ \hline 7 & 49 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$= 2 \times 2 \times 2 \times 3 \times 7 \times 7$

$= 2^3 \times 3 \times 7^2$

Question 7.

If a = 2 and b = 3, find the value of:
(i) $(a + b)^2$
(ii) $(b – a)^3$
(iii) (a x b)a (iv) (a x b)b

Solution:

(i) $(a + b)^2$
$= (2 + 3)^2 = (5)^2 = 5 \times 5 = 25$

(ii) $(b – a)^2$
$= (3 – 2)^2 = (1)^3$
$= 1 \times 1 \times 1 = 1$

(iii) $(a \times b)^a$
$= (2 \times 3)^2 – (6)^2$
$= 6 \times 6 = 36$

(iv) $(a \times b)^b$
$= (2 \times 3)^3 = (6)^3 = 6 \times 6 \times 6 = 216$

Question 8.

Express:
(i) 1024 as a power of 2.
(ii) 343 as a power of 7.
(iii) 729 as a power of 3.

Solution:

(i) 1024 as a power of 2.

2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$= 2^{10}$

(ii) 343 as a power of 7.

$= 7 \times 7 \times 7 = 7^3$

7	343
7	49
7	7
	1

(iii) 729 as a power of 3.

$= 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$

3	729
3	243
3	81
3	27
3	9
3	3
	1

Question 9.

If 27 x 32 = 3^x x 2^y; find the values of x and y.

$27 \times 32 = 3^x \times 2^y$

$27 = 3^x$

3	27
3	9
3	3
	1

Solution:

$27 = 3 \times 3 \times 3$

$= 3^3 = 3^x$

$\therefore\ x = 3^x$

Also, $32 = 2^y$

2	32
2	16
2	8
2	4
2	2
	1

$32 = 2 \times 2 \times 2 \times 2 \times 2$

$= 2^5 = 2^y$

$\therefore\ y = 5$

Question 10.
If 64 x 625 = 2^a x 5^b; find :
(i) the values of a and b.
(ii) 2^b x 5^a

Solution:
(i) the values of a and b.

(i) $64 \times 625 = 2^a \times 5^b$

$64 = 2^a$

2	64
2	32
2	16
2	8
2	4
2	2
	1

$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$64 = 2^6$

$\therefore\ a = 6$

Also, $625 = 5^b$

5	625
5	125
5	25
5	5
	1

$625 = 5 \times 5 \times 5 \times 5$

$625 = 5^4$

$\therefore\ b = 4$

(ii) $2^b \times 5^a$

(ii) $2^b \times 5^a$

$= 2^4 \times 5^6$

$= 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$

$= 16 \times 15625 = 250000$

EXERCISE 5 (B)

Question 1.

Fill in the blanks:

In $5^2 = 25$, base = ……… and index = ……….

If index = 3x and base = 2y, the number = ………

Solution:

(i) In $5^2 = 25$, base = 5 and index = 2

(ii) If index = 3x and base = 2y, the number = $2y^{3x}$

Question 2.

Evaluate:

(i) $2^8 \div 2^3$

(ii) $2^{3 \div} 2^8$

(iii) $(2^6)^0$

(iv) $(3^0)^6$

(v) $8^3 \times 8^{-5} \times 8^4$

(vi) $5^4 \times 5^3 + 5^5$

(vii) $5^4 \div 5^3 \times 5^5$

(viii) $4^4 \div 4^3 \times 4^0$

(ix) $(3^5 \times 4^7 \times 5^8)^0$

Solution:

$$= \frac{4^4}{4^3} = 4^{4-3} = 4^1 = 4$$

(ix) $(3^5 \times 4^7 \times 5^8)^0 = 3^{5 \times 0} \times 4^{7 \times 0} \times 5^{8 \times 0}$

$= 3^0\, 4^0\, 5^0 = 1 \times 1 \times 1 = 1$

(i) $2^8 \div 2^3 = \frac{2^8}{2^3} = 2^{8-3} = 2^5$

(ii) $2^3 \div 2^8 = \frac{2^3}{2^8} = 2^{3-8} = 2^{-5} = \frac{1}{2^5}$

(iii) $(2^6)^0 = 2^{6 \times 0} = 2^0 = 1$

(iv) $(3^0)^6 = 3^{0 \times 6} = 3^0 = 1$

(v) $8^3 \times 8^{-5} \times 8^4 = 8^{3+4-5} = 8^{7-5} = 8^2$

(vi) $5^4 \times 5^3 \div 5^5 = \frac{5^4 \times 5^3}{5^5}$

$= 5^{4+3-5} = 5^{7-5} = 5^2$

(vii) $5^4 \div 5^3 \times 5^5 = \frac{5^4}{5^3} \times 5^5 = 5^{4-3+5} = 5^6$

(viii) $4^4 \div 4^3 \times 4^0 = \frac{4^4}{4^3 4^0} = \frac{4^4}{4^3 \times 1}$

Question 3.

Simplify, giving Solutions with positive index:

(i) $2b^6 \cdot b^3 \cdot 5b^4$ (ii) $x^2y^3 \cdot 6x^5y \cdot 9x^3y^4$

(iii) $(-a^5)(a^2)$ (iv) $(-y^2)(-y^3)$

(v) $(-3)^2 (3)^3$ (vi) $(-4x)(-5x^2)$

(vii) $(5a^2b)(2ab^2)(a^3b)$

(viii) $x^{2a+7} \cdot x^{2a-8}$ (ix) $3^y \cdot 3^2 \cdot 3^{-4}$

(x) $2^{4a} \cdot 3^{3a} \cdot 2^{-a}$ (xi) $4x^2y^2 \div 9x^3y^3$

(xii) $(10^2)^3 (x^8)^{12}$ (xiii) $(a^{10})^{10} (1^6)^{10}$

(xiv) $(n^2)^2 (-n^2)^2$ (xv) $-(3ab)^2 (-5a^2bc^4)^2$

(xvi) $(-2)^2 \times (0)^3 \times (3)^3$

(xvii) $(2a^3)^4 (4a^2)^2$

(xviii) $(4x^2y^3)^3 \div (3x^2y^3)^3$

(xix) $\left(\frac{1}{2x}\right)^3 \times (6x)^2$

(xx) $\left(\frac{1}{4ab^2c}\right)^2 \div \left(\frac{3}{2a^2bc^2}\right)^4$

(xxi) $\frac{(5x^7)^3 \cdot (10x^2)^2}{(2x^6)^7}$

(xxii) $\frac{(7p^2q^9r^5)^2 (4pqr)^3}{(14p^6q^{10}r^4)^2}$

Solution:

(i) $2b^6 \cdot b^3 \cdot 5b^4$

$= 2 \times 5 \times b^{6+3+4} = 10b^{13}$

(ii) $x^2y^3 \cdot 6x^5y \cdot 9x^3y^4$

$= 6 \times 9 \times x^{2+5+3}\, y^{3+1+4} = 54x^{10}y^8$

(iii) $(-a^5)\,(a^2)$

$= (-1 \times a)^5 \times a^2$

$= (-1)^5 \times a^{5+2}$

$= -1 \times a^7 = -a^7$

(iv) $(-y^2)\,(-y^3)$

$= (-1 \times y)^2 \cdot (-1 \times y)^3$

$= (-1)^2 \cdot y^2 \cdot (-1)^3 \times y^3$

$= 1^{2+3} \cdot y^{2+3}$

$= 1^5 y^5 = y^5$

(*v*) $(-3)^2\ (3)^3$

$= (-1 \times 3)^2 \cdot (3)^3$

$= (-1)^2 \times 3^2 \cdot 3^3$

$= -1^2 \cdot 3^{2+3} = 1 \cdot 3^5 = 3^5$

(*vi*) $(-4x)\ (-5x^2)$

$= (-1 \times 4 \times x) \cdot (-1 \times 5 \times x^2)^1$

$= -1 \times 4 \times x \cdot -1 \times 5 \times x^2$

$= -1 \times -1 \times 4 \times 5 \times x^{1+2}$

$= -1^{1+1} \cdot 4^1 \cdot 5^1\ x^3 = 20x^3$

(*vii*) $(5a^2b)\ (2ab^2)\ (a^3b)$

$= 5 \cdot 2 \cdot a^{2+1+3} b^{1+2+1} = 10a^6b^4$

(*viii*) $x^{2a+7} \cdot x^{2a-8}$

$= x^{2a+7+2a-8} = x^{4a-1}$

(*ix*) $3^y \cdot 3^2 \cdot 3^{-4}$

$$= 3^y \cdot \frac{3^2}{3^4} = 3^y = \frac{3 \times 3}{3 \times 3 \times 3 \times 3}$$

$$= 3^y \times \frac{1}{3^2} = 3^{y-2}$$

(*x*) $2^{4a} \cdot 2^{3a} \cdot 2^{-a}$

$= 2^{4a+3a-a} = 2^{7a-a} = 2^{6a}$

(*xi*) $4x^2y^2 \div 9x^3y^3$

$$= \frac{4x^2y^2}{9x^3y^3} = \frac{4x^{2-3}y^{2-3}}{9} = \frac{4x^{-1}y^{-1}}{9}$$

$$= \frac{4}{9xy}$$ (Since index should by positive)

(xii) $(10^2)^3\ (x^8)^{12}$

$= 10^{2 \times 3}\ x^{8 \times 12} = 10^6 x^{96}$

(xiii) $(a^{10})^{10}\ (1^6)^{10}$

$= a^{10 \times 10}\ 1^{6 \times 10} = a^{100}\ 1^{60} = a^{100}$

(xiv) $(n^2)^2\ (-n^2)^2$

$= n^{2 \times 2}\ (-n)^{2 \times 3} = n^4 \times (-n)^6$

$= -n^4 - 1^6\ n^6$

$= -n^{4+6} = -n^{10}$

(xv) $-(3ab)^2\ (-5a^2bc^4)^2$

$= -(3^2a^2b^2) \times (-1)^2 \times 5^2a^{2 \times 2}b^2c^{4 \times 2}$

$= -(3^2a^2b^2)\ (5^2a^4b^2c^8)$

$= -3^25^2a^{2+4}b^{2+2}c^8$

$= -225a^6b^4c^8$

(xvi) $(-2)^2 \times (0)^3 \times (3)^3$

$= 4 \times 0 \times 27 = 0$

(xvii) $(2a^3)^4\ (4a^2)^2$

$= (2a^3)^4\ (2^2a^2)^2$

$= 2^4a^{3 \times 4} \cdot 2^{2 \times 2}\ a^{2 \times 2}$

$= 2^4a^{12} \cdot 2^4a^4$

$= 2^{4+4}a^{12+4}$

$= 2^8a^{16}$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times a^{16}$

$= 256a^{16}$

(xviii) $(4x^2y^3)^3 \div (3x^2y^3)^3$

$$= \frac{4^3x^{2\times3}y^{3\times3}}{3^3x^{2\times3}y^{3\times3}} = \frac{4^3x^6y^9}{3^3x^6y^9} = \frac{4^3}{3^3} = \frac{64}{27}$$

(xix) $\left(\frac{1}{2x}\right)^3 \times (6x)^2$

$$= \frac{1^3}{2^3 \times x^3} \times 6^2 \times x^2$$

$$= \frac{1^3 \times 6^2}{2^3 \times x^{3-2}} = \frac{6^2}{2^3x} = \frac{6 \times 6}{2 \times 2 \times 2 \times x} = \frac{9}{2x}$$

(xx) $\left(\dfrac{1}{4ab^2c}\right)^2 \div \left(\dfrac{3}{2a^2bc^2}\right)^4$

$$= \left(\frac{1}{4ab^2c}\right)^2 \times \left(\frac{2a^2bc^2}{3}\right)^4$$

$$= \frac{1^2}{4^2 a^2 b^{2\times 2}\cdot c^2} \times \frac{2^4 a^{2\times 4}\cdot b^4\cdot c^{2\times 4}}{3^4}$$

$$= \frac{1^2}{3^4} \times a^{8-2} b^{4-4} c^{8-2} \qquad (\because 2^4 = 4^2)$$

$$= \frac{1}{3\times 3\times 3\times 3} a^6 b^0 c^6$$

$$= \frac{1}{81} a^6 c^6 \qquad (\because b^0 = 1)$$

(xxi) $\dfrac{(5x^7)^3\cdot(10x^2)^2}{(2x^6)^7} = \dfrac{5^3 x^{7\times 3}\cdot 10^2\cdot x^{2\times 2}}{2^7\cdot x^{6\times 7}}$

$$= 5^3 \cdot 10^2 \cdot 2^{-7}\, x^{21+4-42}$$

$$= \frac{5^3\times 10^2}{2^7 x^{17}} = \frac{5\times 5\times 5\times 2\times 5\times 2\times 5}{2\times 2\times 2\times 2\times 2\times 2\times 2\cdot x^{17}}$$

$$= \frac{5^5}{2^5 x^{17}} = \frac{3125}{32x^{17}}$$

(xxii) $\dfrac{(7p^2q^9r^5)^2(4pqr)^3}{(14p^6q^{10}r^4)^2}$

$$= \frac{(7^2 p^{2\times 2} q^{9\times 2} r^{5\times 2})(4^3 p^3 q^3 r^3)}{14^2 p^{6\times 2} q^{10\times 2} r^{4\times 2}}$$

$$= \frac{7\times 7 p^4 q^{18} r^{10}\cdot 4\times 4\times 4 p^3 q^3 r^3}{2\times 7\times 2\times 7\times p^{12} q^{20} r^8}$$

$$= p^{4-12+3} q^{18-20+3} r^{10-8+3}\ 4\times 4$$

$$= 16p^{-5}qr^5$$

$$= \frac{16qr^5}{p^5}$$

Question 4.

Simplify and express the Solution in the positive exponent form :

(i) $\dfrac{(-3)^3 \times 2^6}{6 \times 2^3}$ (ii) $\dfrac{(2^3)^5 \times 5^4}{4^3 \times 5^2}$

(iii) $\dfrac{36 \times (-6)^2 \times 3^6}{12^3 \times 3^5}$ (iv) $-\dfrac{128}{2187}$

(v) $\dfrac{a^{-7} \times b^{-7} \times c^5 \times d^4}{a^3 \times b^{-5} \times c^{-3} \times d^8}$

(vi) $(a^3b^{-5})^{-2}$

Solution:

(i) $\dfrac{(-3)^3 \times 2^6}{6 \times 2^3} = \dfrac{(-3)^3 \times 2^6}{2 \times 3 \times 2^3} = \dfrac{(-3)^3 \times 2^6}{3 \times 2^{3+1}}$

$= -(3)^{3-1}\, 2^{6-4} = -(3)^2 2^2 = -3^2 2^2$

(ii) $\dfrac{(2^3)^5 \times 5^4}{4^3 \times 5^2} = \dfrac{2^{3\times5} \times 5^4}{2^3 \times 2^2 \times 5^2}$

$= \dfrac{2^{15} \times 5^4}{2^6 \times 5^2} = 2^{15-6} \times 5^{4-2}$

$= 2^9 \times 5^2$

(iii) $\dfrac{36 \times (-6)^2 \times 3^6}{12^3 \times 3^5} = \dfrac{6 \times 6 \times (-6)^2 \times 3^6}{3^3 \times 4^3 \times 3^5}$

$= \dfrac{(6)^2(-6)^2 \times 3^{6-3-5}}{4^3} = \dfrac{(6)^2(-6)^2 3^{-2}}{4^3}$

$= \dfrac{6^2(-6)^2}{3^2 \times 4^3} = \dfrac{6 \times 6 \times -6 \times -6}{3 \times 3 \times 4 \times 4 \times 4}$

$= \dfrac{9}{4} = \left(\dfrac{3}{2}\right)^2$

(iv) $-\dfrac{128}{2187}$

$$\begin{array}{r|l} 2 & 128 \\ \hline 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 3 & 2187 \\ \hline 3 & 729 \\ \hline 3 & 243 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$= -\frac{2\times2\times2\times2\times2\times2\times2}{3\times3\times3\times3\times3\times3\times3} = -\frac{2^7}{3^7}$$

(v) $\dfrac{a^{-7}\times b^{-7}\times c^5\times d^4}{a^3\times b^{-5}\times c^{-3}\times d^8}$

$= a^{-7-3}\times b^{-7+5}\times c^{5-(-3)}\times d^{4-8}$

$= a^{-10}\times b^{-2}\times c^8\times d^{-4}$

$$= \frac{c^8}{a^{10}\times b^2\times d^4}$$

(vi) $(a^3b^{-5})^{-2} = a^{3\times-2}\, b^{-5\times-2}$

$$= a^{-6}\, b^{10} = \frac{b^{10}}{a^6}$$

Question 5.

Evaluate

(i) $6^{-2}\div(4^{-2}\times3^{-2})$

(ii) $\left[\left(\frac{5}{6}\right)^2\times\frac{9}{4}\right]\div\left[\left(-\frac{3}{2}^2\right)\times\frac{125}{216}\right]$

(iii) $5^3\times3^2+(17)^0\times7^3$

(iv) $2^5\times15^0+(-3)^3-\left(\frac{2}{7}\right)^{-2}$

(v) $(2^2)^0+2^{-4}\div2^{-6}+\left(\frac{1}{2}\right)^{-3}$

(vi) $5^n\times25^{n-1}\div(5^{n-1}\times25^{n-1})$

Solution:

(i) $6^{-2} \div (4^{-2} \times 3^{-2})$

$$= \left(\frac{1}{6}\right)^2 \div \left(\frac{1}{4}\right)^2 \times \left(\frac{1}{3}\right)^2$$

$$= \frac{1}{36} \div \frac{1}{16} \times \frac{1}{9}$$

$$= \frac{1}{36} \div \frac{1}{144}$$

$$36\overline{)144}(4$$
$$\underline{144}$$
$$\underline{\times}$$

$$= \frac{1}{36} \times \frac{144}{1} = 4$$

(ii) $\left[\left(\frac{5}{6}\right)^2 \times \frac{9}{4}\right] \div \left[\left(-\frac{3}{2}^2\right) \times \frac{125}{216}\right]$

$$= \left[\left(\frac{5\times5}{6\times6}\right)\times\frac{9}{4}\right] \div \left[\left(\frac{-3\times-3}{2\times2}\right)\times\frac{125}{216}\right]$$

$$= \left[\frac{25}{36}\times\frac{9}{4}\right] \div \left[\frac{9}{4}\times\frac{125}{216}\right]$$

$$= \left[\frac{25}{4}\times\frac{1}{4}\right] \div \left[\frac{1}{4}\times\frac{125}{24}\right]$$

$$= \left[\frac{25}{16}\right] \div \left[\frac{125}{96}\right]$$

$$= \frac{25}{16} \times \frac{96}{125}$$

$$= \frac{1}{1} \times \frac{6}{5} = 1\frac{1}{5}$$

(iii) $5^3 \times 3^2 + (17)^0 \times 7^3$

$= 5 \times 5 \times 5 \times 3 \times 3 + (17)^0 \times 7 \times 7 \times 7$

$(\because a^0 = 1)$

$= 125 \times 9 + 1 \times 343$

$= 1125 + 343 = 1468$

(iv) $2^5 \times 15^0 + (-3)^3 - \left(\frac{2}{7}\right)^{-2}$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 1 + (-3) \times (-3) \times$

$(-3) - \left(\frac{7}{2}\right) \times \left(\frac{7}{2}\right)$

$$= 32 \times 1 - 27 - \frac{49}{4} \qquad (\because a^0 = 1)$$

$$= \frac{32\times4}{1\times4} - \frac{27\times4}{1\times4} - \frac{49}{4\times1} \qquad (\because \text{LCM} = 4)$$

$$= \frac{128 - 108 - 49}{4} = \frac{-29}{4} = -7\frac{1}{4}$$

(*v*) $(2^2)^0 + 2^{-4} \div 2^{-6} + \left(\frac{1}{2}\right)^{-3}$

$$(4)^0 + \left(\frac{1}{2}\right)^4 \div \left(\frac{1}{2}\right)^6 + \left(\frac{2}{1}\right)^3 \quad (\because a^0 = 1)$$

$$1 + \left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right) \div$$

$$\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right) + \left(\frac{2}{1}\times\frac{2}{1}\times\frac{2}{1}\right)$$

$$1 + \left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times 2\times 2\times 2\times 2\times 2\times 2\right) + 8$$

$$= 1 + 4 + 8 = 13$$

(*vi*) $5^n \times 25^{n-1} \div (5^{n-1} \times 25^{n-1})$

$$= 5^n \times 25^{n-1} \times \frac{1}{5^{n-1} \times 25^{n-1}}$$

$$= 5^n \times \frac{1}{5^{n-1}} = 5^{n-n+1} = 5^1$$

Question 6.
If $m^2 = -2$ and $n = 2$; find the values of:
(i) $m + r^2 - 2mn$
(ii) $m^n + n^m$
(iii) $6m^{-3} + 4n^2$
(iv) $2n^3 - 3m$

Solution:

(i) $m^2 + n^2 - 2mn$

$m = -2, n = 2$

$= (-2)^2 + (2)^2 - 2(-2)(2)$

$= 4 + 4 - (-8)$

$$\begin{array}{r|l} 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

$= 8 + 8 = 16 = (2^4)$

(ii) $m^n + n^m$

$m = -2, n = 2$

$= (-2)^2 + (2)^{-2}$

$= 4 + \frac{1}{2} \times \frac{1}{2}$

$= \frac{4 \times 4}{1 \times 4} + \frac{1}{4}$

$= \frac{16+1}{4} = \frac{17}{4} = 4\frac{1}{4}$

(iii) $6m^{-3} + 4n^2$

$m = -2, n = 2$

$= 6(-2)^{-3} + 4(2)^2$

$= 6 \times \frac{1}{-2} \times \frac{1}{-2} \times \frac{1}{-2} + 4 \times 2 \times 2$

$= \frac{-3}{4} + 16$

$= \frac{-3+16\times 4}{4} = \frac{-3+64}{4} = \frac{61}{4} = 15\frac{1}{4}$

(iv) $2n^3 - 3m$

$m = -2, n = 2$

$= 2(2)^3 - 3(-2)$

$= 2 \times (2 \times 2 \times 2) - 3 \times (-2)$

$= 16 - 3 \times (-2)$

$= 16 + 6 = 22$

CHAPTER - 6
RATIO & PROPORTION

POINTS TO REMEMBER

1. **Ratio**
   A ratio is a method to compare two quantities of the same kind with same unit; by dividing the first quantity by the second. The symbol (:) is used for ratio between two quantities e.g. a : b.
   Note:
   (i) A ratio is a pure number and has no unit.
   (ii) A ratio must always be expressed in its lowest terms in simplest form.
   (iii) If each term of a ratio is multiplied or divided by the same number or quantity, the ratio remains the same.
2. **Proportion :**
   Proportion is equality of two ratios : e.g. a : b = c : d
   i.e. Ratio between first and second is equal to ratio between third and fourth term.
   (ii) a and d are called extreme terms and b and c are called mean terms
   and a x d = b x c
   (iii) Fourth term is called fourth proportional.
3. **Continued Proportion**
   Three quantities are called in continued proportion if the ratio between first and second is equal to the ratio between second and third i. e.
   a, b, c are in continued proportion if a : b = b : c
   b the middle term is called the mean proportional between a and c and c, the third term is called the third proportional to a and b.

EXERCISE 6 (A)

Question 1.
Express each of the given ratio in its simplest form :

(i) 22 : 66 (ii) 1·5 : 2·5 (iii) $6\frac{1}{4} : 12\frac{1}{2}$
(iv) 40 kg : 1 quintal (v) 10 paise : ₹ 1
(vi) 200 m : 5 km (vii) 3 hours : 1 day
(viii) 6 months : $1\frac{1}{3}$ years (ix) $1\frac{1}{3} : 2\frac{1}{4} : 2\frac{1}{2}$

Answer:

$$(i)\ 22 : 66 = \frac{22}{66} = \frac{22 \div 22}{66 \div 22} = \frac{1}{3}$$

(HCF of 22 and 66 = 22)

$$= 1 : 3$$

$$(ii)\ 1{\cdot}5 : 2{\cdot}5 = \frac{1{\cdot}5}{2{\cdot}5} = \frac{15}{25} = \frac{15 \div 5}{25 \div 5} = \frac{3}{5}$$

(HCF of 15, 25 = 5)

$$= 3 : 5$$

$$(iii)\ 6\frac{1}{4} : 12\frac{1}{2} = \frac{25}{4} : \frac{25}{2} = \frac{25}{4} \times \frac{2}{25}$$

$$= \frac{2}{4} = \frac{1}{2} = 1 : 2$$

(iv) 40 kg : 1 quintal = 40 kg : 100 kg

(1 quintal = 100 kg)

$$= \frac{40}{100} = \frac{40 \div 20}{100 \div 20} = \frac{2}{5}$$

(HCF of 40, 100 = 20)

$$= 2 : 5$$

(v) 10 paise : 1 rupee = 10 paise : 100 paise

(1 Re = 100 Paise)

$$= \frac{10}{100} = \frac{1}{10} = 1 : 10$$

(vi) 200 m : 5 km = 200 m : 5000 m

(1 km = 1000 m)

$$= \frac{200}{5000} = \frac{200 \div 200}{5000 \div 200}$$ (HCF of 200, 5000 = 200)

$$= \frac{1}{25} = 1 : 25$$

(vii) 3 hours : 1 day = 3 hours : 24 hours

(1 day = 24 hours)

$$= \frac{3}{24} = \frac{1}{8} = 1 : 8$$

(viii) 6 months : $1\frac{1}{3}$ years = 6 months : $\frac{4}{3} \times 12$ months

= 6 months : 16 months

$$= \frac{6}{16} = \frac{6 \div 2}{16 \div 2} = \frac{3}{8} = 3 : 8$$

(ix) $1\frac{1}{3} : 2\frac{1}{4} : 2\frac{1}{2} = \frac{4}{3} : \frac{9}{4} : \frac{5}{2}$

$$= \frac{16:27:30}{12} \quad \text{(LCM of 3, 4, 2 = 12)}$$

= 16 : 27 : 30

Question 2.
Divide 64 cm long string into two parts in the ratio 5 : 3.

Answer:
Sum of ratios = 5 + 3 = 8
∴ first part = $\frac{5}{8}$ of 64 cm = 40 cm
Second part = $\frac{3}{8}$ of 64 cm = 24 cm

Question 3.
Rs. 720 is divided between x and y in the ratio 4:5. How many rupees will each get?

Answer:
Sol. Total amount = Rs. 720 Ratio between x, y = 4 : 5
Sum of ratios = 4 + 5 = 9
x’s share = $\frac{4}{9}$ of Rs. 720 = Rs. 320
y’s share =$\frac{5}{9}$ of Rs. 720 = Rs. 400

Question 4.
The angles of a triangle are in the ratio 3 :2 : 7. Find each angle.

Answer:
Ratio in angles of a triangle = 3:2:7
Sum of ratios = 3 + 2 + 7=12
Sum of angles of a triangle = 180°
∴ First angle = $\frac{3}{12}$x 180°= 45°
Second angle = $\frac{2}{12}$ x 180°= 30°
Third angle = $\frac{7}{12}$ x 180°= 105°

Question 5.
A rectangular field is 100 m by 80 m. Find the ratio of
(i) length to its breadth
(ii) breadth to its perimeter.

Answer:
Length of field (l) = 100 m
Breadth (b) = 80 m
∴Perimeter = 2 (l + b) = 2 (100 + 80) m = 2 x 180 = 360 m
(i) Ratio between length and breadth
= 100 : 80 = 5 : 4
(Dividing by 20, the HCF of 100 and 80)

(ii) Ratio between breadth and its perimeter
= 80 : 360 = 2 : 9
(Dividing by 40, the HCF of 80 and 360)

Question 6.
The sum of three numbers, whose ratios are 3 $\frac{1}{3}$: 4 $\frac{1}{5}$: 6 $\frac{1}{8}$ is 4917.Find the numbers.

Answer:
Sum of three numbers = 4917

Ratio between them $= 3\frac{1}{3} : 4\frac{1}{5} : 6\frac{1}{8}$

$= \frac{10}{3} : \frac{21}{5} : \frac{49}{8}$

$= \frac{400:504:735}{120}$ (LCM of 3, 5, 8 = 120)

= 400 : 504 : 735

Sum of ratio's = 400 + 504 + 735 = 1639

∴ First number $= \frac{400}{1639}$ of 4917 = 1200

Second number $= \frac{504}{1639}$ of 4917 = 1512

and third number $= \frac{735}{1639}$ of 4917
= 2205

Question 7.
The ratio between two quantities is 3 : the first is Rs. 810, find the second.

Answer:
Ratio between two quantities = 3 : 4
Sum of ratio = 3+4 = 7
∴ Second quantity = Rs. $\frac{810x4}{3}$
= Rs. 270 x 4 = Rs. 1080

Question 8.
Two numbers are in the ratio 5 : 7. Their difference is 10. Find the numbers.

Answer:
Ratio between two numbers = 5:7
Difference = 7-5 = 2
If difference is 2, then first number = 5
and if difference is 10, then first number
= $\frac{5}{2}$ x 10=25
and second number = $\frac{7}{2}$ x 10 = 35

Question 9.
Two numbers are in the ratio 10 : 11. Their sum is 168. Find the numbers.

Answer:
Ratio between two numbers = 10 : 11
Sum of ratios = 10 + 11=21
Total sum = 168
∴first number = $\frac{168}{21}$ x 10 =80
Second number = $\frac{168}{21}$ x 11 =88 Ans.

Question 10.
A line is divided in two parts in the ratio 2.5 : 1.3. If the smaller one is 35T cm, find the length of the line.

Answer:
Ratio between two parts of a line
= 2-5 : 1-3 =25 : 13
Sum of ratios = 25 + 13 = 38
Length of smaller part = 35.1 cm 38
Now length of line = $\frac{38}{13}$ x 35.1 cm
= 38 x 2.7 cm = 102.6 cm

Question 11.
In a class, the ratio of boys to the girls is 7:8. What part of the whole class are girls.

Answer:
Ratio between boys and girls = 7:8
Sum of ratios = 7 + 8 = 15
∴ Girls are $\frac{8}{15}$ of the whole class.

Question 12.
The population of a town is ' 50,000, out of which males are $\frac{1}{3}$ of the whole population. Find the number of females. Also, find the ratio of the number of females to the whole population.

Answer:
Total population = 180,000
Population of males = $\frac{1}{3}$ of 180,000 = 60,000
∴ Population of females = 180,000 – 60,000 = 120,000
Ratio of females to whole population
= 120,000 : 180,000 = 2:3

Question 13.
Ten gram of an alloy of metals A and B contains 7.5 gm of metal A and the rest is metal B. Find the ratio between :
(i) the weights of metals A and B in the alloy.
(ii) the weight of metal B and the weight of the alloy.

Answer:
Total weight of A and B metals = 10 gm A's weight = 7.5 gm B's weight = 10 – 7.5 = 2.5 gm

(i) Ratio between A and B = 7.5 : 2.5
= $\frac{75}{10}$: $\frac{25}{10}$ =3:1

(ii) Ratio between B and total alloy
= 2.5 : 10 = $\frac{25}{10}$: 10
⇒ 25 : 100 = 1 : 4

Question 14.
The ages of two boys A and B are 6 years 8 months and 7 years 4 months respectively. Divide Rs. 3,150 in the ratio of their ages.

Answer:
A’s age = 6 years 8 months
= 6 x 12 + 8 = 72 + 8 = 80 months
B’s age = 7 years 4 months = 7 x 12 + 4 = 84 + 4 = 88 months
∴ Ratio between them = 80 : 88 = 10 : 11
Amount = Rs. 3150
Sum of ratios = 10 + 11 =21
∴A’s share = $\frac{3150x10}{21}$ = 1500 = Rs. 1500

B’s share = $\frac{3150x11}{21}$ = 1650 = Rs. 1650

Question 15.
Three persons start a business and spend Rs. 25,000; Rs. 15,000 atid Rs. 40,000 respectively. Find the share of each out of a profit of Rs. 14,400 in a year.

Answer:
A’s investment = Rs. 25000
B’s investment = Rs. 15000
C’s investment = Rs. 40000
∴ Ratio between their investment
= 25000 : 15000 : 40000
=5 : 3 : 8
Sum of ratios = 5 + 3 + 8=16 Total profit = ₹ 14400
∴ A’s share = $\frac{14400}{16}$ x 5 = ₹ 4500
B’s share = $\frac{14400}{16}$ x 3 = ₹ 2700
C’s share = $\frac{14400}{16}$ x 8 = ₹ 7200

Question 16.
A plot of land, 600 sq m in area, is divided between two persons such that the first person gets three-fifth of what the second gets. Find the share of each.

Answer:

Area of plot of land = 600 sq. meter

Let second's share = x

Then first share = $\frac{3}{5}x$

$\therefore$ Ratio between them

$\frac{3}{5}x : x$

$\Rightarrow \frac{3}{5} : 1 = 3 : 5$

Sum of ratios = 3 + 5 = 8

$\therefore$ Share of first person = $\frac{600}{8} \times 3$

= 225 sq. m

and second share = $\frac{600}{8} \times 5 = 375$ sq. m

Question 17.
Two poles of different heights are standing vertically on a horizontal field. At a particular time, the ratio between the lengths of their shadows is 2 :3. If the height of the smaller pole is 7.5 m, find the height of the other pole.

Answer:

Ratio between the shadows of two poles

= 2 : 3

$\therefore$ Height of smaller pole = 7.5 m

Height of taller pole = $\frac{7.5 \times 3}{2}$

$= \frac{22.5}{2} = 11.25$ m

Question 18.
Two numbers are in the ratio 4 : 7. If their L.C.M. is 168, find the numbers.

Answer:
Given, Ratio in two numbers = 4:7
and their L.C.M. = 168
Let first number = 4x
and second number = 7x
Now, L.C.M. of 4x and 7x
= 4 x 7 x x = 28x
∴ 28x = 168
x = $\frac{168}{28}$
x = 6
∴ Required numbers = 4x and 7x = 4 x 6 = 24 and 7 x 6 = 42

Question 19.
is divided between A and B in such a way that A gets half of B. Find :
(i) the ratio between the shares of A and B.
(ii) the share of A and the share of B.

Answer:
Total amount to be divided between A and B = ₹300

(*i*) A gets half of B

Hence, ratio between A and B = $\frac{1}{2}$

= 1 : 2

(*ii*) Sum of ratios = 1 + 2 = 3

∴ A' shares = $\frac{300 \times 1}{3}$ = ₹100

∴ B' shares = $\frac{300 \times 2}{3}$ = ₹200

Question 20.
The ratio between two numbers is 5 : 9. Find the numbers, if their H.C.F. is 16.

Answer:
Let the first number be 5x and second number be 9x
H.C.F. of 5x and 9x = Largest number common to 5x and 9x = x
Given H.C.F. = 16 ⇒ x = 16
∴Required numbers = 5x and 9x = 5×16 and 9×16 = 80 and 144

Question 21.
A bag contains ₹ 1,600 in the form of ₹10 and ₹20 notes. If the ratio between the numbers of ₹10 and ₹20 notes is 2 : 3; find the total number of notes in all.

Answer:
Total amount in the bag = 1600
It contains notes in the denomination of ₹10 and 20
Ratio between the number of ₹10 and 20 notes is = 2 : 3
Let number of ₹10 note = x
and number of ₹20 notes = y
According to condition,

$$10x + 20y = 1600 \quad \ldots(i)$$

and $x = \frac{2}{3}y \quad \ldots(ii)$

Now, substitute the value of x in eq. (i)

$$10 \times \frac{2}{3}y + 20y = 1600$$

$$\Rightarrow \frac{20}{3}y + 20y = 1600$$

$$\Rightarrow \frac{20+60}{3}y = 1600$$

$$\Rightarrow \frac{80}{3}y = 1600$$

$$\Rightarrow y = \frac{1600 \times 3}{80}$$

$\therefore$ $y = 60$

Now, substitute the value of y in eq. (ii), we get

$$x = \frac{2}{3} \times 60 = 40$$

Total number of notes in all = $x + y$
= 60 + 40 = 100 notes

Question 22.
The ratio between the prices of a scooter and a refrigerator is 4 : 1. If the scooter costs ₹45,000 more than the refrigerator, find the price of the refrigerator.

Answer:
Ratio between the prices of scooter and a refrigerator = 4:1
Cost price of scooter = ₹45,000
Let the cost of scooter = 4x
Cost of refrigerator = 1x
According to condition,
Cost of scooter > Cost of refrigerator
⇒ 4x- 1x = 45000
⇒ 3x = 45000
x = $\frac{45000}{3}$
⇒ x = ₹15000
∴ Price of refrigerator = ₹15000

EXERCISE 6 (B)

Question 1.
Check whether the following quantities form a proportion or not ?

(i) $3x$, $7x$, 24 and 56
(ii) $0\cdot8$, 3, $2\cdot4$ and 9
(iii) $1\frac{1}{2}$, $3\frac{1}{4}$, $4\frac{1}{2}$ and $9\frac{3}{4}$
(iv) $0\cdot4$, $0\cdot5$, $2\cdot9$ and $3\cdot5$
(v) $2\frac{1}{2}$, $5\frac{1}{2}$, $3\cdot0$ and $6\cdot0$

Answer:
(i) $3x$, $7x$, 24 and 56
If these are in proportion, then
$3x \times 56 = 7x \times 24$
$\Rightarrow 168x = 168x$
which is true.
Hence $3x$, $7x$, 24 and 56 are in proportion.
(ii) $0\cdot8$, 3, $2\cdot4$ and 9 are in proportion
if $0\cdot8 \times 9 = 3 \times 2\cdot4$
$\Rightarrow 7\cdot2 = 7\cdot2$
which is true
Hence $0\cdot8$, 3, $2\cdot4$ and 9 are in proportion.

(*iii*) $1\frac{1}{2}, 3\frac{1}{4}, 4\frac{1}{2}$ and $9\frac{3}{4}$ are in proportion

if $1\frac{1}{2} \times 9\frac{3}{4} = 3\frac{1}{4} \times 4\frac{1}{2}$

$\Rightarrow \quad \frac{3}{2} \times \frac{39}{4} = \frac{13}{4} \times \frac{9}{2}$

$\Rightarrow \quad \frac{117}{8} = \frac{117}{8}$ which is true.

Hence $1\frac{1}{2}, 3\frac{1}{4}, 4\frac{1}{2}$ and $9\frac{3}{4}$ are in proportion.

(*iv*) 0·4, 0·5, 2·9 and 3·5 are in proportion

if $\quad 0{\cdot}4 \times 3{\cdot}5 = 0{\cdot}5 \times 2{\cdot}9$

$\Rightarrow \quad 1{\cdot}40 = 1{\cdot}45$

which is not true

Hence 0·4, 0·5, 2·9 and 3·5 are not in proportion.

(*v*) $2\frac{1}{2}, 5\frac{1}{2}$, 3·0 and 6·0 are in proportion

if $\quad 2\frac{1}{2} \times 6{\cdot}0 = 5\frac{1}{2} \times 3{\cdot}0$

$\Rightarrow \quad \frac{5}{2} \times 6{\cdot}0 = \frac{11}{2} \times 3{\cdot}0 \Rightarrow \frac{30}{2} = \frac{33}{2}$

which is not true.

Hence $2\frac{1}{2}, 5\frac{1}{2}$, 3·0 and 6·0 are not in proportion

Question 2.
Find the fourth proportional of

(*i*) 3, 12 and 4 (*ii*) 5, 9 and 45

(*iii*) 2·1, 1·5 and 8·4 (*iv*) $\frac{1}{3}, \frac{2}{5}$ and 8·4

(*v*) 4 hours 40 minutes, 1 hour 10 minutes and 16 hours.

Answer:

(*i*) 4th proportional to 3, 12 and 4

$$= \frac{12 \times 4}{3} = 16$$

(*ii*) Fourth proportional to 5, 9 and 45

$$= \frac{9 \times 45}{5} = 81$$

(*iii*) Fourth proportional to 2·1, 1·5 and 8·4

$$= \frac{1 \cdot 5 \times 8 \cdot 4}{2 \cdot 1} = 1 \cdot 5 \times 4 = 6 \cdot 0$$

(*iv*) Fourth proportional to $\frac{1}{3}, \frac{2}{5}$ and 8·4

$$= \frac{\frac{2}{5} \times 8 \cdot 4}{\frac{1}{3}} = \frac{2}{5} \times 8 \cdot 4 \times \frac{3}{1}$$

$$= \frac{2 \times 84 \times 3}{5 \times 10 \times 1} = \frac{252}{25} = 10 \cdot 08$$

(*v*) Fourth proportional to 4 hours 40 minutes, 1 hour 10 minutes and 16 hours

4 hours 40 minutes = 4 × 60 + 40

= 240 + 40 = 280

1 hour 10 minutes = 1 × 60 + 10

= 60 + 10 = 70 minutes

16 hours = 16 × 60 = 960 minutes

$$\therefore \text{ Fourth proportional } = \frac{70 \times 960}{280}$$

$$= 240 \text{ minutes } = \frac{240}{60} = 4 \text{ hours}$$

Question 3.
Find the third proportional of

(i) 27 and 9 (ii) 2 m 40cm and 40cm

(iii) 1·8 and 0·6 (iv) $\frac{1}{7}$ and $\frac{3}{14}$

(v) 1·6 and 0·8

Answer:

(i) Third proportional to 27 and 9

$$= \frac{9 \times 9}{27} = 3$$

(ii) Third proportional to 2 m 40 cm and 40 cm or 240 cm and 40 cm

$$= \frac{40 \times 40}{240} = \frac{20}{3} = 6\frac{2}{3} \text{ cm}$$

(iii) Third proportional to 1·8 and 0·6

$$= \frac{0{\cdot}6 \times 0{\cdot}6}{1{\cdot}8} = \frac{0{\cdot}36}{1{\cdot}8} = \frac{36}{180}$$

$$= \frac{1}{5} = 0{\cdot}2$$

(iv) Third proportional to $\frac{1}{7}$ and $\frac{3}{14}$

$$= \frac{\frac{3}{14} \times \frac{3}{14}}{\frac{1}{7}} = \frac{9}{196} \times \frac{7}{1} = \frac{9}{28}$$

(v) Third proportional to 1·6 and 0·8

$$= \frac{0{\cdot}8 \times 08}{1{\cdot}6} = \frac{0{\cdot}64}{1{\cdot}6}$$

$$= \frac{64}{160} = \frac{2}{5} = 0{\cdot}4$$

Question 4.
Find the mean proportional between

(i) 16 and 4 (ii) 3 and 27
(iii) 0·9 and 2·5 (iv) 0·6 and 9·6
(v) $\frac{1}{4}$ and $\frac{1}{16}$

Answer:

(i) Mean proportional between 16 and 4

$$= \sqrt{16 \times 4} = \sqrt{64} = 8$$

(ii) Mean proportional between 3 and 27

$$= \sqrt{3 \times 27} = \sqrt{81} = 9$$

(iii) Mean proportional between 0·9 and 2·5

$$= \sqrt{0 \cdot 9 \times 2 \cdot 5}$$

$$= \sqrt{\frac{9}{10} \times \frac{25}{10}} = \sqrt{\frac{225}{100}} = \frac{15}{10}$$

$$= 1 \cdot 5$$

(iv) Mean proportional between 0·6 and 9·6

$$= \sqrt{0 \cdot 6 \times 9 \cdot 6} = \sqrt{\frac{6}{10} \times \frac{96}{10}}$$

$$= \sqrt{\frac{576}{100}} = \frac{24}{10} = 2 \cdot 4$$

(v) Mean proportional between $\frac{1}{4}$ and $\frac{1}{16}$

$$= \sqrt{\frac{1}{4} \times \frac{1}{16}} = \sqrt{\frac{1}{64}} = \frac{1}{8}$$

Question 5.

(i) If A : B = 3 : 5 and B : C = 4 : 7, find A : B : C

(ii) If $x : y = 2 : 3$ and $y : z = 5 : 7$, find $x : y : z$

(iii) If $m : n = 4 : 9$ and $n : s : 3 : 7$, find $m : s$

(iv) If $P : Q = \frac{1}{2} : \frac{1}{3}$ and $Q : R = 1\frac{1}{2} : 1\frac{1}{3}$, find P : R.

(v) If $a : b = 1{\cdot}5 : 3{\cdot}5$ and $b : c = 5 : 6$, find $a : c$.

(vi) If $1\frac{1}{4} : 2\frac{1}{3} = p : q$ and $q : r = 4\frac{1}{2} : 5\frac{1}{4}$; find $p : r$

Answer:

(i) A : B = 3 : 5

$= \frac{3}{5} : 1$ (Dividing by 5)

and B : C = 4 : 7

$= 1 : \frac{7}{4}$ (Dividing by 4)

$\therefore A : B : C = \frac{3}{5} : 1 : \frac{7}{4}$

$= 12 : 20 : 35$

(Multiplying by $5 \times 4 = 20$)

(ii) $x : y = 2 : 3$

$= \frac{2}{3} : 1$ (Dividing by 3)

$y : z = 5 : 7$

$= 1 : \frac{7}{5}$ (Dividing by 5)

$\therefore x : y : z = \frac{2}{3} : 1 : \frac{7}{5}$

$= 10 : 15 : 21$

(Multiplying by $3 \times 5 = 15$)

(iii) $m : n = 4 : 9$

$\frac{m}{n} = \frac{4}{9}$

and $n : s = 3 : 7$

$\therefore \frac{n}{s} = \frac{3}{7}$

$\therefore \frac{m}{n} \times \frac{n}{s} = \frac{4}{9} \times \frac{3}{7}$

$$\frac{m}{s} = \frac{4}{21}$$

$\Rightarrow \quad m : s = 4 : 21$

(iv) $P : Q = \frac{1}{2} : \frac{1}{3}$

$\therefore \frac{P}{Q} = \frac{1}{2} \times \frac{3}{1} = \frac{3}{2}$

and $Q : R = 1\frac{1}{2} : 1\frac{1}{3} = \frac{3}{2} : \frac{4}{3}$

$\therefore \qquad \frac{Q}{R} = \frac{3}{2} \times \frac{3}{4} = \frac{9}{8}$

Now $\frac{P}{Q} \times \frac{Q}{R} = \frac{3}{2} \times \frac{9}{8}$

$\Rightarrow \qquad \frac{P}{R} = \frac{27}{16}$

$\therefore \qquad P : R = 27 : 16$

(v) $a : b = 1{\cdot}5 : 3{\cdot}5$

$\frac{a}{b} = \frac{1{\cdot}5}{3{\cdot}5} = \frac{15}{35} = \frac{3}{7}$

$b : c = 5 : 6$

$\therefore \qquad \frac{b}{c} = \frac{5}{6}$

Now $\qquad \frac{a}{b} \times \frac{b}{c} = \frac{3}{7} \times \frac{5}{6} = \frac{5}{14}$

$\therefore \qquad \frac{a}{c} = \frac{5}{14}$

$\Rightarrow \qquad a : c = 5 : 14$

(vi) $p : q = 1\frac{1}{4} : 2\frac{1}{3} = \frac{5}{4} : \frac{7}{3}$

(vi) $p : q = 1\frac{1}{4} : 2\frac{1}{3} = \frac{5}{4} : \frac{7}{3}$

$$\frac{p}{q} = \frac{5}{4} \times \frac{3}{7} = \frac{15}{28}$$

$$q : r = 4\frac{1}{2} : 5\frac{1}{4} = \frac{9}{2} : \frac{21}{4}$$

$$\frac{q}{r} = \frac{9}{2} \times \frac{4}{21} = \frac{6}{7}$$

$$\therefore \quad \frac{p}{q} \times \frac{q}{r} = \frac{15}{28} \times \frac{6}{7}$$

$$\Rightarrow \quad \frac{p}{r} = \frac{45}{98}$$

$$\therefore \quad p : r = 45 : 98$$

Question 6.
If x: y – 5 :4 and 2 : x = 3 :8, find the value of y.

Answer:

$x : y = 5 : 4$

and $2 : x = 3 : 8$

Then, $\frac{x}{y} = \frac{5}{4}$ ...(*i*)

and $\frac{2}{x} = \frac{3}{8}$ ...(*ii*)

$$\Rightarrow x = \frac{2 \times 8}{3} = \frac{16}{3}$$

Now put the value of x in eq. (*i*)

$$\frac{x}{y} = \frac{5}{4}$$

$$y = x \times \frac{4}{5}$$

$$y = \frac{16}{3} \times \frac{4}{5} = \frac{64}{15}$$

Question 7.
Find the value of x, when 2.5 : 4 = x : 7.5.

Answer:

$2.5 : 4 :: x : 7.5$

$4 \times x = 2.5 \times 7.5$

$$x = \frac{2.5 \times 7.5}{4}$$

$$x = \frac{25 \times 75}{4 \times 100}$$

$$x = \frac{75}{16} = 4\frac{11}{16}$$

Question 8.
Show that 2, 12 and 72 are in continued proportion.

Answer:

Three numbers a, b and c are in continued proportion if, $a : b :: b : c$

The numbers are 2, 12 and 72

$$\frac{a}{b} = \frac{2}{12} = \frac{1}{6}$$

$$\frac{b}{c} = \frac{12}{72} = \frac{1}{6}$$

As, $\frac{a}{b} = \frac{b}{c}$

$\therefore$ 2, 12 and 72 are in continued proportion.

CHAPTER - 7
UNITARY METHOD

The method in which the value of a unit (one) quantity is first calculated to get the value of any other quantity is called the unitary method.
In unitary method, we come across two types of variations :
(i) Direct-variation
(ii) Inverse-variation.

(i) Direct variation : Increase in one quantity causes increase in the other and decrease in one quantity causes decrease in the other.
(ii) Inverse variation : Increase in one quantity causes decrease in the other and decrease in one quantity causes increase in the other.
This is found in the sums of speed, work done etc.

EXERCISE 7 (A)

Question 1.
Weight of 8 identical articles is 4.8 kg. What is the weight of 11 such articles ?

Answer:
Weight of 8 articles = 4.8 kg
Weight of 1 article = $\frac{4.8}{8}$ kg
and weight of 11 articles = $\frac{4.8}{8}$ x 11 kg
= 0.6 x 11 = 6.6 kg

Question 2.
6 books weigh 1 .260 kg. How many books will weigh 3.150 kg ?

Answer:
1 kg 260 g or 1·260 kg, no. of books = 6
and in 1 kg, no. of books = $\frac{6}{1 \cdot 260}$
and in 3·150 kg, no. of books = $\frac{6 \times 3 \cdot 150}{1 \cdot 260}$
$= \frac{6 \times 3150}{1260} = \frac{3150}{210} = 15$ books

Question 3.
8 men complete a work in 6 hours. In how many hours will 12 men complete the same work ?

Answer:
8 men can complete a work in = 6 hours
1 man can complete the work in = 6×8 hours
12 men can complete the work in = $\frac{6x8}{12}$ = 4 hours

Question 4.
If a 25 cm long candle burns for 45 minutes, how long will another candle of the same material and same thickness but 5 cm longer than the previous one, burn ?

Answer:
25 cm long candle burn in = 45 minutes
1 cm long candle will burn in = $\frac{45}{25}$ mintues
25 + 5 = 30 cm long candle will burn in
= $\frac{45x30}{25}$minutes = 54 minutes

Question 5.
A typist takes 80 minutes to type 24 pages. How long will he take to type 87 pages ?

Answer:
For typing 24 pages, time is required = 80 minutes
For typing 1 page, time is required =$\frac{80}{24}$ minutes
and for typing 87 pages, time is required
= $\frac{80x87}{24}$ minutes = 290 minutes

Question 6.
Rs. 750 support a family for 15 days. For how many days will Rs. 2,500 support the same family ?

Answer:
Rs. 750, can support a family for = 15 days
Re. 1 will support for = $\frac{15}{750}$days
and Rs. 2,500 will support for = $\frac{15}{750}$x 2500 days = 50 days

Question 7.
400 men have provisions for 23 weeks. They are joined by 60 men. How long will the provisions last ?

Answer:
400 men have provisions for = 23 weeks
1 man will have provisions for = 23 x 400 weeks
and 400 + 60 = 460 men will have provisions for = $\frac{23x400}{460}$ weeks = 20weeks

Question 8.
200 men have provisions for 30 days. If 50 men left, the same provisions would last for the remaining men, in how many days?

Answer:
200 men have provisions for = 30 days
1 man will have provisions for = 30 x 200 days
200 – 50 = 150 men will have provisions
for = $\frac{30x200}{150}$ days = 40 days

Question 9.
8 men can finish a certain amount of provisions in 40 days. If 2 more men join with them, find for how many days the same amount of provisions be sufficient ?

Answer:
8 men can finish a provision in = 40 days
1 man will finish in = 40 x 8 days
8+2=10 men will finish in = $\frac{40x8}{10}$
= 32 days

Question 10.
If interest on Rs. 200 be Rs. 25 in a certain time, what will be the interest on Rs 750 for the same time ?

Answer:
Interest on Rs. 200 is = Rs. 25

Interest on Rs. 1 will be = Rs. $\frac{25}{200}$

and interest on Rs. 750 will be

$$= \text{Rs. } \frac{25 \times 750}{200} = \text{Rs. } \frac{750}{8} = \text{Rs. } 93 \cdot 75$$

Question 11.
If 3 dozen eggs cost Rs. 90, find the cost of 3 scores of eggs. (1 score = 20)

Answer:
3 dozen = 3 x 12 = 36 eggs,
3 scores = 3 x 20 = 60
The cost of 36 eggs is = Rs. 90
The cost of 1 egg will be = Rs. $\frac{90}{36}$
∴ Cost of 60 eggs will be = Rs. $\frac{90x60}{36}$
= Rs. 150

Question 12.
If the fare for 48 km is Rs. 288, what will be the fare for 36 km ?

Answer:
Fare for 48 km = Rs. 288
fare for 1 km = Rs. $\frac{288x36}{48}$ = Rs. 216

Question 13.
What will be the cost of 3.20 kg of an item, if 3 kg of it costs Rs. 360 ?

Answer:
Cost of 3 kg of an item = Rs. 360

Cost of 1 kg of the item = Rs. $\frac{360}{3}$

∴ Cost of 3·20 kg of the item = Rs. $\frac{360 \times 3\cdot20}{3}$

Question 14.
If 9 lines of a print, in a column of a book contains 36 words. How many words will a column of 51 lines cqntain ?

Answer:
In 9 lines of print, words are = 36

In 1 line of print, words will be = $\frac{36}{9}$

$\therefore$ in 51 lines of print, words will be

$$= \frac{36}{9} \times 51 = 204$$

Question 15.
125 pupil have food sufficient for 18 days. If 25 more pupil join them, how long will the food last now ? What assumption have you made to come to your answer ?

Answer:
Pupils in the beginning = 125
More pupils joined = 25
Total pupils = 125 + 25 = 150
Food is sufficient for 125 pupils for = 18 days
Food will be sufficient for 1 pupil for = 18 x 125 days (less pupil more days)
and food will be sufficient for 150 pupils = $\frac{18x125}{150}$ days (more pupil more days)
= $\frac{18x5}{6}$ 15 days

Question 16.
A carpenter prepares a new chair in 3 days, working 8 hours a day. Atleast how many hours per day must he work in order to make the same chair in 4 days ?

Answer:
A chair is completed in 3 days working per day = 8 hours
Then their will be completed in 1 day working for = 8 x 3 hours per day (less days more hours)
and it will be completed in 4 days working for = $\frac{8x3}{4}$= 6 hours per day.

Question 17.
A man earns ₹5,800 in 10 days. How much will he earn in the month of February of a leap year?

Answer:
Note : Leap years has 29 days in the month of February.
Man earns in 10 days = ₹5800

Man earns in 1 day = ₹$\frac{5800}{10}$

$\therefore$ Man earns in February month of leap year

$$= \frac{5800 \times 29}{10} = ₹16820$$

Question 18.
A machine is used for making rubber balls and makes 500 balls in 30 minutes. How many balls will it make in $3\frac{1}{2}$ hours?

Answer:

$$30 \text{ minutes} = \frac{30}{60} = \frac{1}{2} \text{ hours}$$

In $\frac{1}{2}$ hours ball makes by machine = 500

In 1 hours balls makes by machine

$$= 500 \times \frac{2}{1} = 1000 \text{ balls}$$

$\therefore$ In $3\frac{1}{2}$ $\left(\frac{7}{2}\right)$ hours, Balls make by machine

$$= 500 \times 2 \times \frac{7}{2} = 3500 \text{ balls}$$

Question 19.
In a school's hostel mess, 20 children consume a certain quantity of ration in 6 days. However, 5 children did not return to the hostel after holidays. How long will the same amount of ration last now?

Answer:
Total number of children = 20
20 children consume a certain quantity of ration in = 6 days
1 children consume a certain quantity of ration in = 6 x 20 days
As 5 children did not return to the hostel after holidays.
Then number of children in hostel = 20-5 = 15
Hence, 15 children consume certain quantity 6×20
of ration in = $\frac{6x20}{15}$ days = 8 days

EXERCISE 7 (B)

Question 1.
The cost of $\frac{3}{5}$ kg of ghee is ₹96 ; find the cost of : (*i*) one kg ghee (*ii*) $\frac{5}{8}$ kg ghee.

Answer:
Cost of $\frac{3}{5}$ kg of ghee = ₹96

(*i*) ∴ Cost of 1 kg of ghee = ₹96 × $\frac{5}{3}$ = ₹160

(*ii*) and cost of $\frac{5}{8}$ of ghee = ₹160 × $\frac{5}{8}$ = ₹100

Question 2.
$3\frac{1}{2}$m of cloth costs Rs. 168 ; find the cost of $4\frac{1}{3}$m of the same cloth.

Answer:

Cost of $3\frac{1}{2}$ *i.e.* $\frac{7}{2}$m of cloth = Rs. 168

$\therefore$ Cost of 1 m of cloth = Rs. $168 \times \frac{2}{7}$

= Rs. 48

and cost of $4\frac{1}{3} = \frac{13}{3}$m cloth = Rs. $48 \times \frac{13}{3}$

= Rs. 208

Question 3.
A wrist watch loses 10 sec in every 8 hours; in how much time will it lose 15 sec. ?

Answer:

10 sec. are lost in = 8 hours

$\therefore$ 1 sec will be lost in = $\frac{8}{10}$ hours

and 15 sec. will be lost in = $\frac{8}{10} \times 15$ hours

= 12 hours

Question 4.
In 2 days and 20 hours, a watch gains 20 sec ; find how much time will the watch take to gain 35 sec. ?

Answer:
2 days 20 hours = $2 \times 24 + 20 = 48 + 20$
$= 68$ hours

Now 20 sec. are gained in = 68 hours

$\therefore$ 1 sec. will be gained in = $\frac{68}{20}$ hours

35 sec. will be gained in = $\frac{68}{20} \times 35$
$= 119$ hours $= 119 \div 24$ days
$= 4$ days 23 hours

Question 5.
50 men mow 32 hectares of land in 3 days. How many days will 15 men take to mow it?

Answer:
Land is same in both the cases.
Now 50 men can mow land in = 3 days
$\therefore$ 1 man will mow it in = 3 x 50 days
and 15 men will mow it in = $\frac{3x50}{15}$ = 10 days

Question 6.
The wages of 10 workers for a six days week are Rs, 1,200. What are the one day wages: (i) of one worker ? (ii) of 4 workers?

Answer:

(i) 10 workers can earn in 6 days = Rs. 1,200

1 worker will earn in 6 days = Rs. $\frac{1,200}{10}$

1 worker will earn in 1 day = Rs. $\frac{1,200}{10 \times 6}$

= Rs. 20

(ii) $\therefore$ 4 workers will earn in 1 day = Rs. 20×4

= Rs. 80

Question 7.
If 32 apples weigh 2 kg 800 g. How many apples will there be in a box, containing 35 kg of apples ?

Answer:

Apples in a box = 35 kg

Now, If weight is 2 kg 800 g = 2·800 kg,

then number of apples = 32

if weight is 1 kg, then number of apples

$$= \frac{32}{2{\cdot}800}$$

and if weight is 35 kg, the number of apples

$$= \frac{32 \times 35}{2{\cdot}800}$$

$$= \frac{32 \times 35 \times 1000}{2800} = 400$$

Question 8.
A truck uses 20 litres of diesel for 240 km. How many litres will be needed for 1200 km?

Answer:
For 240 km, diesel is needed = 20 litres
∴ for 1 km, diesel will be needed 20

Question 9.
A garrison of 1200 men has provisions for 15 days. How long will the provisions last if the garrison be increased by 600 men ?

Answer:
1200 men has provision for = 15 days
1 man will have that provision for = 15 x 1200 days
∴1200 + 600 = 1800 men will has that provisions for = $\frac{15x1200}{1800}$ days
= 10 days

Question 10.
A camp has provisions for 60 pupil for 18 days. In how many days, the same provisions will finish off if the strength of the camp is increased to 72 pupil ?

Answer:
60 pupil have provision for = 18 days 1 pupil will have provision for = 18 x 60 days (less pupils more days)
and 72 pupils will have provision for = $\frac{18x60}{72}$ days
= 15 days.

EXERCISE 7 (C)

Question 1.
A can do a piece of work in 6 days and B can do it in 8 days. How long will they take to complete it together ?

Answer:

A can do a work in = 6 days

$\therefore$ A's one day's work $= \frac{1}{6}$

B can do the same work in = 8 days

$\therefore$ B is one day's work $= \frac{1}{8}$

$\therefore$ A and B's both one day's work

$= \frac{1}{6} + \frac{1}{8} = \frac{4+3}{24} = \frac{7}{24}$

$\therefore$ Both A and B can do the same work in

$\frac{24}{7} = 3\frac{3}{7}$ days

Question 2.
A and B working together can do a piece of work in 10 days B alone can do the same work in 15 days. How long will A alone take to do the same work ?

Answer:

A and B together can do a work in 10 days
and B can do the same work in 15 days

A and B's one day's work $= \frac{1}{10}$

and B's one day's work $= \frac{1}{15}$

$\therefore$ A's one day's work $= \frac{1}{10} - \frac{1}{15}$

$= \frac{3-2}{30} = \frac{1}{30}$

Hence A can do the same work in
= 30 days

Question 3.

A can do a piece of work in 4 days and B can do the same work in 5 days. Find, how much work can be done by them working together in : (i) one day (ii) 2 days. What part of work will be left, after they have worked together for 2 days ?

Answer:

A can do a piece of work in 4 days and B can do the same work in 5 days.

$\therefore$ A's one day's work $= \frac{1}{4}$

and B's one day's work $= \frac{1}{5}$

(i) A and B's both one day's work $= \frac{1}{4} + \frac{1}{5}$

$$= \frac{5+4}{20} = \frac{9}{20}$$

(ii) A and B's 2 day's work $= \frac{9}{20} \times 2 = \frac{9}{10}$

$\therefore$ work left after 2 day's $= 1 - \frac{9}{10}$

$$= \frac{10-9}{10} = \frac{1}{10}$$

Question 4.
A and B take 6 hours and 9 hours respectively to complete a work. A works for 1 hour and then B works for two hours.
(i) How much work is done in these 3 hours ?
(ii) How much work is still left ?

Answer:

A takes 6 hours to finish work

and B take 9 hours to finish the same work

$\therefore$ A's 1 hour's work $= \frac{1}{6}$

and B's 1 hour's work $= \frac{1}{9}$

and B's 2 hours work $= \frac{1}{9} \times 2 = \frac{2}{9}$

(*i*) Now A's 1 hours work + B's 2 hours work

$$= \frac{1}{6} + \frac{2}{9} = \frac{3+4}{18} = \frac{7}{18}$$

(*ii*) Work left $= 1 - \frac{7}{18} = \frac{18-7}{18} = \frac{11}{18}$

Question 5.
A, B and C can do a piece of work in 12, 15 and 20 days respectively. How long will they take to do it working together ?

Answer:

A can do a piece of work in 12 days

B can do the same work in 15 days

C can do the same work in 20 days

$\therefore$ A's 1 day's work $= \frac{1}{12}$

B's 1 day's work $= \frac{1}{15}$

C's 1 day's work $= \frac{1}{20}$

$\therefore$ (A + B + C)'s together1 day's work

$$= \frac{1}{12} + \frac{1}{15} + \frac{1}{20}$$

$$= \frac{5+4+3}{60} = \frac{12}{60} = \frac{1}{5}$$

$\therefore$ They can do the work in 5 days.

Question 6.
Two taps can fill a cistern in 10 hours and 8 hours respectively. A third tap can empty it in 15 hours. How long will it take to fill the empty cistern, if all of them are opened together ?

Answer:

First tap can fill a cistern in 10 hours

Second tap can fill the cistern in 8 hours

Third tap can empty the cistern in 15 hours

$\therefore$ First tap's 1 hour's work $= \frac{1}{10}$

Second tap's 1 hour's work $= \frac{1}{8}$

and third tap's 1 hour's work $= \frac{1}{15}$

If all of them are opened together, then

their one hour's work $= \frac{1}{10} + \frac{1}{8} - \frac{1}{15}$

$$= \frac{12 + 15 - 8}{120} = \frac{27 - 8}{120} = \frac{19}{120}$$

$\therefore$ They can fill the cistern in $= \frac{120}{19}$ hours

$$= 6\frac{6}{19} \text{ hours}$$

Question 7.
Mohit can complete a work in 50 days, whereas Anuj can complete the same work in 40 days.
Find:
(i) work done by Mohit in 20 days.
(ii) work left after Mohit has worked on it for 20 days.
(iii) time taken by Anuj to complete the remaining work.

Answer:

Mohit can complete a work in 50 days

and Anuj can complete the same work in 40 days

$\therefore$ Mohit's one day's work $= \frac{1}{50}$

and Anuj's one day's work $= \frac{1}{40}$

(*i*) Mohit's 20 day's work

$= \frac{1}{50} \times 20 = \frac{2}{5}$

(*ii*) Work left $= 1 - \frac{2}{5} = \frac{5-2}{5} = \frac{3}{5}$

Anuj can do $\frac{3}{5}$ work in $= 40 \times \frac{3}{5}$ days

$= 24$ days

Question 8.

Joseph and Peter can complete a work in 20 hours and 25 hours respectively. Find :

(i) work done by both together in 4 hrs.

(ii) work left after both worked together for 4 hrs.

(iii) time taken by Peter to complete the remaining work.

Answer:

Joseph can do a work in = 20 hours

Peter can do the same work in = 25 hours

Now Joseph's 1 hour's work $= \frac{1}{20}$

and Peter's 1 hour's work $= \frac{1}{25}$

Both's 1 hour's work $= \frac{1}{20} + \frac{1}{25} = \frac{5+4}{100} = \frac{9}{100}$

(*i*) Both's 4 hours work $= \frac{9}{100} \times 4 = \frac{9}{25}$

(*ii*) Work left over $= 1 - \frac{9}{25}$

$= \frac{25-9}{25} = \frac{16}{25}$ **Ans.**

(*iii*) Peter can do $\frac{16}{25}$ work in $= 25 \times \frac{16}{25}$

$= 16$ hours

Question 9.

A is able to complete $\frac{1}{3}$ of a certain work in 10 hrs and B is able to complete $\frac{2}{5}$ of the same work in 12 hrs.
Find:
(i) how much work can A do in 1 hour ?
(ii) how much work can B do in 1 hour ?
(iii) in how much time will the work be completed, if both work together.

Answer:

$$\text{A can do } \frac{1}{3} \text{ of a work in} = 10 \text{ hours}$$

$$\therefore \text{ A can do full work in} = \frac{10 \times 3}{1} = 30 \text{ hours}$$

$$\text{B can do } \frac{2}{5} \text{ of the work in} = 12 \text{ hours}$$

$$\therefore \text{ B can do the whole work in} = \frac{12 \times 5}{2} = 30 \text{ hours}$$

(*i*) Now A's 1 hour's work $= \frac{1}{30}$

(*ii*) B's 1 hour's work $= \frac{1}{30}$

(*iii*) Both's 1 hour's work

$$= \frac{1}{30} + \frac{1}{30} = \frac{2}{30} = \frac{1}{15}$$

$\therefore$ Both can finish the work in 15 hours

Question 10.

Shaheed can prepare one wooden chair in 3 days and Shaif can prepare the same chair in 4 days. If they work together, in how many days will they prepare :
(i) one chair ?
(ii)14 chairs of the same kind?

Answer:

Shaheed's 1 day's work = $\frac{1}{3}$

and Shaif's 1 day's work = $\frac{1}{4}$

Both one day's work = $\frac{1}{3} + \frac{1}{4} = \frac{4+3}{12} = \frac{7}{12}$

$\therefore$ Both can prepare the chair in = $\frac{12}{7}$ days

= $1\frac{5}{7}$ days

One chair is prepared in = $1\frac{5}{7}$ days

$\therefore$ 14 chairs will be prepared in = $\frac{12}{7} \times 14$

= 24 days

Question 11.

A, B and C together finish a work in 4 days. If A alone can finish the same work in 8 days and B in 12 days, find how long will C take to finish the work.

Answer:

A, B and C finish work together in = 4 days.

A, B and C finish work together in 1 day = $\frac{1}{4}$

A' one day work = $\frac{1}{8}$

B's one day work = $\frac{1}{12}$

$\therefore$ C's one day work = $\frac{1}{4} - \left(\frac{1}{8} + \frac{1}{12}\right)$

$$= \frac{6-(3+2)}{24} = \frac{1}{24}$$

$\therefore$ C can finish the work in = 24 days

CHAPTER - 8
PERCENTAGE

POINTS TO REMEMBER

1. The cent means hundred. Therefore percent means after hundred and notation % is used for it.
2. **To express an ordinary given statement as percent.**
   (i) Express the given statement as a fraction.
   (ii) Convert this fraction into an equivalent fraction with denominator 100.
   Therefore to express a fraction or a decimal as percent, multiply it by 100.
3. **To Express-One quantity as a percent of the other.**
   (i) If necessary, convert with the quantitities into the same units.
   (ii) From the fraction with the number to be compared as numerator and the number with which it is to be compared as denominator.
   (iii) Multiply the fraction obtained by 100 and at the same time write the percent sign (%).

EXERCISE 8 (A)

Question 1.
Express each of the following as percent :

(i) $\frac{3}{4}$ (ii) $\frac{2}{3}$

(iii) $0{\cdot}025$ (iv) $0{\cdot}125$

(v) $\frac{3}{8}$ (vi) $0{\cdot}25$

Solution :

(i) $\frac{3}{4} = \frac{3}{4} \times 100 = 75\%$

(ii) $\frac{2}{3} = \frac{2}{3} \times 100 = \frac{200}{3} = 66\frac{2}{3}\%$

(iii) $0{\cdot}025 = \frac{25}{1000} \times 100 = \frac{25}{10}\% = 2{\cdot}5\%$

(iv) $0{\cdot}125 = \frac{125}{1000} \times 100 = \frac{125}{10} = 12{\cdot}5\%$

(v) $\frac{3}{8} = \frac{3}{8} \times 100 = \frac{75}{2} = 37\frac{1}{2}\%$

(vi) $0{\cdot}25 = \frac{25}{100} \times 100 = 25\%$

Question 2.

Express the following percentages as fractions and as decimal numbers :

(i) $7\frac{1}{2}\%$ (ii) $2\cdot50\%$ (iii) $0\cdot02\%$

(iv) 175% (v) 5%

(vi) 25%

Solution :

(i) $7\frac{1}{2}\% = \frac{15}{2 \times 100} = \frac{3}{40}$

$= \frac{15}{200} = 0\cdot075$

$$\begin{array}{r|l} & 0\cdot075 \\ 200 & 15\cdot000 \\ & -1400 \\ \hline & 1000 \\ & -1000 \\ \hline & \times \end{array}$$

(ii) $2\cdot50\% = \frac{250}{100 \times 100} = \frac{1}{40}$

$= \frac{250}{10000} = 0\cdot0250$

$= 0\cdot025$

(iii) $0\cdot02\% = \frac{0\cdot02}{100}$

$= \frac{2}{100 \times 100} = \frac{2}{10000} = \frac{1}{5000}$

$= \frac{2}{10000} = 0\cdot0002$

(iv) $175\% = \frac{175}{100} = \frac{7}{4} = 1\cdot75$

(v) $5\% = \frac{5}{100} = \frac{1}{20}$ and $\frac{5}{100} = 0\cdot05$

(vi) $25\% = \frac{25}{100} = \frac{1}{4} = 0\cdot25$

Question 3.

What percent is :
(i) 16 hours of 2 days ?
(ii) 40 paisa of Rs. 2 ?
(iii) 25 cm of 4 metres
(iv) 600 gm of 5 kg ?

Solution :

Sol. (*i*) 16 hours of 2 days

$$= \frac{16}{2 \times 24} = \frac{16}{48} \times 100\%$$

$$= \frac{100}{3}\% = 33\frac{1}{3}\%$$

(*ii*) 40 paise of Rs. 2

$$= \frac{40}{2 \times 100} = \frac{40}{200} \times 100\%$$

$= 20\%$ **Ans.**

(*iii*) 25 cm of 4 metres

$$= \frac{25}{4 \times 100} = \frac{1}{16} \times 100\%$$

$$= \frac{25}{4}\% = 6\frac{1}{4}\%$$

(*iv*) 600 gm of 5 kg $= \frac{600}{5 \times 1000} \times 100\%$

$= 12\%$

Question 4.
Find the value of:

(i) 5% of Rs. 350 (ii) 10% of Rs. 400·40

(iii) 1% of Rs. 500 (iv) $12\frac{1}{2}\%$ of 80 kg

(v) $\frac{5}{8}\%$ of Rs. 600 (vi) $33\frac{1}{3}\%$ of 27 m

Solution :

(i) 5% of Rs. 350

$= \text{Rs. } 350 \times \frac{5}{100} = \text{Rs. } \frac{35}{2}$

$= \text{Rs. } 17 \cdot 50$

(ii) 10% of Rs. 400·40 $= \text{Rs. } 400 \cdot 40 \times \frac{10}{100}$

$= \text{Rs. } 40 \cdot 04$

(iii) 1% of Rs. 500 $= \text{Rs. } 500 \times \frac{1}{100}$

$= \text{Rs. } 5$

(iv) $12\frac{1}{2}\%$ of 80 kg $= 80 \text{ kg} \times \frac{25}{2 \times 100}$

$= 10 \text{ kg}$

(v) $\frac{5}{8}\%$ of Rs. 600 $= \text{Rs. } 600 \times \frac{5}{8 \times 100}$

$= \text{Rs. } \frac{15}{4} = \text{Rs. } 3 \cdot 75$

(vi) $33\frac{1}{3}\%$ of 27 m

$= 27 \text{ m} \times \frac{100}{3 \times 100}$

$= 9 \text{ m}$

Question 5.
In a class of 60 children, 30% are girls. How many boys are there ?

Solution :
Total children = 60,
Girls = 30%
∴Total girls = 30% of 60 = 60 x $\frac{30}{100}$= 18
∴ No. of boys = 60 – 18 = 42

Question 6.
In an election, two candidates A and B contested. A got 60% of the votes. The total votes polled were 8000. How many votes did each get ?

Solution :
Total number of votes polled = 8000
A got 60% of the votes
A got total votes = 60% of 8000 = 8000 x $\frac{60}{100}$ = 4800
∴ B got total votes = 8000 – 4800 = 3200

Question 7.
A person saves 12% of his salary every month. If his salary is ₹2,500, find his expenditure.

Solution :
Total salary = ₹2500
Saving = 12% of the salary
∴ Total savings = 12% of ₹2500
= ₹2500 x$\frac{12}{100}$ = ₹300
∴Total expenditure = ₹2500 – ₹300 = ₹2200

Question 8.
Seeta got 75% marks out of a total of 800. How many marks did she lose ?

Solution :
Total marks = 800
Marks Seeta got = 75% of total marks
∴ Total marks Seeta got = 75% of 800
= 800 x $\frac{75}{100}$ = 600
∴ Marks Seeta lose = 800 – 600 = 200

Question 9.
A shop worth ₹25,000 was insured for 95% of its value. How much would the owner get in case of any mishappening ?

Solution :
Value of shop =₹25,000
Insured amount = 95% of total value
=95% of ₹25,000
= ₹25,000 x $\frac{95}{100}$
= ₹ 23,750

Question 10.
A class has 30 boys and 25 girls. What is the percentage of boys in the class ?

Solution :
No. of boys = 30
No. of girls = 25
Total number of children = 30 + 25 = 55
∴Percentage of boys in the class
= $\frac{30}{55}$ x 100
= $\frac{600}{11}$=54$\frac{6}{11}$ %

Question 11.
Express :
(i) 3 $\frac{2}{5}$ as a percent
(ii) 0.0075 as percent
(iii) 3 : 20 as percent
(iv) 60 cm as percent of 1 m 25 cm
(v) 9 hours as a percent of 4 days.

Solution :
(*i*) $3\frac{2}{5}$ as a percent

$3\frac{2}{5} = \frac{3\times5+2}{5} = \frac{17}{5}$

Now, convert $\frac{17}{5}$ as a percent

$= \frac{17}{5} \times 100 = 340\%$

(*ii*) 0.0075 as percent
.0075 × 100 = 0.75%

or $\frac{0.0075}{10000} \times 100 = 0.75\%$

(*iii*) 3 : 20 as percent

$$= \frac{3}{20} \times 100 = 15\%$$

(*iv*) 60 cm as percent of 1 m 25 cm

60 cm as percent of $(1 \times 100 + 25)$ cm

$\because$ 1 metre = 100 cm

$$= \frac{60}{125} \times 100 = 12 \times 4 = 48\%$$

(*v*) 9 hours as a percent of 4 days

= 1 days = 24 hours

$\therefore$ 4 days = $4 \times 24 = 96$ hours

$$= \frac{9}{96} \times 100 = \frac{75}{8}\%$$

$$\text{or } 9\frac{3}{8}\%$$

Question 12.
(i) Find 2% of 2 hours 30 min.
(ii) What percent of 12 kg is 725 gm?

Solution :

(*i*) 2% of 2 hours 30 min

$\because$ 1 hour = 60 minutes

$\therefore$ 2 hours 30 min. = 2 × 60 min. + 30 min.

$= 120 + 30 = 150$ min

$$= 150 \times \frac{2}{100} = \frac{30}{10}$$

= 3 minutes

(*ii*) 12 kg is 725 gm

1 kg = 1000 gm

$\therefore$ 12 kg = 12 × 1000 = 12000 gm

$$\frac{725}{12000} \times 100 = \frac{725}{120}$$

$$= \frac{145}{24}\% \text{ or } 6\frac{10}{24}\%$$

EXERCISE 8 (B)

Question 1.

Deepak bought a basket of mangoes containing 250 mangoes 12% of these were found to be rotten. Of the remaining, 10% got crushed. How many mangoes were in good condition ?

Solution :

Total mangoes = 250

Rotten mangoes = 12% of 250

$$= 250 \times \frac{12}{100} = 30$$

Remaining mangoes = 250 – 30 = 220

Mangoes which were crushed = 10% of 220

$$= 220 \times \frac{10}{100} = 22$$

$\therefore$ Balance = 220 – 22 = 198

Hence 198 mangoes were in good condition.

Question 2.

In a Maths Quiz of 60 questions, Chandra got 90% correct answers and Ram got 80% correct answers. How many correct answers did each give ?
What percent is Ram's correct answers to Chandra's correct answers ?

Solution :

No. of total questions = 60

Chandra got correct answers of the questions

= 90% of 60

$$= \frac{60 \times 90}{100} = 54$$

Ram got correct answers of the questions

= 80% of 60

$$= 60 \times \frac{80}{100} = 48$$

$\therefore$ Percentage of Ram's correct answer of

$$\text{that of Chandra's} = \frac{48}{54} \times 100 = \frac{800}{9}\%$$

$$= 88\frac{8}{9}\%$$

Question 3.

In an examination, the maximum marks are 900. A student gets 33% of the maximum marks and fails by 45 marks. What is the passing mark ? Also, find the pass percentage.

Solution :

Maximum marks = 900

A student got 33% of 900 marks

$$= 900 \times \frac{33}{100} = 297$$

No. of marks by which he failed = 45

$\therefore$ Pass marks = 297 + 45 = 342

$$\text{Percentage of pass marks} = \frac{342 \times 100}{900}$$

$$= 38\%$$

Question 4.

In a train, 15% people travel in first class, 35% travel in second class. The balance travel in the A.C. class ? Calculate the percentage of A.C. class travellers ?

Solution :

Let no. of people = 100

No. of people in first class = 15

and no. of people travel in second class

= 35

$\therefore$ Balance = 100 – (15 + 35) = 100 – 50 = 50

$\therefore$ Percent of people travel in AC class

= 50%

Question 5.
A boy eats 25% of the cake and gives away 35% of it to his friends. What percent of the cake is still left with him ?

Solution :

Let total cake = 100

Cake which was eaten by the boy = 25

Cake which was given to his friends = 35

$\therefore$ Balance cake $= 100 - (25 + 35)$

$= 100 - 60 = 40$

Hence he has 40% of the cake with him.

Question 6.
What is the percentage of vowels in the English alphabet ?

Solution :

There are 5 vowels in 26 English alphabets

$$\therefore \text{Percentage of vowels} = \frac{5 \times 100}{26}$$

$$= \frac{250}{13} = 19\frac{3}{13}\%$$

Question 7.

(*i*) $6\frac{1}{4}\%$ of what number is 375 ?

(*ii*) 0·2% of a number is 5. Find the number.

(*iii*) 30 is $16\frac{2}{3}\%$ of a number. Find the number.

Solution :

(*i*) Let number be x.

Then $6\frac{1}{4}\%$ of $x = 375$

$\Rightarrow \frac{25}{4 \times 100}$ of $x = 375$

$\Rightarrow \frac{1}{16}x = 375$

$\therefore\ x = \frac{375 \times 16}{1} = 6000$

Hence number = 6000

(*ii*) Let number = x

then $0{\cdot}2\%$ of $x = 5$

$\Rightarrow \frac{2}{10 \times 100}$ of $x = 5 \quad \Rightarrow \frac{1}{500}$ of $x = 5$

$\Rightarrow \quad x = \frac{5 \times 500}{1}$

$\Rightarrow \quad x = 2500$

$\therefore$ Number = 2500

(*iii*) Let the number = x

then $16\frac{2}{3}\%$ of $x = 30$

$\Rightarrow \frac{50}{3 \times 100}$ of $x = 30 \quad \Rightarrow \frac{1}{6}$ of $x = 30$

$\Rightarrow x = 30 \times 6 = 180$

Hence number = 180

Question 8.

The money spent on the repairs of a house was 1% of its value. If the repair, costs Rs. 5,000, find the cost of the house.

Solution :

Let cost of house $= x$

Then cost of repairs $= 1\%$ of x

$\therefore$ 1% of $x = 5000$

$$\Rightarrow \frac{1}{100} \times x = 5000 \Rightarrow x = 5,000 \times \frac{100}{1}$$

$$x = 5,00,000$$

Hence cost of house = Rs. 5,00,000

Question 9.

In a school out of300 students, 70% are girls and 30% are boys. If 30 girls leave and no new boy is admitted, what is the new percentage of girls in the school ?

Solution :

Total number of children in a school = 300

No. of boys = 30% of 300

$$= \frac{30}{100} \times 300 = 90$$

and no. of girls = 70% of 300

$$= \frac{70}{100} \times 300 = 210$$

Now no. of girls left = 30

$\therefore$ No. of girls after leaving 30 girls

$$= 210 - 30 = 180$$

and No. of children in the school

$$= 180 + 90 = 270$$

$$\therefore \% \text{ of girls now} = \frac{180}{270} \times 100 = \frac{200}{3}\%$$

$$= 66\frac{2}{3}\%$$

Question 10.

Kumar bought a transistor for Rs. 960. He paid $12\frac{1}{2}$ % cash money. The rest he agreed to pay in 12 equal monthly instalments. How much will he pay each month ?

Solution :

Price of transistor = Rs. 960

Amount paid in cash = $12\frac{1}{2}\%$ of Rs. 960

$$= \frac{25}{2 \times 100} \times 960 = \text{Rs. } 120$$

Balance amount = Rs. 960 – Rs. 120

= Rs. 840

No. of instalments = 12

$\therefore$ Amount of each instalment

= Rs. 840 ÷ 12 = Rs. 70

Question 11.

An ore contains 20% zinc. How many kg of ore will be required to get 45 kg of zinc ?

Solution :

In an ore, zinc = 20%

Let quantity of ore = x

$\therefore$ 20% of x = 45 kg

$$\Rightarrow \frac{20}{100} \times x = 45 \quad \Rightarrow \frac{x}{5} = 45$$

$$\Rightarrow x = 45 \times 5 = 225$$

$\therefore$ quantity of ore = 225 kg

EXERCISE 8 (C)

Question 1.

The salary of a man is increased from Rs. 600 per month to Rs. 850 per month. Express the increase in salary as percent.

Solution :

Original salary of a man = Rs. 600

Increased salary = Rs. 850

$\therefore$ Amount of increase = Rs. 850 – 600

= Rs. 250

$$\text{Percentage increase} = \frac{250 \times 100}{600} = \frac{125}{3}$$

$$= 41\frac{2}{3}\%$$

Question 2.

Increase :

(i) 60 by 5%

(ii) 20 by 15%

(iii) 48 by 121 %

(iv) 80 by 140%

(v) 1000 by 3.5%

Solution :

(*i*) Rate of increase = 5%

$$\therefore \text{Total increase} = 5\% \text{ of } 60 = \frac{5}{100} \times 60$$

$$= 3$$

$\therefore$ Increased number = 60 + 3 = 63

(ii) Increase on 20 at the rate of 15%

$$= 20 \times \frac{15}{100} = 3$$

$\therefore$ Increased number = 20 + 3 = 23

(iii) Increase on 48 by $12\frac{1}{2}\% = 48 \times \frac{25}{2}\%$

$$= 48 \times \frac{25}{2 \times 100} = 48 \times \frac{1}{8} = 6$$

$\therefore$ Increased number = 48 + 6 = 54

(iv) Increase on 80 by 140% $= 80 \times \frac{140}{100}$

$= 112$

$\therefore$ Increased number = 80 + 112 = 192

(v) Increase on 1000 by 3·5% $= 1000 \times \frac{3 \cdot 5}{100}$

$$= 1000 \times \frac{35}{10 \times 100} = 35$$

$\therefore$ Increased number = 1000 + 35

= 1035

Question 3.

Decrease :

(i)80 by 20%

(ii) 300 by 10%

(iii) 50 by 12.5%

Solution :

(i) Decrease on 80 by 20% $= 80 \times \frac{20}{100} = 16$

$\therefore$ Decreased number = 80 – 16 = 64

(ii) Decrease on 300 by 10% $= 300 \times \frac{10}{100} = 30$

$\therefore$ Decreased number = 300 – 30 = 270

(iii) Decrease on 50 by 12·5% $= 50 \times \frac{12 \cdot 5}{100}$

$$= \frac{50 \times 125}{10 \times 100} = \frac{25}{4} = 6 \cdot 25\%$$

$\therefore$ Decrease number = 50 – 6·25

= 43·75

Question 4.
What number :
(i) When increased by 10% becomes 88 ?
(ii) When increased by 15% becomes 230 ?
(iii) When decreased by 15% becomes 170 ?
(iv) When decreased by 40% becomes 480 ?
(v) When increased by 100% becomes 100 ?
(vi) When decreased by 50% becomes 50 ?

Solution :

(*i*) Let the number be = 100

Increase = 10% = 10

$\therefore$ Increased number = 100 + 10

= 110

If increased number is 110, then original number = 100

and if increased number is 88, then original

number $= \frac{100}{110} \times 88 = 80$

(*ii*) Let the number be = 100

Increase = 15% = 15

$\therefore$ Increased number = 100 + 15 = 115

If increased number is 115, then original number = 100

and if increased number is 230, then original

number $= \frac{100 \times 230}{115} = 200$

(*iii*) Let the number be = 100

Decrease = 15% = 15

$\therefore$ Decreased number = 100 – 15 = 85

If decreased number is 85, then original number = 100

and if decreased number is 170, then original

number $= \frac{100}{85} \times 170 = 200$

(*iv*) Let the number be = 100

Decrease = 40% = 40

$\therefore$ Decreased number = 100 – 40 = 60

If decreased number is 60, then original number = 100

and if decreased number is 480, then original

number = $\frac{100 \times 480}{60}$ = 800

(*v*) Let the number be = 100

Increase = 100% = 100

$\therefore$ Increased number = 100 + 100 = 200

If increased number is 200, then original number = 100

and if increased number is 100, then original

number = $\frac{100 \times 100}{200}$ = 50

(*vi*) Let the number be = 100

Decrease = 50% = 50

$\therefore$ Decreased number = 100 – 50 = 50

If decreased number is 50, then original number = 100

and if decreased number is 50, then original

number = $\frac{100 \times 50}{50}$ = 100

Question 5.

The price of a car is lowered by 20% to Rs. 40,000. What was the original price ? Also, find the reduction in price.

Solution :

Let original price of the car = Rs. 100

Reduction = 20%’ = Rs. 20

∴ Reduced price = Rs. 100 – 20 = Rs. 80

If reduced price is Rs. 80, then original price = Rs. 100

and if reduced price is Rs. 40,000 then original price = $\frac{Rs. 100x40000}{80}$

= Rs. 50,000
and reduction = Rs. 50000 – Rs. 40000
= Rs. 10,000

Question 6.
If the price of an article is increased by 25%, The increase is Rs. 10. Find the new price.

Solution :
Let the price of an article = Rs. 100
Increase = 25%
∴Increase = Rs. 25
If an increased price = Rs. 100 + 25 = Rs. 125
If increase is Rs. 25 then new price = Rs. 125
and if increase is Rs. 10, then new price = Rs. $\frac{125x10}{25}$
= Rs. 50

Question 7.
If the price of an article is reduced by 10%, the reduction is Rs. 40. What is the old price ?

Solution :
Let the original (old) price = Rs. 100
Reduction = 10% = Rs. 10
∴If reduction is Rs. 10, then old price = Rs. 100
and if reduction is Rs. 40, then old price = Rs. $\frac{100x40}{10}$ = Rs. 400

Question 8.
The price of a chair is reduced by 25%. What is the ratio of:
(i) Change in price to the old price.
(ii) Old price to the new price.

Solution :
Let old (original) price of a chair = Rs. 100
Reduction = 25% = Rs. 25
∴Reduced price = Rs. 100 – Rs. 25 = Rs. 75
(i) Ratio between change in price and old price = 25 : 100
= 1:4 (Dividing by 25)
(ii) Ratio between old price and new price = 100 : 75
= 4:3 (Dividing by 25)

Question 9.

If x is 20% less than y, find :

(i) $\frac{x}{y}$ (ii) $\frac{y-x}{y}$ (iii) $\frac{x}{y-x}$

Solution :

Let $y = 100$

then reduction $= 20\% = 20$

then $x = 100 - 20 = 80$

(i) $\frac{x}{y} = \frac{80}{100} = \frac{4}{5}$ (Dividing by 20)

(ii) $\frac{y-x}{y} = \frac{100-80}{100} = \frac{20}{100} = \frac{1}{5}$

(Dividing by 20)

(iii) $\frac{x}{y-x} = \frac{80}{100-80} = \frac{80}{20} = \frac{4}{1} = 4$

(Dividing by 20)

Question 10.

If x is 30% more than y; find :

(i) $\frac{x}{y}$ (ii) $\frac{y+x}{x}$ (iii) $\frac{y}{y-x}$

Solution :

Let $y = a$

Then $x = a \times \frac{100+30}{100} = a \times \frac{130}{100} = \frac{13}{10}a$

Now, (i) $\frac{x}{y} = \frac{a}{\frac{13}{10}a} = \frac{a \times 10}{13a} = \frac{10}{13}$

(ii) $\frac{y+x}{x} = \frac{a + \frac{13}{10}a}{\frac{13}{10}a}$

$= \frac{(10+13)a}{10 \times \frac{13}{10}a} = \frac{23\,a}{10} \times \frac{10}{13\,a} = \frac{23}{13}$

(iii) $\frac{y}{y-x} = \frac{a}{a - \frac{13}{10}a} = \frac{a}{\frac{-3}{10}a}$

$= \frac{a \times 10}{-3a} = -\frac{10}{3}$

Question 11.
The weight of a machine is 40 kg. By mistake it was weighed as 40.8 kg. Find the error percent.

Solution :

Actual weight of machine = 40 kg

Errored weight = 40·8 kg

$\therefore$ Error in weight = 40·8 – 40 = 0·8 kg

$$\text{Error \%} = \frac{0\cdot8 \times 100}{40} = \frac{8 \times 100}{10 \times 40} = 2\%$$

Question 12.
From a cask, containing 450 litres of petrol, 8% of the petrol was lost by leakage and evaporation. How many litres of petrol was left in the cask ?

Solution :

Original petrol in the cask = 450 litres

Leakage and evaporation = 8%

$$\therefore \text{Lost petrol} = 8\% \text{ of } 450 \text{ litres} = \frac{8 \times 450}{100}$$

$$= 36 \text{ litres}$$

Question 13.
An alloy consists of 13 parts of copper, 7 parts of zinc and 5 parts of nickel. What is the percentage of each metal in the alloy?

Solution :

Copper = 13 parts, Zinc = 7 parts

Nickel = 5 parts

Total alloy = 13 + 7 + 5 = 25 parts

$$\text{Now, percentage of copper} = \frac{13}{25} \times 100 = 52\%$$

$$\text{Percentage of zinc} = \frac{7}{25} \times 100 = 28\%$$

$$\text{and percentage of nickel} = \frac{5}{25} \times 100 = 20\%$$

Question 14.

In an examination, first division marks are 60%. A student secures 538 marks and misses the first division by 2 marks. Find the total marks of the examination.

Solution :

Percentage for first division = 60%

A student secures 538 marks but misses the first division by 2 marks.

$\therefore$ Marks for first division = 538 + 2 = 540

$\therefore$ 60% of total marks = 540

$$\Rightarrow \frac{60}{100} \times \text{total marks} = 540$$

$$\Rightarrow \text{total marks} = \frac{540 \times 100}{60} = 900$$

Question 15.

Out of 1200 pupils in a school, 900 are boys and the rest are girls. If 20% of the boys and 30% of the girls wear spectacles, find :
(i) how many pupils in all, wear spectacles ?
(ii) what percent of the total number of pupils wear spectacles ?

Solution :

Total number of pupils = 1200

No. of boys = 900

and no. of girls = 1200 – 900 = 300

No. of boys who wear spectacles

$$= 20\% \text{ of } 900 = \frac{20}{100} \times 900 = 180$$

No. of girls who wear spectacles

$$= 30\% \text{ of } 300 = \frac{30}{100} \times 300 = 90$$

(*i*) $\therefore$ Total number of pupils who wear spectacles = 180 + 90 = 270

(*ii*) Percentage of pupils who wear spectacles

$$= \frac{270 \times 100}{1200} = \frac{270}{12} = 22 \cdot 5\%.$$

Question 16.

Out of 25 identical bulbs, 17 are red, 3 are black and the remaining are yellow. Find the difference between the numbers of red and yellow bulbs and express this difference as percent.

Solution :

Total number of bulbs = 25

Number of red bulbs = 17

Number of black bulbs = 3

= 17 + 3 = 20

$\therefore$ Number of yellow bulbs = 25 – 20 = 5 bulbs

Difference between the number of red and yellow bulbs

= No. of red bulbs – No. of yellow bulbs

= 17 – 5 = 12

$\therefore$ Percentage difference

$$= \frac{\text{Difference in red and yellow bulbs}}{\text{Total number of bulbs}}$$

$$= \frac{12}{25} \times 100 = 48\%$$

Question 17.

A number first increases by 20% and then decreases by 20%. Find the percentage increase or decrease on the whole.

Solution :

Let the number be 100

In 1^{st} condition,

Increase = 20% of 100

$$= \frac{20}{100} \times 100 = 20$$

$\therefore$ Number after this increase = 100 + 20

= 120

In 2^{nd} condition,

Decrease = 20% of 120

$$= \frac{20}{100} \times 120 = 24$$

$\therefore$ Number after this decrease = 120 – 24 = 96

$\Rightarrow$ Total decrease on the whole = 24 – 20 = 4

and the percentage decrease on the whole

$$= \frac{4}{100} \times 100 = 4\%$$

Question 18.
A number is first decreased by 40% and then again decreased by 60%. Find the percentage increase or decrease on the whole.

Solution :

Let the number be 100

In first condition :

Decrease = 40% of 100

$$= \frac{40}{100} \times 100 = 40$$

$\therefore$ Number after this decrease

$$= 100 - 40 = 60$$

In Second condition :

Decrease = 60% of 40

$$= \frac{60}{100} \times 40 = 24$$

$\therefore$ Number after the 2nd decrease

$$= 40 - 24 = 16$$

$\Rightarrow$ Total decrease on the whole

$$= 40 + 24 = 64$$

$\therefore$ Percentage decrease on the whole

$$= \frac{64}{100} \times 100 = 64\%$$

Question 19.
If 150% of a number is 750, find 60% of this number.

Solution :

Let the required number be x

Now, 150% of $x = 750$

$$\Rightarrow \frac{150}{100} \times x = 750$$

$$\Rightarrow x = \frac{750 \times 100}{150} = 500$$

Hence, the required number = 500

Now, 60% of 500 = $500 \times \frac{60}{100} = 300$

CHAPTER - 9
PROFIT, LOSS & DISCOUNT

POINTS TO REMEMBER

1. The Cost Price (C.P.) of an article is the price at which the article is bought.
2. The Selling Price (S.P.) of an article is the price at which the article is sold.
3. If **Selling Price** of an article is more than its cost price ; it is sold at a **profit (gain)**
   Profit = Selling Price-Cost Price
   i.e., Profit (gain) = S.P. – C.P. and S.P. = C.P. + Gain
4. If Selling Price of an article is less than its cost price ; it is sold at a loss.
   Loss = Cost Price – Selling Price
   i.e., **Loss = C.P. – S.P. and S.P. = C.P. — Loss**
5. Profit percent and loss percent are always calculated on cost price (C.P.) only.
   i.e., **(i)** Profit % = $\frac{Profit}{C.P.}$ x 100% and
   (ii) Loss % = $\frac{Loss}{C.P.}$ x 100%
6. **Selling Price** = Marked price – Discount
   i.e., S.P. = M.P.—(piscount
   Note : (i) Discount is calculated on marked price (M.P.)
   (ii) Marked price is also written as List price.

EXERCISE 9 (A)

Question 1.
Find the gain or loss percent, if
(i) C.P. = Rs. 200 and S.P.: = Rs. 224
(ii) C.P. = Rs. 450 and S.P. = Rs. 400
(iii) C.P. = Rs. 550 and gain = Rs . 22
(iv) CP. = Rs. 216 and loss = Rs. 72
(v) S.P. = Rs. 500 and loss : = Rs. 100
(vi) S.P. = Rs. 12 and profit = Rs. 4
(vii) C.P. = Rs. 5 and gain = 60 P

Solution:

(*i*) C.P. = Rs. 200 and S.P. = Rs. 224

$\therefore$ Gain = S.P. – C.P.

= Rs. 224 – Rs. 200 = Rs. 24

$$\text{Gain \%} = \frac{\text{gain} \times 100}{\text{C.P.}} = \frac{24 \times 100}{200} = 12\%$$

(*ii*) C.P. = Rs. 450 and S.P. = Rs. 400

$\therefore$ Loss = C.P. – S.P.

= Rs. 450 – Rs. 400 = Rs. 50

$$\text{Loss \%} = \frac{\text{Loss} \times 100}{\text{C.P.}}$$

$$= \frac{50 \times 100}{450} = \frac{100}{9} = 11\frac{1}{9}\%$$

(*iii*) C.P. = Rs. 550 and gain = Rs. 22

$\therefore$ S.P. = C.P. + gain

= Rs. 550 + Rs. 22 = Rs. 572

$$\text{Gain \%} = \frac{\text{Gain} \times 100}{\text{C.P.}} = \frac{22 \times 100}{550} = 4\%$$

(*iv*) C.P. = Rs. 216 and loss = Rs. 72

$\therefore$ S.P. = C.P. – loss

= Rs. 216 – Rs. 72 = Rs. 144

$$\text{Loss \%} = \frac{\text{Loss} \times 100}{\text{C.P.}} = \frac{72 \times 100}{216} = \frac{100}{3}$$

$$= 33\frac{1}{3}\%$$

(*v*) S.P. = Rs. 500 and loss = Rs. 100

$\therefore$ C.P. = S.P. + loss

= Rs. 500 + Rs. 100 = Rs. 600

$$\text{Loss \%} = \frac{\text{Loss} \times 100}{\text{C.P.}} = \frac{100 \times 100}{600}$$

$$= \frac{50}{3} = 16\frac{2}{3}\%$$

(*vi*) S.P. = Rs. 12 and profit = Rs. 4

$\therefore$ C.P. = S.P. – gain = Rs. 12 – Rs. 4 = Rs. 8

$$\text{Gain \%} = \frac{\text{Gain} \times 100}{\text{S.P.}} = \frac{4 \times 100}{8} = 50\%$$

(*vii*) C.P. = Rs. 5 and gain = 60 P

$\therefore$ S.P. = C.P. + gain = Rs. 5 + 60 P.

= Rs. 5·60

$$\text{Gain \%} = \frac{\text{Gain} \times 100}{\text{C.P.}} = \frac{0{\cdot}60 \times 100}{5}$$

$$= \frac{60 \times 100}{100 \times 5} = 12\%$$

Question 2.
Find the selling price, if:
(i) C.P. = Rs. 500 and gain = 25%
(ii) C.P. = Rs. 60 and loss = 12 1/2%
(iii) C.P. = Rs. 150 and loss = 20%
(iv) C.P. = Rs. 80 and gain = 2.5%

Solution:

(i) C.P. = Rs. 500, gain = 25%

$$\text{S.P.} = \frac{\text{C.P.}\,(100 + \text{gain\%})}{100}$$

$$= \frac{500\,(100 + 25)}{100} = \text{Rs.}\frac{500 \times 125}{100}$$

$$= \text{Rs. } 625$$

(ii) C.P. = Rs. 60

$$\text{Loss} = 12\frac{1}{2}\% = \frac{25}{2}\%$$

$$\therefore \text{S.P.} = \frac{\text{C.P.}\,(100 - \text{Loss\%})}{100}$$

$$= \frac{60\left(100 - \frac{25}{2}\right)}{100} = \frac{60\left(\frac{200-25}{2}\right)}{100}$$

$$= \frac{60 \times 175}{2 \times 100} = \text{Rs.}\frac{105}{2} = \text{Rs. } 52{\cdot}50$$

(iii) C.P. = Rs. 150, Loss = 20%

$$\therefore \text{S.P.} = \frac{\text{C.P.}\,(100 - \text{loss\%})}{100}$$

$$= \frac{150\,(100 - 20)}{100} = \frac{150 \times 80}{100}$$

$$= \text{Rs. } 120$$

(iv) C.P. = Rs. 80, gain = 2·5%

$$\therefore \text{S.P.} = \frac{\text{C.P.}\,(100 + \text{gain \%})}{100}$$

$$= \frac{80\,(100 + 2{\cdot}5)}{100} = \frac{80 \times 102{\cdot}5}{100}$$

$$= \frac{\text{Rs. } 80 \times 1025}{100 \times 10} = \text{Rs.}82$$

Question 3.
Rohit bought a tape-recorder for Rs. 1,500 and sold it for Rs. 1,800. Calculate his profit or loss percent.

Solution:
C.P. of tape-recorder = Rs. 1500
S.P. = Rs. 1800

$\therefore$ Gain = S.P. – C.P.

= Rs. 1800 – Rs. 1500 = Rs. 300

$$\text{Gain \%} = \frac{\text{Gain} \times 100}{\text{C.P.}} = \frac{300 \times 100}{1500}$$

= 20%

Question 4.
An article bought for Rs. 350 is sold at a profit of 20%. Find its selling price.

Solution:
C.P. of article = Rs. 350
Profit = 20%

$$\therefore \text{S.P.} = \frac{\text{C.P.}\ (100 + \text{profit\%})}{100}$$

$$= \text{Rs. } \frac{350\ (100 + 20)}{100} = \text{Rs. } \frac{350 \times 120}{100}$$

= Rs. 420

Question 5.
An old machine is bought for Rs. 1,400 and is sold at a loss of 15%. Find its selling price.

Solution:
C.P. of the machine = Rs. 1400
Loss = 15%

$$\therefore \text{S.P.} = \frac{\text{C.P.}\ (100 - \text{Loss\%})}{100}$$

$$= \frac{1400\ (100 - 15)}{100} = \text{Rs. } \frac{1400 \times 85}{100}$$

= Rs. 1190

Question 6.
Oranges are bought at 5 for Rs. 10 and sold at 6 for Rs. 15. Find profit or loss as percent.

Solution:
L.C.M. of 5 and 6 = 30

Let 30 oranges are bought

$\therefore$ C.P. of 30 oranges $= \frac{30 \times 10}{5} =$ Rs. 60

and S.P. of 30 oranges $= \frac{30 \times 15}{6}$

= Rs. 75

Gain = S.P. – C.P.

= Rs. 75 – Rs. 60 = Rs. 15

$\therefore$ Gain % $= \frac{\text{gain} \times 100}{\text{C.P.}}$

$= \frac{15 \times 100}{60} = 25\%$

Question 7.
A certain number of articles are bought at 3 for Rs. 150 and all of them are sold at 4 for Rs. 180. Find the loss or gain as percent.

Solution:
L.C.M. of 3 and 4 = 12

Let 12 articles are bought

$\therefore$ C.P. of 12 articles = Rs. $\frac{150 \times 12}{3}$

= Rs. 600

and S.P. of 12 articles = Rs. $\frac{180 \times 12}{4}$

= Rs. 540

Loss = C.P. – S.P.

= Rs. 600 – Rs. 540 = Rs. 60

Loss % $= \frac{\text{Loss} \times 100}{\text{C.P.}} = \frac{60 \times 100}{600}$

$= 10\%$

Question 8.
A vendor bought 120 sweets at 20 p each. In his house, 18 were consumed and he sold the remaining at 30 p each. Find his profit or loss as percent.

Solution:

Quantity of sweets bought = 120

$$\therefore \text{ C.P. of 120 sweets } = \frac{120 \times 20}{100}$$

$$= \text{Rs. } 24$$

No. of sweets consumed = 18

Balance sweets = 120 – 18 = 102

$\therefore$ S.P. of 102 sweets

$$= \frac{102 \times 30}{100} = \frac{3060}{100} = \text{Rs. } 30{\cdot}60$$

Gain = S.P. – C.P.

$$= \text{Rs. } 30{\cdot}60 - \text{Rs. } 24 = \text{Rs. } 6{\cdot}60$$

$$\text{Gain \%} = \frac{\text{gain} \times 100}{\text{C.P.}} = \frac{6{\cdot}60 \times 100}{24}$$

$$= \frac{660 \times 100}{100 \times 24} = \frac{55}{2} = 27{\cdot}5\%$$

Question 9.
The cost price of an article is Rs. 1,200 and selling price is $\frac{5}{4}$ times of its cost price. Find:
(i) selling price of the article
(ii) profit or loss as percent.

Solution:

Cost price (C.P.) = Rs. 1200

$$\therefore \text{ S.P.} = \frac{5}{4} \text{ of C.P.}$$

$$= \frac{5}{4} \times 1200 = \text{Rs. } 1500$$

$\therefore$ Gain = S.P. – C.P.

$$= \text{Rs. } 1500 - \text{Rs. } 1200 = \text{Rs. } 300$$

$$\therefore \text{ Gain\%} = \frac{\text{gain} \times 100}{\text{C.P.}} = \frac{300 \times 100}{1200}$$

$$= 25\%$$

Question 10.

The selling price of an article is Rs. 1,200 and cost price is $\frac{5}{4}$ times of its selling price,
find :
(i) cost price of the article ;
(ii) profit or loss as percent.

Solution:

(i) S.P. of an article = Rs. 1200

$$\therefore \text{ C.P.} = \frac{5}{4} \text{ of S.P.} = \frac{5}{4} \times 1200 = \text{Rs. } 1500$$

(ii) Loss = C.P. – S.P.

$$= \text{Rs. } 1500 - \text{Rs. } 1200 = \text{Rs. } 300$$

$$\text{Loss\%} = \frac{\text{Loss} \times 100}{\text{C.P.}} = \frac{300 \times 100}{1500}$$

$$= \frac{100}{5} = 20\%$$

EXERCISE 9 (B)

Question 1.
Find the cost price, if:
(i) S.P. = Rs. 21 and gain = 5%
(ii) S.P. = Rs. 22 and loss = 12%
(iii) S.P. = Rs. 340 and gain = Rs. 20
(iv) S.P. = Rs. 200 and loss = Rs. 50
(v) S.P. = Re. 1 and loss = 5 p.

Solution:

(i) S.P. = Rs. 21, Gain = 5%

$$\therefore \text{C.P.} = \frac{\text{S.P.}\times 100}{100+\text{gain\%}} = \frac{21\times 100}{100+5}$$

$$= \frac{21\times 100}{105} = \text{Rs. } 20$$

(ii) S.P. = Rs. 22, loss = 12%

$$\therefore \text{C.P.} = \frac{\text{S.P.}\times 100}{100-\text{loss\%}} = \frac{22\times 100}{100-12}$$

$$= \frac{22\times 100}{88} = \text{Rs.} 25$$

(iii) S.P. = Rs. 340, Gain = Rs. 20

$\therefore$ C.P. = S.P. – Gain

= Rs. 340 – Rs. 20 = Rs. 320

(iv) S.P. = Rs. 200

Loss = Rs. 50

$\therefore$ C.P. = S.P. + loss

= Rs. 200 + Rs. 50 = Rs. 250

(v) S.P. = Re 1, Loss = 5 paise

$\therefore$ C.P. = S.P. + loss

= Re 1 + 5 p

= Rs. 1·05

Question 2.
By selling an article for Rs. 810, a loss of 10 percent is suffered. Find its cost price.

Solution:

S.P. of an article = Rs. 810

Loss = 10 %

$$\therefore \text{C.P.} = \frac{\text{S.P.} \times 100}{100 - \text{Loss\%}} = \frac{810 \times 100}{100 - 10}$$

$$= \frac{810 \times 100}{90} = \text{Rs. } 900$$

Question 3.
By selling a scooter for Rs. 9,200, a man gains 15%. Find the cost price of the scooter.

Solution:

S.P. of the scooter = Rs. 9200

Gain = 15%

$$\therefore \text{C.P.} = \frac{\text{S.P.} \times 100}{100 + \text{gain\%}} = \frac{9200 \times 100}{100 + 15}$$

$$= \frac{9200 \times 100}{115} = \text{Rs.} 8000$$

Question 4.
On selling an article for Rs. 2,640, a profit of 10 percent is made. Find
(i) cost price of the article
(ii) new selling price of it, in order to gain 15%

Solution:

S.P. of an article = Rs. 2640

Gain = 10%

(i) $\therefore$ C.P. $= \dfrac{\text{S.P.} \times 100}{100 + \text{gain}\%}$

$$= \text{Rs.} \frac{2640 \times 100}{100 + 10}$$

$$= \text{Rs.} \frac{2640 \times 100}{110} = \text{Rs.} 2400$$

(ii) In second case, Gain = 15%

$$\therefore \text{S.P.} = \frac{\text{C.P.}\ (100 + \text{gain}\ \%)}{100}$$

$$= \text{Rs.} \frac{2400\ (100 + 15)}{100}$$

$$= \text{Rs.} \frac{2400 \times 115}{100} = \text{Rs. } 2760$$

Question 5.
A T.V. set is sold for Rs. 6800 at a loss of 15%. Find
(i)cost price of the T.V. set.
(ii)new selling price of it, in order to gain 12%

Solution:

S.P. of the T.V. set = Rs. 6800
Loss = 15%

$$(i)\ \therefore \text{C.P.} = \frac{\text{S.P.} \times 100}{100 - \text{loss\%}} = \text{Rs.} \frac{6800 \times 100}{100 - 15}$$

$$= \text{Rs.} \frac{6800 \times 100}{85} = \text{Rs. } 8000$$

(ii) In second case, gain = 12 %

$$\therefore \text{S.P.} = \frac{\text{C.P. } (100 + \text{gain } \%)}{100}$$

$$= \text{Rs.} \frac{8000\ (100 + 12)}{100} = \text{Rs.} \frac{8000 \times 112}{100}$$
$$= \text{Rs. } 8960$$

Question 6.
A fruit seller bought mangoes at Rs. 90 per dozen and sold them at a loss of 8 percent. How much will a customer pay for.
(i) one mango
(ii) 40 mangoes

Solution:

C.P. of 1 dozen or 12 mangoes = Rs. 90
Loss = 8%

$\therefore$ S.P. of 1 dozen or 12 mangoes

$$= \frac{\text{C.P.} \times (100 - \text{loss\%})}{100}$$

$$= \text{Rs.} \frac{90(100 - 8)}{100} = \frac{90 \times 92}{100} = \frac{828}{10} = \text{Rs. } 82.80$$

$$(i)\ \text{S.P. of 1 mango} = \frac{82.80}{12} = \text{Rs.}6.90$$

(ii) S.P. of 40 mangoes = Rs. 6.90 × 40
= Rs. 276

Question 7.
By selling two transistors for Rs. 00 each, a shopkeeper gains 20 percent on one transistor and loses 20 percent on the other.
Find :
(i) C.P. of each transistor
(ii) total C.P. and total S.P. of both the transistors
(iii) profit or loss percent on the whole.

Solution:

S.P. of first transistor = Rs. 600

Gain = 20%

(i) $\therefore$ C.P. $= \dfrac{S.P. \times 100}{100 + \text{gain}\%} = \dfrac{600 \times 100}{100 + 20}$

$= \dfrac{600 \times 100}{120} =$ Rs. 500

S.P. of the second transistor = Rs. 600

Loss = 20%

$\therefore$ C.P. of the other transistor

$= \dfrac{S.P. \times 100}{100 - \text{loss}\ \%} = \dfrac{600 \times 100}{100 - 20}$

$=$ Rs. $\dfrac{600 \times 100}{80} =$ Rs. 750

$\therefore$ C.P. of the two transistors are Rs. 500 and Rs. 750

(ii) Total C.P. of both the transistors

= Rs. 500 + Rs. 750 = Rs. 1250

and total S.P. of both the transistors

= Rs. 600 + Rs. 600 = Rs. 1200

(iii) Total loss = C.P. – S.P.

= 1250 – 1200 = Rs. 50

$\therefore$ loss% $= \dfrac{\text{loss} \times 100}{C.P.} = \dfrac{50 \times 100}{1250} = 4\%$

Question 8.
Mangoes are bought at 20 for Rs. 60. If they are sold at $33\frac{1}{3}$ percent profit.
Find:
(i) selling price of each mango.
(ii) S.P. of 8 mangoes.

Solution:

$$\text{C.P. of 20 mangoes} = \text{Rs. } 60$$

$$\text{Gain} = 33\frac{1}{3}\% = \frac{100}{3}\%$$

$$\therefore \text{ S.P. of 20 mangoes} = \frac{\text{C.P.} \times (100 + \text{gain}\%)}{100}$$

$$= \frac{60\left(100 + \frac{100}{3}\right)}{100} = \text{Rs.} \frac{60 \times 400}{100 \times 3} = \text{Rs. } 80$$

$$(i) \text{ S.P. of 1 mango} = \text{Rs.} \frac{80}{20} = \text{Rs.} 4$$

$$(ii) \text{ S.P. of 8 mangoes} = \text{Rs. } 4 \times 8 = \text{Rs. } 32$$

Question 9.
Find the cost price of an article, which is sold for Rs. 4050 at a loss of 10%. Also, find the new selling price of the article which must give a profit of 8%.

Solution:

$$\text{S.P. of an article} = \text{Rs. } 4050$$

$$\text{Loss} = 10\%$$

$$(i) \therefore \text{ C.P. of the article} = \frac{\text{S.P.} \times 100}{100 - \text{loss}\%}$$

$$= ₹\frac{4050 \times 100}{100 - 10} = ₹\frac{4050 \times 100}{90}$$

$$= ₹4500$$

(*ii*) When gain = 8%

∴ New S.P. of the article

$$= \frac{\text{C.P.}(100 + \text{gain\%})}{100}$$

$$= ₹\frac{4500\,(100 + 8)}{100}$$

$$= ₹\frac{4500 \times 108}{100} = ₹4860$$

Question 10.

By selling an article for ₹825, a man loses $\frac{1}{3}$ equal to j of its selling price.
Find :
(i) the cost price of the article,
(ii) the profit percent or the loss percent made, if the same article is sold for ₹1265.

Solution:

S.P. of an article = ₹825

$$\text{Loss} = \frac{1}{3} \text{ of S.P.} = \frac{1}{3} \times 825 = ₹275$$

(*i*) ∴ C.P. = S.P. + Loss

= ₹825 + ₹275 = ₹1100

(*ii*) In second case,

S.P. = ₹1265

∴ Gain = S.P. – C.P.

= ₹1265 – ₹1100 = ₹165

$$\text{Gain \%} = \frac{\text{gain} \times 100}{\text{C.P.}}$$

$$= \frac{165 \times 100}{1100} = 15\%$$

Question 11.
Find the loss or gain as percent, if the C.P. of 10 articles, all of the same kind, is equal to S.P. of 8 articles.

Solution:

C.P. of 10 articles = S.P. of 8 articles = ₹80 (suppose)

$\therefore$ C.P. of 1 article = $\frac{80}{10}$ = ₹ 8

and S.P. of 1 article = $\frac{80}{8}$ = ₹ 10

$\therefore$ Gain = S.P. – C.P. = ₹ 10 – ₹ 8 = ₹ 2

$$\text{Gain } \% = \frac{\text{Gain} \times 100}{\text{C.P.}}$$

$$= \frac{2 \times 100}{8} = 25\%$$

Question 12.
Find the loss or gain as percent, if the C.P. of 8 articles, all of the same kind, is equal to S.P. of 10 articles.

Solution:

C.P. of 8 articles = S.P. of 10 articles = ₹80 (suppose)

$\therefore$ C.P. of 1 article = $\frac{80}{8}$ = ₹ 10

and S.P. of 1 article = $\frac{80}{10}$ = ₹ 8

$\therefore$ Loss = C.P. – S.P. = ₹ 10 – ₹ 8 = ₹ 2

$$\text{Loss } \% = \frac{\text{Loss} \times 100}{\text{C.P.}}$$

$$= \frac{2 \times 100}{10} = 20\%$$

Question 13.
The cost price of an article is 96% of its selling price. Find the loss or the gain as percent on the whole.

Solution:

Let S.P. = ₹100

C.P. = 96% of S.P.

$$= ₹\frac{96}{100} \times 100 = ₹96$$

$\therefore$ Gain = ₹100 – ₹96 = ₹4

and, Gain percent

$$= \frac{\text{Gain} \times 100}{\text{C.P.}}$$

$$= \frac{4}{96} \times 100\%$$

$$= \frac{25}{6} \text{ or } 4\frac{1}{6}\%$$

Question 14.
The selling price of an article is 96% of its cost price. Find the loss or the gain as percent on the whole.

Solution:

Let C.P. = ₹100

S.P. = 96% of C.P.

$$= ₹\frac{96}{100} \times 100 = ₹96$$

$\therefore$ Loss = ₹100 – ₹96 = ₹4

and loss percent $= \frac{\text{Loss} \times 100}{\text{C.P.}}$

$$= \frac{4}{100} \times 100\%$$

$$= 4\ \%$$

Question 15.

Hundred oranges are bought for ₹350 and all of them are sold at the rate of ₹48 per dozen. Find the profit percent or loss percent made.

Solution:

$\because$ C.P. of one orange $= ₹\frac{350}{100} = ₹3.50$

and S.P. of one orange $= ₹\frac{48}{12} = ₹4$

Clearly, Gain $= ₹4 - ₹3.50 = ₹0.50$

and Gain percent $= \frac{\text{Gain} \times 100}{\text{C.P.}}$

$= ₹\frac{0.50}{3.50} \times 100\% = 14\frac{2}{7}\%$

Question 16.

Oranges are bought at 100 for ?80 and all of them are sold at ₹80 for ₹100. Find the loss or gain as percent in this transaction.

Solution:

$\because$ C.P. of one orange $= ₹\frac{80}{100} = ₹0.8$

and S.P. of one orange $= ₹\frac{100}{80} = ₹1.25$

Clearly, profit $= ₹1.25 - ₹0.8 = ₹0.45$

and profit% $= ₹\frac{0.45}{0.8} \times 100$

$= \frac{45}{8} \times \frac{100 \times 10}{100}$

$= \frac{450}{8} = 56.25\%$

$\therefore$ Profit% $= 56.25\%$

Question 17.
An article is bought for ₹5,700 and ₹1,300 is spent on its repairing, transportion, etc. For how much should this article be sold in order to gain 20% on the whole.

Solution:

C.P. of an article = ₹5700

Amount spent on repair = ₹1300

Total cost price (C.P.) = ₹5700 + ₹1300

= ₹7000

Gain = 20%

$\therefore$ S.P. of an article = $\frac{\text{C.P.} \times (100 + \text{Gain\%})}{100}$

$= ₹\frac{7000 \times (100 + 20)}{100}$

$= ₹\frac{7000 \times 120}{100} = ₹\frac{840000}{100} = ₹8400$

$\therefore$ Selling price of an article (S.P.) = ₹8400

EXERCISE 9 (C)

Question 1.
A machine is marked at ₹5000 and is sold at a discount of 10%. Find the selling price of the machine.

Solution:

Marked price (M.P.) of the machine

= ₹5000

Rate of discount = 10%

$\therefore$ Amount of discount = $₹5000 \times \frac{10}{100} = ₹500$

$\therefore$ Selling price = M.P. – discount

= ₹5000 – ₹500 = ₹4500

Question 2.
shopkeeper marked a dinner set for ₹1000. He sold it at ₹900, what percent discount did he give ?

Solution:

Marked price of a dinner set = ₹1000

and selling price (S.P.) = ₹900

∴ Amount of discount = Rs. 1000 – ₹900
= ₹100

$$\therefore \text{ Discount percent } = \frac{\text{Discount } \times 100}{\text{M.P.}}$$

$$= \frac{100 \times 100}{1000} = 10\%$$

Question 3.
A pair of shoes marked at ₹320, are sold at a discount of 15 percent.
Find :
(i) discount
(ii) selling price of the shoes.

Solution:

Marked Price (M.P.) of shoes = ₹320

Rate of discount = 15%

$$(i) \ \therefore \text{ Amount of discount} = ₹\frac{320 \times 15}{100} = ₹48$$

(ii) Selling price = M.P. – Discount
= ₹320 – 48 = ₹272

Question 4.
The list price of an article is ₹450 and it is sold for ₹360.
Find :
(i) discount
(ii) discount percent

Solution:

List price (M.P.) of an article = ₹450

Selling price = ₹360

(*i*) $\therefore$ Amount of discount = M.P. – S.P.

₹450 – ₹360 = ₹90

(*ii*) Discount percent. $= \dfrac{\text{Discount} \times 100}{\text{M.P.}}$

$$= \frac{90 \times 100}{450} = 20\%$$

Question 5.

A shopkeeper buys an article for₹300. He increases its price by 20% and then gives 10% discount on the new price. Find:
(i) the new price (marked price) of the article.
(ii) the discount given by the shopkeeper.
(iii) the selling price.
(iv) profit percent made by the shopkeeper.

Solution:

C.P. of an article = ₹300

Increase in price = 20%

(*i*) $\therefore$ Marked price (M.P.)

$$= \frac{\text{C.P.} \times (100 + \text{increase\%})}{100}$$

$$= ₹\frac{300(100+20)}{100}$$

$$= ₹\frac{300 \times 120}{100} = ₹360$$

(*ii*) Rate of discount = 10%

Amount of discount

$$= ₹\frac{360 \times 10}{100} = ₹36$$

(*iii*) $\therefore$ Selling price = M.P. – discount

= ₹360 – 36 = ₹324

(*iv*) Net profit to the shopkeeper

= S.P. – C.P. = Rs. 324 – 300 = Rs. 24

$$\text{Gain \%} = \frac{\text{gain} \times 100}{\text{C.P.}} = \frac{24 \times 100}{300} = 8\%$$

Question 6.

A car is marked at Rs. 50,000. The dealer gives 5% discount on first Rs. 20,000 and 2% discount on the remaining Rs. 30,000.
Find :
(i) the total discount.
(ii) the price charged by the dealer.

Solution:

Marked Price (M.P.) of a car = Rs. 50,000

Discount at the rate of 5% on first

Rs. 20,000 = Rs. $\frac{20,000 \times 5}{100}$ = Rs. 1000

Discount at the rate of 2% on remaining

Rs. 30,000 = Rs. $\frac{30,000 \times 2}{100}$ = Rs 600

(i) $\therefore$ Total discount = Rs. 1000 + Rs. 600
= Rs. 1600

(ii) Price charged by the dealer
= Rs. 50,000 – Rs. 1600 = Rs. 48400

Question 7.

A dealer buys a T.V. set for Rs. 2500. He marks it at Rs. 3,200 and then gives a discount of 10% on it.
Find :
(i) the selling price of the T.V. set
(ii) the profit percent made by the dealer.

Solution:

C.P. of a T.V. set = Rs. 2,500, M.P.=Rs. 3,200

Rate of discount = 10%

$\therefore$ Total discount = Rs. 3,200 $\times \frac{10}{100}$ = Rs. 320

(i) Selling price = Rs. 3,200 – Rs. 320 = Rs. 2,880

(ii) Gain = S.P. – C.P.
= Rs. 2,880 – Rs. 2,500 = Rs. 380

$$\therefore \text{Gain } \% = \frac{\text{gain} \times 100}{\text{C.P.}} = \frac{380 \times 100}{2500}$$

$$= \frac{76}{5} = 15\frac{1}{5}\% = 15{\cdot}2\%$$

Question 8.

A sells his goods at 15% discount. Find the price of an article which is sold for Rs. 680.

Solution:

S.P. of an article = Rs. 680

Rate of discount = 15%

Let M.P. of the article = Rs. 100

$\therefore$ S.P. = Rs. 100 − 15 = Rs. 85

If S.P. is Rs. 85, then M.P. = Rs. 100

and if S.P. is Rs. 680, then M.P.

$$= \text{Rs. } \frac{100 \times 680}{85} = \text{Rs. } 800$$

Question 9.

A shopkeeper allows 20% discount on the marked price of his articles. Find the marked price of an article for which he charges Rs. 560.

Solution:

Let Marked Price (M.P.) = Rs. 100

Discount = 20%

$\therefore$ S.P. of that article = Rs. 100 − 20 = Rs. 80.

If S.P. is Rs. 80, then marked price = Rs. 100

and if S.P. is Rs. 560, then marked price

$$= \text{Rs. } \frac{100 \times 560}{80} = \text{Rs. } 700$$

Question 10.
An article is bought for Rs. 1,200 and Rs. 100 is spent on its transportation, etc. Find :
(i) the total C.P. of the article.
(ii) the selling price of it in order to gain 20% on the whole.

Solution:

C.P. of an article = Rs. 1200

Amount spent on transportation = Rs. 100

(*i*) Total C.P. of that article = Rs. 1200 + 100
= Rs. 1300

(*ii*) Gain = 20%, $\text{S.P.} = \dfrac{\text{C.P.} \times (100 + \text{gain\%})}{100}$

$$= \frac{1300 \times (100 + 20)}{100} = \frac{1300 \times 120}{100} = \text{Rs. } 1560$$

Question 11.
40 pens are bought at 4 for Rs. 50 and all of them are sold at 5 for Rs. 80
Find :
(i) C.P. of one pen.
(ii) S/P. of one pen.
(iii) Profit made by selling one pen.
(iv) Profit percent made by selling one pen.
(v) C.P. of 40 pens
(vi) S.P. of 40 pens.
(vii) Profit made by selling 40 pens.
(viii) Profit percent made by selling 40 pens. Are the results of parts (iv) and (viii) same? What conclusion do you draw from the above result ?

Solution:

(*i*) C.P. of 4 pens = Rs. 50

$\therefore$ C.P. of 40 pens = $\frac{50 \times 40}{4}$ = Rs. 500

and C.P. of 1 pen = $\frac{500}{40}$ = Rs. $\frac{25}{2}$ = Rs. 12.50

(*ii*) S.P. of pens = Rs. 80

$\therefore$ S.P. of 1 pen = Rs. $\frac{80}{5}$ = Rs. 16

(*iii*) Profit on one pen = S.P. – C.P.
= Rs. 16.00 – 12.50 = Rs. 3.50

(*iv*) Profit percent = $\frac{\text{Profit} \times 100}{\text{C.P.}}$

$= \frac{3.50 \times 100}{12.50} = \frac{350 \times 100}{1250} = 28\%$

(*v*) C.P. of 40 pens = 40 × 12.50 = Rs. 500

(*vi*) S.P. of 40 pens = 40 × 16 = Rs. 640

(*vii*) Profit on 40 pens = S.P. – C.P.
= Rs. 640 – 500 = Rs. 140

(*viii*) Profit on 40 pens = $\frac{\text{Profit} \times 100}{\text{C.P.}} = \frac{140 \times 100}{500}$

= 28%

Yes, the results of (*iv*) and (*viii*) are same.
We see that profit of on equal number of articles remains the same.

Question 12.
The C.P. of 5 identical articles is equal to S.P. of 4 articles. Calculate the profit percent or loss percent made if all the articles bought are sold.

Solution:

C.P. of 5 articles = S.P. of 4 articles

Let C.P. of 5 articles = S.P. of 4 articles= Rs. 100

$$\therefore \text{ C.P. of 1 articles} = \text{Rs. } \frac{100}{5} = \text{Rs. } 20$$

$$\text{and S.P. of 1 article} = \text{Rs. } \frac{100}{4} = \text{Rs. } 25$$

$\therefore$Profit = S.P. – C.P. = Rs. [illegible] – Rs. 20 = Rs. 5

$$\therefore \text{ Profit } \% = \frac{\text{Profit} \times 100}{\text{C.P.}} = \frac{5 \times 100}{20} = 25\%$$

Question 13.
The C.P. of 8 pens is same as S.P. of 10 pens. Calculate the profit or loss percent made, if all the pens bought are considered to be sold

Solution:

C.P. of 8 pens = S.P. of 10 pens = Rs. 100 (suppose)

$$\therefore \text{ C.P. of 1 pen} = \frac{100}{8} = \text{Rs. } 12.50$$

$$\text{and S.P. of 1 pen} = \frac{100}{10} = \text{Rs. } 10$$

$\therefore$Loss = C.P.–S.P. = Rs. 12.50 – Rs. 10=Rs. 2.50

$$\text{Loss } \% = \frac{\text{Loss} \times 100}{\text{C.P.}} = \frac{2.50 \times 100}{12.50}$$

$$= \frac{250 \times 100 \times 100}{1250 \times 100} = 20\%$$

Question 14.
A certain number of articles are bought at Rs. 450 per dozen and all of them are sold at a profit of 20%. Find the S.P. of:
(i) one article
(ii)seven articles.

Solution:

C.P. of 1 dozen articles = Rs. 450

Profit = 20%

$$\therefore \text{ S.P.} = \frac{\text{C.P.} \times (100 + \text{Profit})}{100}$$

$$= \text{Rs. } \frac{450 \times (100 + 20)}{100} = \text{Rs. } \frac{450 \times 120}{100} = \text{Rs. } 540$$

(i) $\therefore$ S.P. of 1 article = Rs. $\frac{540}{12}$ = Rs. 45

(ii) S.P. of 7 articles = Rs. 45 × 7 = Rs. 315

Question 15.
An article is marked 60% above the cost price and sold at 20% discount. Find the profit percent made.

Solution:

Let cost price of an article = Rs. 100

$\therefore$ Marked price = Rs. 100 + 60 = Rs. 160

Rate of discount = 20%

$$\therefore \text{ S.P.} = \frac{\text{M.P.} \times (100 - \text{Discount\%})}{100}$$

$$= \frac{160 \times (100 - 20)}{100} = \frac{160 \times 80}{100} = \text{Rs. } 128$$

Profit = S.P. – C.P.

= Rs. 128 – 100 = Rs. 28

$$\therefore \text{ Profit \%} = \frac{\text{Profit} \times 100}{\text{C.P.}} = \frac{28 \times 100}{100} = 28\%$$

CHAPTER - 10
SIMPLE INTEREST

POINTS TO REMEMBER

1. Simple Interest : $\dfrac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$

(*i*) or S.I. or Interest $= \dfrac{P \times R \times T}{100}$ where, P = Principal, R = Rate % and T = Time or pe

(*ii*) $P = \dfrac{S.I. \times 100}{R \times T}$ (*iii*) $R = \dfrac{S.I. \times 100}{P \times T}$ and (*iv*) $T = \dfrac{S.I. \times 100}{P \times R}$

2. Amount ; Principal + S. Interest

i.e. $A = P + S.I.$ or $A = P + I = P + \dfrac{PRT}{100}$

Question 1.

Find the S.I. and amount on :

(i) Rs. 150 for 4 years at 5% per year.

(ii) Rs. 350 for $3\frac{1}{2}$ years at 8% p.a.

(iii) Rs. 620 for 4 months at 8 p. per rupee per month.

(iv) Rs. 3,380 for 30 months at $4\frac{1}{2}$% p.a.

(v) 600 from July 12 to Dec. 5 at 10% p.a.

(vi) Rs. 850 from 10th March to 3rd August at $2\frac{1}{2}$% p.a.

(vii) Rs. 225 for 3 years 9 months at 16% p.a.

Solution:

(*i*) P = Rs. 150, R = 5% per year

T = 4 years

$\therefore$ S.I. $= \dfrac{P.R.T.}{100} = \dfrac{150 \times 5 \times 4}{100}$

$= \text{Rs. } 30$

and amount $= P + S.I.$

$= \text{Rs. } 150 + \text{Rs. } 30 = \text{Rs. } 180$

(*ii*) $P = \text{Rs. } 350$, $R = 8\%$ p.a.

$$T = 3\frac{1}{2} \text{ years} = \frac{7}{2} \text{ years}$$

$$\therefore \ S.I. = \frac{P \times R \times T}{100} = \frac{350 \times 8 \times 7}{100 \times 2}$$

$= \text{Rs. } 98$

Amount $= P + S.I. = \text{Rs. } 350 + \text{Rs. } 98$

$= \text{Rs. } 448$

(*iii*) $P = \text{Rs. } 620$

$R = 8$ p. per rupee per month $= 8\%$ p.m.

$T = 4$ months

$$\therefore \ S.I. = \frac{P.R.T.}{100} = \frac{620 \times 8 \times 4}{100}$$

$$= \text{Rs.}\frac{19840}{100} = \text{Rs. } 198{\cdot}40$$

$\therefore$ Amount = P + S.I.

= Rs. 620 + Rs. 198·40 = Rs. 818·40

(*iv*) Principal (P) = Rs. 3380

$$\text{Rate} = 4\frac{1}{2}\% \text{ p.a.} = \frac{9}{2}\%$$

$$\text{Period} = 30 \text{ months} = \frac{30}{12} \text{ years}$$

$$\text{S.I.} = \frac{PRT}{100} = \frac{3380 \times 9 \times 30}{100 \times 2 \times 12}$$

$$= \text{Rs. } \frac{1521}{4} = \text{Rs. } 380.25$$

Amount = P + S.I.

= Rs. 3380 + 380.25 = Rs. 3760.25

(*v*) P = Rs. 600, R = 10% p.a.

T = July 12 to Dec. 5

July	= 19 Days
Aug.	= 31 Days
Sep.	= 30 Days
Oct.	= 31 Days
Nov.	= 30 Days
Dec.	= 05 Days
Total	146 Days

$$= \frac{146}{365} \text{ years} = \frac{2}{5} \text{ years}$$

$$\therefore \text{ S.I.} = \frac{\text{P.R.T.}}{100} = \frac{600 \times 10 \times 2}{100 \times 5} = \text{Rs. } 24$$

$$\therefore \text{ Amount} = \text{P} + \text{S.I.} = \text{Rs. } 600 + \text{Rs. } 24$$

$$= \text{Rs. } 624$$

(*vi*) P = Rs. 850,

$$\text{R} = 2\frac{1}{2}\% = \frac{5}{2}\% \text{ p.a.}$$

T = 10th march to 3rd Aug.

March	= 21 days
April	= 30 days
May	= 31 days
June	= 30 days
July	= 31 days
Aug.	= 03 days
Total	146 days

$$= \frac{146}{365} = \frac{2}{5} \text{ years}$$

$$\therefore \text{ S.I.} = \frac{\text{P.R.T.}}{100} = \frac{850 \times 5 \times 2}{100 \times 2 \times 5} = \frac{850}{100}$$

$$= \text{Rs. } 8{\cdot}50$$

$$\therefore \text{ Amount} = \text{P} + \text{S.I.}$$

$$= \text{Rs. } 850 + \text{Rs. } 8{\cdot}50 = \text{Rs. } 858{\cdot}50$$

(*vii*) P = Rs. 225, R = 16% p.a.

T = 3 years 9 months

$$= 3\frac{9}{12} = 3\frac{3}{4} \text{ years} = \frac{15}{4} \text{ years}$$

$$\therefore \text{ S.I.} = \frac{\text{P.R.T.}}{100} = \frac{225 \times 16 \times 15}{100 \times 4} = \text{Rs. } 135$$

$$\therefore \text{ Amount} = \text{P} + \text{S.I.} = \text{Rs. } 225 + \text{Rs. } 135$$

$$= \text{Rs. } 360$$

Question 2.
On what sum of money does the S.I. for 10 years at 5% become Rs. 1,600 ?

Solution:

$$S.I. = Rs.\ 1600,\ R = 5\%\ p.a.$$
$$T = 10\ years$$
$$\therefore\ P = \frac{S.I. \times 100}{R \times T} = \frac{1600 \times 100}{5 \times 10} = Rs.\ 3200$$

Question 3.
Find the time in which Rs. 2,000 will amount to Rs. 2,330 at 11% p.a. ?

Solution:

$$Amount\ (A) = Rs.\ 2,330$$
$$Principal\ (P) = Rs.\ 2,000$$
$$\therefore\ S.I. = A - P = Rs.\ 2,330 - Rs.\ 2,000$$
$$= Rs.\ 330$$
$$R = 11\%\ p.a.$$
$$\therefore\ Time = \frac{S.I. \times 100}{P \times R} = \frac{330 \times 100}{2000 \times 11}$$
$$= \frac{3}{2} = 1\frac{1}{2}\ years$$

Question 4.
In what time will a sum of money double it self at 8% p.a ?

Solution:

$$Let\ the\ principal\ (P) = ₹100$$
$$\therefore\ Amount\ (A) = ₹100 \times 2 = ₹200$$
$$\therefore\ S.I. = A - P = ₹200 - ₹100$$
$$= ₹100$$
$$Rate\ (R) = 8\%\ p.a.$$
$$\therefore\ Time = \frac{S.I. \times 100}{P \times R} = \frac{100 \times 100}{100 \times 8}$$
$$= \frac{25}{2} = 12\frac{1}{2}\ years$$

Question 5.

In how many years will be ₹870 amount to ₹1,044, the rate of interest being $2\frac{1}{2}$ % p.a ?

Solution:

Principal (P) = ₹870

Amount (A) = ₹1044

$\therefore$ S.I. = P – A = ₹1044 – ₹870

= ₹174

Rate (R) = $2\frac{1}{2} = \frac{5}{2}$% p.a.

$$\therefore \text{ Time } = \frac{\text{S.I.} \times 100}{\text{P} \times \text{R}} = \frac{174 \times 100 \times 2}{870 \times 5}$$

= 8 years.

Question 6.

Find the rate percent if the S.I. on ₹275 is 2 years is ₹22.

Solution:

Principal (P) = ₹275, S.I. = ₹22

Time = 2 years

$$\therefore \text{ Rate } = \frac{\text{S.I.} \times 100}{\text{P} \times \text{T}} = \frac{22 \times 100}{275 \times 2}$$

= 4% p.a.

Question 7.
Find the sum which will amount to ₹700 in 5 years at 8% rate p.a.

Solution:
Amount = ₹700, Rate (R) = 8% p.a.
Time (T) = 5 years
Let principal (P) = ₹100

then $S.I. = \frac{P.R.T.}{100} = \frac{100 \times 8 \times 5}{100} = ₹40$

$\therefore$ Amount (A) = P + S.I.

= ₹100 + 40 = ₹140

If amount is ₹140, then principal = ₹100
and, if amount is Rs. 700, then principal

$= ₹\frac{100 \times 700}{140} = ₹500$

Question 8.
What is the rate of interest, if ₹3,750 amounts to ₹4,650 in 4 years ?

Solution:
Principal (P) = ₹3,750
Amount (A) = ₹4,650
$\therefore$ S.I. = A – P = ₹4,650 – 3,750
= ₹900
Time (T) = 4 years.

$\therefore$ Rate $= \frac{S.I. \times 100}{P \times T} = \frac{900 \times 100}{3750 \times 4}$
= 6% p.a.

Question 9.
In 4 years, ₹6,000 amount to ₹8,000. In what time will ₹525 amount to ₹700 at the same rate ?

Solution:

In first case, Principal (P) = ₹6,000

Amount (A) = ₹8,000

$\therefore$ S.I. = A – P = ₹8,000 – ₹6,000

= ₹2000

Time (T) = 4 years

$$\therefore R = \frac{S.I. \times 100}{P \times T} = \frac{2000 \times 100}{6000 \times 4}$$

$$= \frac{25}{3}\% = 8\frac{1}{3}\% \text{ p.a.}$$

In second case, Principal (P) = ₹525

Amount (A) = ₹700

$\therefore$ S.I. = A – P = ₹700 – ₹525

= ₹175

$$\text{Rate (R)} = \frac{25}{3}\% \text{ of p.a.}$$

$$\therefore \text{Time} = \frac{S.I. \times 100}{P \times R} = \frac{\text{Rs. } 175 \times 100 \times 3}{525 \times 25} = 4 \text{ years}$$

Question 10.
The interest on a sum of money at the end of $2\frac{1}{2}$ years is $\frac{4}{5}$ of the sum. What is the rate percent ?

Solution:

Let the sum (P) = Rs. 100

$$\therefore \text{S.I.} = \text{Rs. } 100 \times \frac{4}{5} = \text{Rs. } 80$$

$$\text{Period (T)} = 2\frac{1}{2} = \frac{5}{2} \text{ years.}$$

$$\therefore \text{Rate} = \frac{S.I. \times 100}{P \times T} = \frac{80 \times 100 \times 2}{100 \times 5} = 32\% \text{ p.a.}$$

Question 11.
What sum of money lent out at 5% for 3 years will produce the same interest as Rs. 900 lent out at 4% for 5 years ?

Solution:

In second case, Principal (P) = Rs. 900

Rate (R) = 4%, Time (T) = 5 years

$$\therefore \text{S.I.} = \frac{P \times R \times T}{100} = \frac{900 \times 4 \times 5}{100} = \text{Rs. } 180$$

In first case, S.I. = Rs. 180

Rate = 5%, Time = 3 years

$$\therefore \text{Sum} = \frac{\text{S.I.} \times 100}{R \times T} = \frac{180 \times 100}{5 \times 3} = \text{Rs. } 1200.$$

Question 12.
A sum of Rs. 1,780 become Rs. 2,136 in 4 years,
Find :
(i) the rate of interest.
(ii) the sum that will become Rs. 810 in 7 years at the same rate of interest ?

Solution:

(*i*) In first case, Principal (P) = Rs. 1,780

Amount (A) = Rs. 2,136

$\therefore$ S.I. = A – P = Rs. 2,136 – 1,780 = Rs. 356

Time (T) = 4 years

$$\therefore \quad \text{Rate} = \frac{\text{S.I.} \times 100}{P \times T} = \frac{356 \times 100}{1780 \times 4} = 5\% \text{ p.a.}$$

(*ii*) In second case, Let principal (P) = Rs. 100

Rate (R) = 5% p.a., Time (T) = 7 years

$$\therefore \text{S.I.} = \frac{P \times R \times T}{100} = \frac{100 \times 5 \times 7}{100} = \text{Rs. } 35$$

$\therefore$ Amount = P + S.I. = Rs. 100 + 35 = Rs. 135

If amount is Rs. 135, then principal = Rs. 100

and if amount is Rs. 810, then principal

$$= \text{Rs.} \frac{100 \times 810}{135} = \text{Rs. } 600$$

Question 13.
A sum amounts to Rs. 2,652 in 6 years at 5% p.a. simple interest.
Find :
(i) the sum
(ii) the time in which the same sum will double itself at the same rate of interest.

Solution:

.(*i*) In first case, Let principal (P) = Rs. 100

Rate (R) = 5% p.a., Time (T) = 6 years

$$\therefore \text{ S.I.} = \frac{P \times R \times T}{100} = \frac{100 \times 5 \times 6}{100} = \text{Rs.} 30$$

and, amount = Rs. 100 + Rs. 30 = Rs. 130

If amount is Rs. 130, then principal = Rs. 100

and, if amount is Rs. 2652, then principal

$$= \frac{100 \times 2,652}{130} = \text{Rs. } 2040$$

In second case, Let sum (P) = Rs. 100

Amount (A) = Rs. 100 × 2 = Rs. 200

S.I. = A – P = Rs. 200 – 100 = Rs. 100

Rate = 5% p.a.

$$\text{Time} = \frac{\text{S.I.} \times 100}{P \times R} = \frac{100 \times 100}{100 \times 5} = 20 \text{ years}$$

Question 14.
P and Q invest Rs. 36,000 and Rs. 25,000 respectively at the same rate of interest per year. If at the end of 4 years, P gets Rs. 3,080 more interest than Q; find the rate of interest.

Solution:

P's investment (P_1) = Rs. 36000

and Q's investment (P_2) = Rs. 25000

Period (T) = 4 years, Let rate of interest = x %

$$\text{Q's interest} = \text{Rs. } \frac{36000 \times x \times 4}{100}$$

$$= \text{Rs. } 1440x \qquad \left(\because \text{S.I.} = \frac{\text{PRT}}{100}\right)$$

$$\text{and Q's interest} = \frac{25000 \times x \times 4}{100} = \text{Rs. } 1000x$$

Difference in their interest

$= \text{Rs.}(1440 - 1000)\,x = \text{Rs. } 440x$

But difference = Rs. 3080

$$\therefore 440x = 3080 \Rightarrow x = \frac{3080}{440} \Rightarrow x = 7\%$$

$\therefore$ Rate of interest = 7% p.a.

Question 15.
A sum of money is lent for 5 years at R% simple interest per annum. If the interest earned be one-fourth of the money lent, find the value of R.

Solution:

Let the sum (P) = ₹100

$$\therefore \text{ S.I.} = \frac{1}{4} \times ₹100 = ₹25$$

Period (T) = 5 years

$$\therefore \text{ Rate\%} = \frac{\text{S.I.} \times 100}{\text{P} \times \text{T}} = \frac{25 \times 100}{100 \times 5} = 5\%$$

Question 16.
The simple interest earned on a certain sum in 5 years is 30% of the sum. Find the rate of interest.

Solution:

Let the sum (P) = ₹100

$$\text{S.I.} = \frac{30}{100} \times ₹100 = ₹30$$

Period (T) = 5 years

$$\therefore \text{ Rate} = \frac{\text{S.I.} \times 100}{\text{P} \times \text{T}} = \frac{30 \times 100}{100 \times 5} = 6\%$$

CHAPTER - 11
FUNDAMENTAL CONCEPTS

POINTS TO REMEMBER

1. **Constants and Variables :** The numbers which has fixed value is called constant and same at English alphabet which can be assigned any value according to the requirement is called variables.
2. **Term :** A term is a number, (constant), a variable or a combination of numbers and variables.
3. **Algebraic Expression :** An algebraic expression is a collection of one or more terms, which are separated from each other by addition (+) or subtraction (-) signs.
4. **Types of algebraic expressions :**
   (i) Monomial : It has only one term
   (ii) Binomial : It has two terms
   (iii) Trinomial : It has three terms
   (iv) Multinomial : It has more than three terms
   (v) Polynomial : It has two or more than two terms.

   Note : An expression of the type $\frac{2}{5}$ does not form a monomial unless JC is not equal to zero.
5. **Product:** When two or more quantities are multiplied together, the result is called their product.
6. **Factors :** Each of the quantities (numbers or variables) multiplied together to form a term is called a factor of the given term.
7. **Co-efficient:** In a monomial, any factor or group of factors of a term is called the co-efficient of the remaining part of the monomial.
8. **Degree of a monomial:** The degree of a monomial is the exponent of its variable or the sum of the exponents of its variables.
9. **Degree of a polynomial:** The degree of a polynomial is the degree of its highest degree term.
10. **Like and unlike terms** : Terms having the same literal co-efficients or alphabetic letters are called like terms ; whereas the terms with different literal co-efficients are called unlike terms.
11. **Addition and subtraction** : Addition and subtraction of only like terms is possible by adding or subtracting the numerical co-efficients.
12. **Multiplication and division** :
    (A) Multiplication :
    (i) Multiplications of monomials.
    (a) Multiply the numerical co-efficient together
    (ii) Multiply the literal co-efficients separately together.
    (iii) Combine the like terms.
    (B) Division :
    (i) Dividing a polynomial by a monomial Divide each term of the polynomial by monomial and simplify each fractions.

(ii) While dividing one polynomial by another polynomial ; arrange the terms of both the dividend and the divisior both in descending or in ascending order of their powers and then divide.

SOME IMPORTANT POINTS

TYPES OF BRACKETS:
The name of different types of brackets and the order in which they are removed is shown below:
(a) ____ ; Bar (Vinculum) bracket
(b) ( ); Circular bracket .
(c) { } ; Curly bracket and then
(d) [ ]; square bracket

EXERCISE 11 (A)

Question 1.
Separate constant terms and variable terms from tile following :

$$(i)\ 8, x, 6xy, 6+x, -5xy^2, 15az^2, \frac{32z}{xy}, \frac{y^2}{3x}$$

Solution:

Constant is only 8 others are variables

Question 2.
Constant is only 8 others are variables
(i) $2x \div 15$
(ii) $ax+9$
(iii) $3x^2 \times 5x$
(iv) $5 + 2a-3b$
(v) $2y - \frac{7}{3} z \div x$
(vi) $3p$ x $q \div z$
(vii) $12z \div 5x + 4$
(viii) $12 - 5z - 4$
(ix) $a^3 - 3ab^2$ x c

Answer:

(i) $2x \div 15 = \dfrac{2x}{15}$

It is a monomial as it has one term.

(ii) $ax + 9$: It is binomial

($\because$ It has two terms)

(iii) $3x^2 \times 5x = 15x^3$: It is monomial

($\because$ It has one term)

(iv) $5 + 2a - 3b$: It is trinomial

($\because$ It has three terms)

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(v) $2y - \dfrac{7}{3}z \div x = 2y - \dfrac{7z}{3x}$: It is binomial

($\because$ It has two terms)

(vi) $3p \times q \div z = \dfrac{3pq}{z}$: It is monomial

($\because$ It has one term)

(vii) $12z \div 5x + 4 = \dfrac{12z}{5x} + 4$: It is binomial

($\because$ It has two terms)

(viii) $12 - 5z - 4 = 8 - 5z$: It is binomial

($\because$ It has two terms)

(ix) $a^3 - 3ab^2 \times c = a^3 - 3ab^2c$: It is binomial

($\because$ It has two terms)

Question 3.
Write the coefficient of:
(i) xy in $-3axy$
(ii) z^2 in p^2yz^2
(iii) mn in $-mn$
(iv) 15 in $-15p^2$

Solution:
(i) Co-efficient of xy in $-3axy = -3a$
(ii) Co-efficient of z^2 in $p^2yz^2 = p^2y$

(iii) Co-efficient of mn in – mn = – 1
(iv) Co-efficient of 15 in – $15p^2$ is $-p^2$

Question 4.
For each of the following monomials, write its degree :
(i) 7y
(ii) – x^2y
(iii) xy^2z
(iv) – $9y^2z^3$
(v) 3 m^3n^4
(vi) – $2p^2q^3r^4$

Solution:
(i) Degree of 7y = 1
(ii) Degree of – x^2y = 2+1=3
(iii) Degree of xy^2z = 1 + 2 + 1 = 4
(iv) Degree of – $9y^2z^3$ = 2 + 3 = 5
(v) Degree of $3m^3n^4$ = 3 + 4 = 7
(vi) Degree of – $2p^2q^3r^4$ = 2 + 3 + 4 = 9

Question 5.
Write the degree of each of the following polynomials :
(i) $3y^3$-x^2y^2 + 4x
(ii) p^3q^2 – $6p^2q^5$ + p^4q^4
(iii) – $8mn^6$+ $5m^3n$
(iv) 7 – $3x^2y$ + y^2
(v) 3x – 15
(vi) $2y^2z$ + $9yz^3$

Solution:
(i) The degree of $3y^3$ – x^2y^2+ 4x is 4 as x^2
y^2 is the term which has highest degree.
(ii) The degree of p^3q^2 – $6p^2q^5$-p^4q^4 is 8 as p^4 q^4 is the term which has highest degree.
(iii) The degree of- $8mn^6$ + $5m^3n$ is 7 as – $8mx^6$ is the term which has the highest degree.
(iv) The degree of 7 – $3x^2$ y + y^2 is 3 as – $3x^2y$ is the term which has the highest degree.
(v) The degree of 3x – 15 is 1 as 3x is the term which is highest degree.
(vi) The degree of $2y^2$ z + 9y z^3 is 4 as $9yz^3$ has the highest degree.

Question 6.
Group the like term together :
(i) $9x^2$, xy, – $3x^2$, x^2 and – 2xy
(ii) ab, – a^2b, – 3ab, $5a^2b$ and – $8a^2b$
(iii) 7p, 8pq, – 5pq – 2p and 3p

Solution:
(i) $9x^2$, $-3x^2$ and x^2 are like terms
xy and – 2xy are like terms
(ii) ab, – 3ab, are like terms,
$-a^2b$, $5a^2b$, $-8a^2b$ are like terms
(iii) 7p, – 2p and 3p are like terms,
8pq, – 5pq are like terms.

Question 7.
Write numerical co-efficient of each of the followings :
(i) y
(ii) -y
(iii) $2x^2y$
(iv) $-8xy^3$
(v) $3py^2$
(vi) $-9a^2b^3$

Solution:
(i) Co-efficient of y = 1
(ii) Co-efficient of-y = – 1
(iii) Co-efficient of 2x2y is = 2
(iv) Co-efficient of – 8xy3 is = – 8
(v) Co-efficient of Ipy2 is = 3
(vi) Co-efficient of – 9a2b3 is = – 9

Question 8.
In $-5x^3y^2z^4$; write the coefficient of:
(i) z^2
(ii) y^2
(iii) yz^2
(iv) x^3y
(v) $-xy^2$
(vi) $-5xy^2z$
Also, write the degree of the given algebraic expression.

Solution:
$-5x^3y^2z^4$
(i) Co-efficient of z2 is $-5x^3y^2z^2$
(ii) Co-efficient of y2 is $-5x^3z^4$
(iii) Co-efficient of yz^2 is $-5x^3yz^2$
(iv) Co-efficient of x^3y is $-5yz^4$
(v) Co-efficient of $-xy^2$ is $5x^2z^4$
(vi) Co-efficient of $-5xy^2z$ is x^2z^3
Degree of the given expression is 3 + 2 + 4 = 9

EXERCISE 11 (B)

Question 1.

Fill in the blanks :

(i) $8x + 5x =$ ………

(ii) $8x – 5x =$……..

(iii) $6xy^2 + 9xy^2 =$……..

(iv) $6xy^2 – 9xy^2 =$ ………

(v) The sum of 8a, 6a and 5b = ……..

(vi) The addition of 5, 7xy, 6 and 3xy = …………

(vii) $4a + 3b – 7a + 4b =$ ……….

(viii) $– 15x + 13x + 8 =$ ………

(ix) $6x^2y + 13xy^2 – 4x^2y + 2xy^2 =$ ……..

(x) $16x^2 – 9x^2 =$ and $25xy^2 – 17xy^2 =$………

Solution :

(i) $8x + 5x = 13x$

(ii) $8x - 5x = 3x$

(iii) $6xy^2 + 9xy^2 = 15xy^2$

(iv) $6xy^2 - 9xy^2 = -3xy^2$

(v) The sum of $8a$, $6a$ and $5b$

$= 8a + 6a + 5b = 14a + 5b$

(vi) The addition of 5, $7xy$, 6 and $3xy$

$= 5 + 6 + 7xy + 3xy = 11 + 10xy$

(vii) $4a + 3b - 7a + 4b$

$= 4a - 7a + 3b + 4b = -3a + 7b = 7b - 3a$

(viii) $-15x + 13x + 8$

$= -2x + 8 = 8 - 2x$

(ix) $6x^2y + 13xy^2 - 4x^2y + 2xy^2$

$= 6x^2y - 4x^2y + 13xy^2 + 2xy^2 = 2x^2y + 15xy^2$

(x) $16x^2 - 9x^2 = 7x^2$ and

$25xy^2 - 17xy^2 = 8xy^2$

Question 2.
Add :
(i)- 9x, 3x and 4x
(ii) $23y^2$, $8y^2$ and – $12y^2$
(iii) 18pq – 15pq and 3pq

Solution:

$$(i)\ -9x + 3x + 4x$$
$$= -9x + 7x = -2x$$
$$(ii)\ 23y^2 + 8y^2 - 12y^2$$
$$= 31y^2 - 12y^2 = 19y^2$$
$$(iii)\ 18pq - 15pq + 3pq$$
$$= 18pq + 3pq - 15pq = 21pq - 15pq = 6pq$$

Question 3.
Simplify :
(i) 3m + 12m – 5m
(ii) $7n^2 - 9n^2 + 3n^2$
(iii) 25zy—8zy—6zy
(iv) $-5ax^2 + 7ax^2 - 12ax^2$
(v) – 16am + 4mx + 4am – 15mx + 5am

Solution:

$$(i)\ 3m + 12m - 5m = 15m - 5m = 10m$$
$$(ii)\ 7n^2 - 9n^2 + 3n^2$$
$$= 7n^2 + 3n^2 - 9n^2 = 10n^2 - 9n^2 = n^2$$
$$(iii)\ 25zy - 8zy - 6zy$$
$$= 25zy - 14zy = 11zy$$
$$(iv)\ -5ax^2 + 7ax^2 - 12ax^2$$
$$= -5ax^2 - 12ax^2 + 7ax^2$$
$$= -17ax^2 + 7ax^2 = -10ax^2$$
$$(v)\ -16am + 4mx + 4am - 15mx + 5am$$
$$= -16am + 4am + 5am + 4mx - 15mx$$
$$= -16am + 9am + 4mx - 15mx = -7am - 11mx$$

Question 4.
Add :
(i) a + i and 2a + 3b
(ii) 2x + y and 3x – 4y
(iii)- 3a + 2b and 3a + b
(iv) 4 + x, 5 – 2x and 6x

Solution:

(i) $a + b$ and $2a + 3b$ (ii) $2x + y$ and $3x - 4y$

(iii) $-3a + 2b$ and $3a + b$ (iv) $4 + x, 5 - 2x$ and $6x$

(i) $a + b + 2a + 3b$

$= a + 2a + b + 3b = 3a + 4b$

(ii) $2x + y + 3x - 4y$

$= 2x + 3x + y - 4y = 5x - 3y$

(iii) $-3a + 2b + 3a + b$

$= -3a + 3a + 2b + b = 0 + 3b = 3b$

(iv) $4 + x + 5 - 2x + 6x$

$= x - 2x + 6x + 4 + 5$

$= 7x - 2x + 9 = 5x + 9$

Question 5.
Find the sum of:
(i) 3x + 8y + 7z, 6y + 4z- 2x and 3y – 4x + 6z
(ii) 3a + 5b + 2c, 2a + 3b-c and a + b + c.
(iii) $4x^2$+ 8xy – $2y^2$ and 8xy – $5y^2$ + x^2
(iv) $9x^2$ – 6x + 7, 5 – 4x and 6 – $3x^2$
(v) $5x^2$ – 2xy + $3y^2$ and – $2x^2$ + 5xy + $9y^2$
and $3x^2$ -xy- $4y^2$
(vi) a^2 + b^2 + 2ab, $2b^2$ + c^2 + 2bc
and $4c^2$-a^2 + 2ac
(vii) 9ax – 6bx + 8, 4ax + 8bx – 7
and – 6ax – 46x – 3
(viii) abc + 2 ba + 3 ac, 4ca – 4ab + 2 bca
and 2ab – 3abc – 6ac
(ix) $4a^2$ + $5b^2$ – 6ab, 3ab, $6a^2$ – $2b^2$
and $4b^2$ – 5 ab
(x) x^2 + x – 2, 2x – $3x^2$ + 5 and $2x^2$ – 5x + 7
(xi) $4x^3$ + $2x^2$ – x + 1, $2x^3$ – $5x^2$– 3x + 6, x^2 + 8 and $5x^3$ – 7x

Solution:

(i) $3x + 8y + 7z + 6y + 4z - 2x$

$+ 3y - 4x + 6z$

$= 3x - 2x - 4x + 8y + 6y + 3y + 7z + 4z + 6z$

$= 3x - 6x + 17y + 17z$

$= -3x + 17y + 17z$

(ii) $3a + 5b + 2c + 2a + 3b - c + a + b + c$

$= 3a + 2a + a + 5b + 3b + b + 2c - c + c$

$= 6a + 9b + 3c - c = 6a + 9b + 2c$

(iii) $4x^2 + 8xy - 2y^2$ and $8xy - 5y^2 + x^2$

$4x^2 + 8xy - 2y^2 + 8xy - 5y^2 + x^2$

$= 4x^2 + x^2 + 8xy + 8xy - 2y^2 - 5y^2$

$= 5x^2 + 16xy - 7y^2$

(iv) $9x^2 - 6x + 7, 5 - 4x$ and $6 - 3x^2$

$9x^2 - 6x + 7 + 5 - 4x + 6 - 3x^2$

$= 9x^2 - 3x^2 - 6x - 4x + 7 + 5 + 6$

$= 6x^2 - 10x + 18$

(v) $5x^2 - 2xy + 3y^2, -2x^2 + 5xy + 9y^2$

and $3x^2 - xy - 4y^2$

$5x^2 - 2xy + 3y^2 - 2x^2 + 5xy + 9y^2$

$+ 3x^2 - xy - 4y^2$

$= 5x^2 - 2x^2 + 3x^2 - 2xy + 5xy - xy$

$+ 3y^2 + 9y^2 - 4y^2$

$= 8x^2 - 2x^2 + 5xy - 3xy + 12y^2 - 4y^2$

$= 6x^2 + 2xy + 8y^2$

(*vi*) $a^2 + b^2 + 2ab,\ 2b^2 + c^2 + 2bc$

and $4c^2 - a^2 + 2ac$

$a^2 + b^2 + 2ab + 2b^2 + c^2 + 2bc$

$+ 4c^2 - a^2 + 2ac$

$= a^2 - a^2 + b^2 + 2b^2 + c^2 + 4c^2 + 2ab$

$+ 2bc + 2ac$

$= 3b^2 + 5c^2 + 2ab + 2bc + 2ac$

(*vii*) $9ax - 6bx + 8,\ 4ax + 8bx - 7$

and $- 6ax - 4bx - 3$

$9ax - 6bx + 8 + 4ax + 8bx - 7$

$- 6ax - 4bx - 3$

$= 9ax + 4ax - 6ax - 6bx + 8bx - 4bx$

$+ 8 - 7 - 3$

$= 13ax - 6ax + 8bx - 10bx + 8 - 10$

$= 7ax - 2bx - 2$

(*viii*) $abc + 2ba + 3ac,\ 4ca - 4ab + 2bca$

and $2ab - 3abc - 6ac$

$abc + 2ab + 3ca + 4ca - 4ab + 2abc$

$+ 2ab - 3abc - 6ca$

$= abc + 2abc - 3abc + 2ab - 4ab + 2ab$

$+ 3ca + 4ca - 6ca$

$= 3abc - 3abc + 4ab - 4ab + 7ca - 6ca$

$= 0 + 0 + ca = ca$

(ix) $4a^2 + 5b^2 - 6ab, 3ab, 6a^2 - 2b^2$

and $4b^2 - 5ab$

$4a^2 + 5b^2 - 6ab + 3ab + 6a^2 - 2b^2$
$+ 4b^2 - 5ab$

$= 4a^2 + 6a^2 + 5b^2 - 2b^2 + 4b^2 - 6ab$
$+ 3ab - 5ab$

$= 10a^2 + 9b^2 - 2b^2 - 11ab + 3ab$

$= 10a^2 + 7b^2 - 8ab$

(x) $x^2 + x - 2, 2x - 3x^2 + 5$ and $2x^2 - 5x + 7$

$= x^2 + x - 2 + 2x - 3x^2 + 5 + 2x^2 - 5x + 7$

$= x^2 - 3x^2 + 2x^2 + x + 2x - 5x - 2 + 5 + 7$

$= 3x^2 - 3x^2 + 3x - 5x - 2 + 12$

$= 0 - 2x + 10$

$= - 2x + 10$

(xi) $4x^3 + 2x^2 - x + 1, 2x^3 - 5x^2 - 3x + 6, x^2 + 8$

and $5x^3 - 7x$

$4x^3 + 2x^2 - x + 1 + 2x^3 - 5x^2 - 3x + 6$
$+ x^2 + 8 + 5x^3 - 7x$

$= 4x^3 + 2x^3 + 5x^3 + 2x^2 - 5x^2 + x^2 - x$
$- 3x - 7x + 1 + 6 + 8$

$= 11x^3 + 3x^2 - 5x^2 - 11x + 15$

$= 11x^3 - 2x^2 - 11x + 15$

Question 6.
Find the sum of:
(i) x and 3y
(ii) -2a and +5
(iii) – $4x^2$ and +7x
(iv) +4a and -7b
(v) x^3+3x^2y and $2y^2$
(vi) 11 and -by

Solution:

(i) $x + 3y$ (ii) $- 2a + 5$

(iii) $- 4x^2 + 7x$ (iv) $4a - 7b$

(v) $x^3 + 3x^2y + 2y^2$ (vi) $11 - by$

Question 7.
The sides of a triangle are 2x + 3y, x + 5y and 7x – 2y, find its perimeter.

Solution:
Sides of a triangle are $2x + 3y, x + 5y, 7x - 2y$
$\therefore$ Perimeter = sum of three sides of the triangle
$= 2x + 3y + x + 5y + 7x - 2y$
$= 2x + x + 7x + 3y + 5y - 2y$
$= 10x + 8y - 2x = 10x + 6y$

Question 8.
The two adjacent sides of a rectangle are 6a + 96 and 8a – 46. Find its, perimeter.

Solution
Sides of a rectangle are $6a + 9b$ and $8a - 4b$
Let, length $= 6a + 9b$
and breadth $= 8a - 4b$
$\therefore$ Perimeter = 2 (length + breadth)
$= 2\,(6a + 9b + 8a - 4b)$
$= 2\,(14a + 5b) = 28a + 10b$

Question 9.
Subtract the second expression from the first:
(i) $2a + b, a + b$ (ii) $-2b + 2c, b + 3c$
(iii) $5a + b, -6b + 2a$ (iv) $a^3 - 1 + a, 3a - 2a^2$
(v) $p + 2, 1$
(vi) $x + 2y + z, -x - y - 3z$
(vii) $3a^2 - 8ab - 2b^2, 3a^2 - 4ab + 6b^2$
(viii) $4pq - 6p^2 - 2q^2, 9p^2$
(ix) $10abc, 2a^2 + 2abc - 4b^2$
(x) $a^2 + ab + c^2, a^2 - d^2$

Solution:

(i) $(2a + b) - (a + b)$

$= 2a + b - a - b = 2a - a + b - b$

$= a + 0 = a$.

(ii) $(-2b + 2c) - (b + 3c)$

$= -2b + 2c - b - 3c$

$= -2b - b + 2c - 3c$

$= -3b - c$

(iii) $(5a + b) - (-6b + 2a)$

$= 5a + b + 6b - 2a$

$= 5a - 2a + b + 6b$

$= 3a + 7b$

(iv) $(a^3 - 1 + a) - (3a - 2a^2)$

$= a^3 - 1 + a - 3a + 2a^2$

$= a^3 + 2a^2 - 2a - 1$

(v) $(p + 2) - 1 = p + 2 - 1 = p + 1$

(vi) $(x + 2y + z) - (-x - y - 3z)$

$= x + 2y + z + x + y + 3z$

$= x + x + 2y + y + z + 3z$

$= 2x + 3y + 4z$

(vii) $(3a^2 - 8ab - 2b^2) - (3a^2 - 4ab + 6b^2)$

$= 3a^2 - 8ab - 2b^2 - 3a^2 + 4ab - 6b^2$

$= 3a^2 - 3a^2 - 2b^2 - 6b^2 - 8ab + 4ab$

$= 0 - 8b^2 - 4ab$

$= -4ab - 8b^2$

(viii) $(4pq - 6p^2 - 2q^2) - (9p^2)$

$= 4pq - 6p^2 - 2q^2 - 9p^2$

$= 4pq - 15p^2 - 2q^2$

(ix) $10abc - (2a^2 + 2abc - 4b^2)$

$$= 10abc - 2a^2 - 2abc + 4b^2$$
$$= 10abc - 2abc - 2a^2 + 4b^2$$
$$= 8abc - 2a^2 + 4b^2$$

(x) $(a^2 + ab + c^2) - (a^2 - d^2)$

$$= a^2 + ab + c^2 - a^2 + d^2$$
$$= a^2 - a^2 + ab + c^2 + d^2$$
$$= ab + c^2 + d^2$$

Question 10.

Subtract:

(i) $4x$ from $8 - x$

(ii) $-8c$ from $c + 3d$

(iii) $-5a - 2b$ from $b + 6c$

(iv) $4p + p^2$ from $3p^2 - 8p$

(v) $5a - 3b + 2c$ from $4a - b - 2c$

(vi) $-xy + yz - zx$ from $xy - yz + xz$

(vii) $2x^2 - 7xy - y^2$ from $3x^2 - 5xy + 3y^2$

(viii) $a^2 - 3ab - 6b^2$ from $2b^2 - a^2 + 2ab$

(ix) $4x^2 - 5x^2y + y^2$ from $-3y^2 + 5xy^2 - 7x^2 - 9x^2y$

(x) $6m^3 + 4m^2 + 7m - 3$ from $3m^3 + 4$

Solution:

(i) $4x$ from $8 - x$

$(8 - x) - 4x = 8 - x - 4x = 8 - 5x$

(ii) $-8c$ from $c + 3d$

$(c + 3d) - (-8c)$

$c + 3d + 8c = 9c + 3d$

(iii) $-5a - 2b$ from $b + 6c$

$(b + 6c) - (-5a - 2b)$

$= b + 6c + 5a + 2b = 5a + 3b + 6c$

(iv) $4p + p^2$ from $3p^2 - 8p$

$(3p^2 - 8p) - (4p + p^2)$

$= 3p^2 - 8p - 4p - p^2 = 2p^2 - 12p$

(v) $5a - 3b + 2c$ from $4a - b - 2c$

$(4a - b - 2c) - (5a - 3b + 2c)$

$= 4a - b - 2c - 5a + 3b - 2c$

$= 4a - 5a - b + 3b - 2c - 2c$

$= -a + 2b - 4c$

(vi) $-xy + yz - zx$ from $xy - yz + xz$

$(xy - yz + zx) - (-xy + yz - xz)$

$= xy - yz + zx + xy - yz + xz$

$= xy + xy - yz - yz + zx + xz$

$= 2(xy - yz + zx)$

(vii) $2x^2 - 7xy - y^2$ from $3x^2 - 5xy + 3y^2$

$(3x^2 - 5xy + 3y^2) - (2x^2 - 7xy - y^2)$

$= 3x^2 - 5xy + 3y^2 - 2x^2 + 7xy + y^2$

$= 3x^2 - 2x^2 - 5xy + 7xy + 3y^2 + y^2$

$= x^2 + 2xy + 4y^2$

(viii) $a^2 - 3ab - 6b^2$ from $2b^2 - a^2 + 2ab$

$(2b^2 - a^2 + 2ab) - (a^2 - 3ab - 6b^2)$

$= 2b^2 - a^2 + 2ab - a^2 + 3ab + 6b^2$

$= -a^2 - a^2 + 2b^2 + 6b^2 + 2ab + 3ab$

$= -2a^2 + 8b^2 + 5ab$

$= 8b^2 + 5ab - 2a^2$

(ix) $4x^2 - 5x^2y + y^2$ from $-3y^2 + 5xy^2 - 7x^2 - 9x^2y$

$(-3y^2 + 5xy^2 - 7x^2 - 9x^2y) - (4x^2 - 5x^2y + y^2)$

$= -3y^2 + 5xy^2 - 7x^2 - 9x^2y - 4x^2 + 5x^2y - y^2$

$= -3y^2 - y^2 + 5xy^2 - 7x^2 - 4x^2 - 9x^2y + 5x^2y$

$= -4y^2 + 5xy^2 - 11x^2 - 4x^2y$

(x) $6m^3 + 4m^2 + 7m - 3$ from $3m^3 + 4$

$(3m^3 + 4) - (6m^3 + 4m^2 + 7m - 3)$

$= 3m^3 + 4 - 6m^3 - 4m^2 - 7m + 3$

$= 3m^3 - 6m^3 - 4m^2 - 7m + 4 + 3$

$= -3m^3 - 4m^2 - 7m + 7$

Question 11.
Subtract – $5a^2$ – 3a + 1 from the sum of $4a^2$ + 3 – 8a and 9a – 7.

Solution:
Sum of $4a^2 + 3 - 8a$ and $9a - 7$
$= 4a^2 + 3 - 8a + 9a - 7 = 4a^2 + a - 4$
$\therefore (4a^2 + a - 4) - (-5a^2 - 3a + 1)$
$= 4a^2 + a - 4 + 5a^2 + 3a - 1$
$= 4a^2 + 5a^2 + a + 3a - 4 - 1$
$= 9a^2 + 4a - 5$

Question 12.
By how much does $8x^3$ – $6x^2$ + 9x – 10 exceed $4x^3$ + $2x^2$ + 7x -3 ?

Solution:
$8x^3 - 6x^2 + 9x - 10$ exceeds $4x^3 + 2x^2 + 7x - 3$
$= (8x^3 - 6x^2 + 9x - 10) - (4x^3 + 2x^2 + 7x - 3)$
$= 8x^3 - 6x^2 + 9x - 10 - 4x^3 - 2x^2 - 7x + 3$
$= 8x^3 - 4x^3 - 6x^2 - 2x^2 + 9x - 7x - 10 + 3$
$= 4x^3 - 8x^2 + 2x - 7$

Question 13.
What must be added to $2a^3$ + 5a – a^2 – 6 to get a^2 – a – a^3 + 1 ?

Solution:
We get, the required result by subtracting
$2a^3 - a^2 + 5a - 6$ from $-a^3 + a^2 - a + 1$
$= (-a^3 + a^2 - a + 1) - (2a^3 - a^2 + 5a - 6)$
$= -a^3 + a^2 - a + 1 - 2a^3 + a^2 - 5a + 6$
$= -a^3 - 2a^3 + a^2 + a^2 - a - 5a + 1 + 6$
$= -3a^3 + 2a^2 - 6a + 7$

Question 14.
What must be subtracted from $a^2 + b^2 + lab$ to get $-4ab + 2b^2$?

Solution:
We get, the required result by subtracting $-4ab + 2b^2$ from $a^2 + b^2 + 2ab$.

$= a^2 + b^2 + 2ab - (-4ab + 2b^2)$
$= a^2 + b^2 + 2ab + 4ab - 2b^2$
$= a^2 + b^2 - 2b^2 + 2ab + 4ab$
$= a^2 - b^2 + 6ab$

Question 15.
Find the excess of $4m^2 + 4n^2 + 4p^2$ over $m^2 + 3n^2 - 5p^2$

Solution:
The required result will be by subtracting $m^2 + 3n^2 - 5p^2$ from $4m^2 + 4n^2 + 4p^2$

$= 4m^2 + 4n^2 + 4p^2 - (m^2 + 3n^2 - 5p^2)$
$= 4m^2 + 4n^2 + 4p^2 - m^2 - 3n^2 + 5p^2$
$= 4m^2 - m^2 + 4n^2 - 3n^2 + 4p^2 + 5p^2$
$= 3m^2 + n^2 + 9p^2$

Question 16.
By how much is $3x^3 - 2x^2y + xy^2 - y^3$ less than $4x^3 - 3x^2y - 7xy^2 + 2y^3$

Solution:
We can get the required result by subtracting $3x^3 - 2x^2y + xy^2 - y^3$ from $4x^3 - 3x^2y - 7xy^2 + 2y^3$

$= (4x^3 - 3x^2y - 7xy^2 + 2y^3) - (3x^3 - 2x^2y + xy^2 - y^3)$
$= 4x^3 - 3x^2y - 7xy^2 + 2y^3 - 3x^3 + 2x^2y - xy^2 + y^3$
$= 4x^3 - 3x^3 - 3x^2y + 2x^2y - 7xy^2 - xy^2 + 2y^3 + y^3$
$= x^3 - x^2y - 8xy^2 + 3y^3$

Question 17.

Subtract the sum of 3a2 – 2a + 5 and a2 – 5a – 7 from the sum of 5a2 -9a + 3 and 2a – a2 – 1

Solution:

Sum of $3a^2 - 2a + 5$ and $a^2 - 5a - 7$

$= 3a^2 - 2a + 5 + a^2 - 5a - 7$

$= 3a^2 + a^2 - 2a - 5a + 5 - 7$

$= 4a^2 - 7a - 2$

and sum of $5a^2 - 9a + 3$ and $2a - a^2 - 1$

$= 5a^2 - 9a + 3 + 2a - a^2 - 1$

$= 5a^2 - a^2 - 9a + 2a + 3 - 1$

$= 4a^2 - 7a + 2$

Now $(4a^2 - 7a + 2) - (4a^2 - 7a - 2)$

$= 4a^2 - 7a + 2 - 4a^2 + 7a + 2$

$= 4a^2 - 4a^2 - 7a + 7a + 2 + 2$

$= 0 + 0 + 4 = 4.$

Question 18.

The perimeter of a rectangle is 28x3+ 16x2 + 8x + 4. One of its sides is 8x2 + 4x. Find the other side

Solution:

Perimeter of a rectangle $(2l + 2b)$

$= 28x^3 + 16x^2 + 8x + 4$

Let one side $(l) = 8x^2 + 4x$

$\therefore\ 2l = 2(8x^2 + 4x) = 16x^2 + 8x$

$\therefore\ 2b = (28x^3 + 16x^2 + 8x + 4) - (16x^2 + 8x)$

$= 28x^3 + 16x^2 + 8x + 4 - 16x^2 - 8x$

$= 28x^3 + 4$

$$\therefore \text{ Other side } (b) = \frac{28x^3 + 4}{2} = 14x^3 + 2$$

Question 19.
The perimeter of a triangle is $14a^2 + 20a + 13$. Two of its sides are $3a^2 + 5a + 1$ and $a^2 + 10a – 6$. Find its third side.

Solution:
Perimeter of a triangle $= 14a^2 + 20a + 13$
Sum of two sides
$= 3a^2 + 5a + 1 + a^2 + 10a - 6$
$= 3a^2 + a^2 + 5a + 10a + 1 - 6$
$= 4a^2 + 15a - 5$
$\therefore$ Third side $= (14a^2 + 20a + 13) - (4a^2 + 15a - 5)$
$= 14a^2 + 20a + 13 - 4a^2 - 15a + 5$
$= 14a^2 - 4a^2 + 20a - 15a + 13 + 5$
$= 10a^2 + 5a + 18$

Question 20.
$x = 4a^2 + b^2 - 6ab$
$y = 3b^2 - 2a^2 + 8ab$
$z = 6a^2 + 8b^2 - 6ab$
(i) $x + y + z = 4a^2 + b^2 - 6ab + 3b^2 - 2a^2 + 8ab + 6a^2 + 8b^2 - 6ab$
$= 4a^2 - 2a^2 + 6a^2 + b^2 + 3b^2 + 8b^2 - 6ab + 8ab - 6ab$
$= 10a^2 - 2a^2 + 12b^2 - 12ab + 8ab$
$= 8a^2 + 12b^2 - 4ab$
(ii) $x - y - z = (4a^2 + b^2 - 6ab) - (3b^2 - 2a^2 + 8ab) - (6a^2 + 8b^2 - 6ab)$
$= 4a^2 + b^2 - 6ab - 3b^2 + 2a^2 - 8ab - 6a^2 - 8b^2 + 6ab$
$= 4a^2 + 2a^2 - 6a^2 + b^2 - 3b^2 - 8b^2 - 6ab - 8ab + 6ab$
$= 6a^2 - 6a^2 + b^2 - 11b^2 - 14ab + 6ab$
$= -10b^2 - 8ab$

Solution:
If $x = 4a^2 + b^2 - 6ab$, $y = 3b^2 - 2a^2 + 8ab$
and $z = 6a^2 + 8b^2 - 6ab$ find :
(i) $x + y + z$ (ii) $x - y - z$

Question 21.

If $m = 9x^2 - 4xy + 5y^2$ and $n = -3x^2 + 2xy - y^2$ find :

(i) $2m - n$

(ii) $m + 2n$

(iii) $m - 3n$.

Solution:

$m = 9x^2 - 4xy + 5y^2$

$n = -3x^2 + 2xy - y^2$

(i) $2m - n = 2(9x^2 - 4xy + 5y^2) - (-3x^2 + 2xy - y^2)$

$= 18x^2 - 8xy + 10y^2 + 3x^2 - 2xy + y^2$

$= 18x^2 + 3x^2 - 8xy - 2xy + 10y^2 + y^2$

$= 21x^2 - 10xy + 11y^2$

(ii) $m + 2n = (9x^2 - 4xy + 5y^2) + 2(-3x^2 + 2xy - y^2)$

$= 9x^2 - 4xy + 5y^2 - 6x^2 + 4xy - 2y^2$

$= 9x^2 - 6x^2 - 4xy + 4xy + 5y^2 - 2y^2$

$= 3x^2 + 3y^2$

(iii) $m = 9x^2 - 4xy + 5y^2$

$n = -3x^2 + 2xy - y^2$

Now,

$m - 3n = 9x^2 - 4xy + 5y^2 - 3(-3x^2 + 2xy - y^2)$

$= 9x^2 - 4xy + 5y^2 + 9x^2 - 6xy + 3y^2$

$= 18x^2 - 10xy + 8y^2$

Question 22.
Simplify:

(i) $3x + 5(2x + 6) - 7x$

(ii) $3(4y - 10) + 2(y - 1)$

(iii) $-(7 + 6x) - 7(x + 2)$

(iv) $x - (x - y) - y - (y - x)$

(v) $4x + 7y - [5y - 8] - 2x$

(vi) $-2m + 5 + 4(m - 3)$

(vii) $2x - y + 5 - (x - y)$

(viii) $2(x - y) - (x - 8)$

(ix) $4(3x - 8) - 3(5x + 3) - 2(6x - 8)$

(x) $5(x - 4) - 3(x - 4) + 7(x - 4)$

Solution:

(i) $3x + 5(2x + 6) - 7x$

$\Rightarrow 3x + 10x + 30 - 7x$

$\Rightarrow 3x + 10x - 7x + 30$

$\Rightarrow 13x - 7x + 30$

$\Rightarrow 6x + 30$

(ii) $3(4y - 10) + 2(y - 1)$

$\Rightarrow 12y - 30 + 2y - 2$

$\Rightarrow 12y + 2y - 30 - 2$

$\Rightarrow 14y - 32$

(iii) $-(7 + 6x) - 7(x + 2)$

$\Rightarrow -7 - 6x - 7x - 14$

$\Rightarrow -7x - 6x - 7 - 14$

$\Rightarrow -13x - 21$

(iv) $x - (x - y) - y - (y - x)$

$\Rightarrow x - x + y - y - y + x$

$\Rightarrow 2x - x - 2y + y$

$\Rightarrow x - y$

(v) $4x + 7y - [5y - 8] - 2x$

$\Rightarrow 4x + 7y - 5y + 8 - 2x$

$\Rightarrow 4x - 2x + 7y - 5y + 8$

$\Rightarrow 2x + 2y + 8$

(vi) $-2m + 5 + 4(m - 3)$

$\Rightarrow -2m + 5 + 4m - 12$

$\Rightarrow -2m + 4m + 5 - 12$

$\Rightarrow 2m - 7$

(vii) $2x - y + 5 - (x - y)$

$\Rightarrow 2x - y + 5 - x + y$

$\Rightarrow 2x - x + 5$

$\Rightarrow x + 5$

(viii) $2(x - y) - (x - 8)$

$\Rightarrow 2x - 2y - x + 8$

$\Rightarrow 2x - x - 2y + 8$

$\Rightarrow x - 2y + 8$

(ix) $4(3x - 8) - 3(5x + 3) - 2(6x - 8)$

$\Rightarrow 12x - 32 - 15x - 9 - 12x + 16$

$\Rightarrow 12x - 15x - 12x - 32 - 9 + 16$

$\Rightarrow 12x - 27x - 41 + 16$

$\Rightarrow -15x - 25$

(x) $5(x - 4) - 3(x - 4) + 7(x - 4)$

$\Rightarrow 5x - 20 - 3x + 12 + 7x - 28$

$\Rightarrow 5x + 7x - 3x - 20 - 28 + 12$

$\Rightarrow 12x - 3x - 48 + 12$

$\Rightarrow 9x - 36$

EXERCISE 11 (C)

Question 1.

Multiply:

(i) $3x$, $5x^2y$ and $2y$

(ii) 5, $3a$ and $2ab^2$

(iii) $5x + 2y$ and $3xy$

(iv) $6a - 5b$ and $-2a$

(v) $4a + 5b$ and $4a - 5b$

(vi) $9xy + 2y^2$ and $2x - 3y$

(vii) $-3m^2n + 5mn - 4mn^2$ and $6m^2n$

(viii) $6xy^2 - 7x^2y^2 + 10x^3$ and $-3x^2y^3$

Solution:

(i) Product of $3x$, $5x^2y$ and $2y$

$= 3x + 5x^2y \times 2y$

$= 3 \times 5 \times 2 \times x \times x^2 \times y \times y$

$= 30x^3y^2$

(ii) Product of 5, $3a$ and $2ab^2$

$= 5 \times 3a \times 2ab^2$

$= 5 \times 3 \times 2 \times a \times ab^2$

$= 30a^2b^2$

(iii) Product of $5x + 2y$ and $3xy$

$= 3xy\,(5x + 2y)$

$= 3xy \times 5x + 3xy \times 2y$

$= 15x^2y + 6xy^2$

(iv) Product of $6a - 5b$ and $-2a$

$= -2a\,(6a - 5b)$

$= -2a \times 6a + (-2a)(-5b)$

$= -12a^2 + 10ab$

(v) Product of $4a + 5b$ and $4a - 5b$

$= 16a^2 - 25b^2$

$$\begin{array}{r} 4a + 5b \\ \times 4a - 5b \\ \hline 16a^2 + 20ab \qquad\qquad \\ -20ab - 25b^2 \\ \hline 16a^2 \qquad\qquad - 25b^2 \\ \hline \end{array}$$

(vi) Product of $9xy + 2y^2$ and $2x - 3y$

$= 18x^2y - 23xy^2 - 6y^3$

$$\begin{array}{r} 9xy + 2y^2 \\ \times\ 2x - 3y \\ \hline 18x^2y + 4xy^2 \qquad\quad \\ -27xy^2 - 6y^3 \\ \hline 18x^2y - 23xy^2 - 6y^3 \\ \hline \end{array}$$

(vii) Proudct of $-3m^2n + 5mn - 4mn^2$ and $6m^2n$

$= 6m^2n(-3m^2n + 5mn - 4mn^2)$

$= 6m^2n \times (-3m^2n) + 6m^2n \times 5mn + 6m^2n \times (-4mn^2)$

$= -18m^4n^2 + 30m^3n^2 - 24m^3n^3$

(viii) Product of $6xy^2 - 7x^2y^2 + 10x^3$ and $-3x^2y^3$

$= -3x^2y^3(6xy^2 - 7x^2y^2 + 10x^3)$

$= -3x^2y^3 \times 6xy^2 + (-3x^2y^3)(-7x^2y^2) + (-3x^2y^3) \times 10x^3$

$= -18x^3y^5 + 21x^4y^5 - 30x^5y^3$

Question 2.

Copy and complete the following multi-plications :

(i) $\begin{array}{r} 3a + 2b \\ \times \quad -3xy \\ \hline \end{array}$

(ii) $\begin{array}{r} 9x + 5y \\ \times \quad -3xy \\ \hline \end{array}$

(iii) $\begin{array}{r} 3xy - 2x^2 - 6x \\ \times \quad -5x^2y \\ \hline \end{array}$

(iv) $\begin{array}{r} a + b \\ \times \ a + b \\ \hline \end{array}$

(v) $\begin{array}{r} ax - b \\ \times\ 2ax + 2b^2 \\ \hline \end{array}$

(vi) $\begin{array}{r} 2a - b + 3c \\ \times \quad 2a - 4b \\ \hline \end{array}$

(vii) $\begin{array}{r} 3m^2 + 6m - 2n \\ \times \quad 5n - 3m \\ \hline \end{array}$

(viii) $\begin{array}{r} 6 - 3x + 2x^2 \\ \times\ 1 + 5x - x^2 \\ \hline \end{array}$

(ix) $\begin{array}{r} 4x^3 - 10x^2 + 6x - 8 \\ \times \quad 3 + 2x - x^2 \\ \hline \end{array}$

Solution:

(i)

$$\begin{array}{r} 3a + 2b \\ \times \quad -3xy \\ \hline -9axy - 6bxy \\ \hline \end{array}$$

(ii)

$$\begin{array}{r} 9x - 5y \\ \times \quad -3xy \\ \hline -27x^2y + 15xy^2 \\ \hline \end{array}$$

(iii)

$$\begin{array}{r} 3xy - 2x^2 - 6x \\ \times \quad -5x^2y \\ \hline -15x^3y^2 + 10x^4y + 30x^3y \\ \hline \end{array}$$

(iv)

$$\begin{array}{r} a + b \\ \times\ a + b \\ \\ a^2 + ab \qquad \\ ab + b^2 \\ \hline a^2 + 2ab + b^2 \\ \hline \end{array}$$

(v)

$$\begin{array}{l} \quad ax - b \\ \times\ 2ax + 2b^2 \\ \hline 2a^2x^2 - 2abx + 2ab^2x - 2b^3 \\ \hline \end{array}$$

(vi)

$$\begin{array}{l} 2a - b + 3c \\ \times \quad 2a - 4b \\ \hline 4a^2 - 2ab + 6ac \\ \qquad - 8ab + 4b^2 - 12bc \\ \hline 4a^2 - 10ab + 6ac + 4b^2 - 12bc \\ \hline \end{array}$$

(vii)

$$\begin{array}{l} 3m^2 + 6m - 2n \\ \times \qquad 5n - 3m \\ \hline 15m^2n + 30mn - 10n^2 - 9m^3 - 18m^2 \\ \qquad + 6mn \\ \hline 15m^2n + 36mn - 10n^2 - 9m^3 - 18m^2 \\ \hline \end{array}$$

(viii)

$$\begin{array}{l} 6 - 3x + 2x^2 \\ \times 1 + 5x - x^2 \\ \hline 6 - 3x + 2x^2 \\ \quad + 30x - 15x^2 + 10x^3 \\ \qquad - 6x^2 + 3x^3 - 2x^4 \\ \hline 6 + 27x - 19x^2 + 13x^3 - 2x^4 \\ \hline \end{array}$$

(ix)

$$\begin{array}{r} 4x^3 - 10x^2 + 6x - 8 \\ \times \qquad 3 + 2x - x^2 \\ \hline 12x^3 - 30x^2 + 18x - 24 \\ 8x^4 - 20x^3 + 12x^2 - 16x \qquad \\ 4x^5 + 10x^4 - 6x^3 + 8x^2 \qquad\qquad\qquad \\ \hline 4x^5 + 18x^4 - 14x^3 - 10x^2 + 2x - 24 \\ \hline \end{array}$$

Question 3.
Evaluate :

(i) $(c + 5)(c - 3)$ (ii) $(3c - 5d)(4c - 6d)$

(iii) $\left(\frac{1}{2}a + \frac{1}{2}b\right)\left(\frac{1}{2}a - \frac{1}{2}b\right)$

(iv) $(a^2 + 2ab + b^2)(a + b)$

(v) $(3x - 1)(4x^3 - 2x^2 + 6x - 3)$

(vi) $(4m - 2)(m^2 + 5m - 6)$

(vii) $(8 - 12x + 7x^2 - 6x^3)(5 - 2x)$

(viii) $(4x^2 - 4x + 1)(2x^3 - 3x^2 + 2)$

(ix) $(6p^2 - 8pq + 2q^2)(-5p)$

(x) $-4y(15x + 12y - 8z)(x - 2y)$

(xi) $(a^2 + b^2 + c^2 - ab - bc - ca)(a + b + c)$

Solution:

(i) $(c + 5)(c - 3) = c(c - 3) + 5(c - 3)$
$= c^2 - 3c + 5c - 15$
$= c^2 + 2c - 15$

(ii) $(3c - 5d)(4c - 6d)$
$= 3c(4c - 6d) - 5d(4c - 6d)$
$= 12c^2 - 18cd - 20cd + 30d^2$
$= 12c^2 - 38cd + 30d^2$

(iii) $\left(\frac{1}{2}a + \frac{1}{2}b\right)\left(\frac{1}{2}a - \frac{1}{2}b\right)$

$= \frac{1}{2a}\left(\frac{1}{2}a - \frac{1}{2}b\right) + \frac{1}{2}b\left(\frac{1}{2}a - \frac{1}{2}b\right)$

$= \frac{1}{4}a^2 - \frac{1}{4}ab + \frac{1}{4}ab - \frac{1}{4}b^2$

$= \frac{1}{4}a^2 - \frac{1}{4}b^2$

(iv) $(a^2 + 2ab + b^2)(a + b)$
$= a(a^2 + 2ab + b^2) + b(a^2 + 2ab + b^2)$
$= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3$
$= a^3 + 3a^2b + 3ab^2 + b^3$

(v) $(3x - 1)(4x^3 - 2x^2 + 6x - 3)$
$= 3x(4x^3 - 2x^2 + 6x - 3) - 1$
$(4x^3 - 2x^2 + 6x - 3)$
$= 12x^4 - 6x^3 + 18x^2 - 9x - 4x^3 + 2x^2 - 6x + 3$
$= 12x^4 - 6x^3 - 4x^3 + 18x^2 + 2x^2 - 9x - 6x + 3$
$= 12x^4 - 10x^3 + 20x^2 - 15x + 3$

(vi) $(4m - 2)(m^2 + 5m - 6)$

$= 4m(m^2 + 5m - 6) - 2(m^2 + 5m - 6)$

$= 4m^3 + 20m^2 - 24m - 2m^2 - 10m + 12$

$= 4m^3 + 20m^2 - 2m^2 - 24m - 10m + 12$

$= 4m^3 + 18m^2 - 34m + 12$ **Ans.**

(vii) $(8 - 12x + 7x^2 - 6x^3)(5 - 2x)$

$= 5(8 - 12x + 7x^2 - 6x^3)$
$\quad - 2x(8 - 12x + 7x^2 - 6x^3)$

$= 40 - 60x + 35x^2 - 30x^3 - 16x + 24x^2$
$\quad - 14x^3 + 12x^4$

$= 40 - 60x - 16x + 35x^2 + 24x^2 - 30x^3$
$\quad - 14x^3 + 12x^4$

$= 40 - 76x + 59x^2 - 44x^3 + 12x^4$

(viii) $(4x^2 - 4x + 1)(2x^3 - 3x^2 + 2)$

$= 4x^2(2x^3 - 3x^2 + 2) - 4x(2x^3 - 3x^2 + 2)$
$\quad + 1(2x^3 - 3x^2 + 2)$

$= 8x^5 - 12x^4 + 8x^2 - 8x^4 + 12x^3 - 8x + 2x^3$
$\quad - 3x^2 + 2$

$= 8x^5 - 12x^4 - 8x^4 + 12x^3 + 2x^3 + 8x^2$
$\quad - 3x^2 - 8x + 2$

$= 8x^5 - 20x^4 + 14x^3 + 5x^2 - 8x + 2$

(ix) $(6p^2 - 8pq + 2q^2)(-5p)$

$= -5p \times 6p^2 - 5p \times (-8pq) - 5p(2q^2)$

$= -30p^3 + 40p^2q - 10pq^2$

(x) $-4y(15 + 12y - 8z)(x - 2y)$

$= -4y(x - 2y)(15x + 12y - 8z)$

$= (-4xy + 8y^2)(15x + 12y - 8z)$

$= -4xy(15x + 12y - 8z) + 8y^2$
$\quad (15x + 12y - 8z)$

$= -60x^2y - 48xy^2 + 32xyz + 120xy^2$
$\quad + 96y^3 - 64y^2z$

$= -60x^2y - 48xy^2 + 120xy^2 - 64y^2z + 96y^3$
$\quad + 32xyz$

$= -60x^2y + 72xy^2 - 64y^2z + 96y^3 + 32xyz$

(xi) $(a^2 + b^2 + c^2 - ab - bc - ca)(a + b + c)$

$$= a(a^2 + b^2 + c^2 - ab - bc - ca) + b(a^2 + b^2 + c^2 - ab - bc - ca) + c(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= a^3 + ab^2 + ac^2 - a^2b - abc - ca^2 + a^2b + b^3 + bc^2 - ab^2 - b^2c - abc + a^2c + b^2c + c^3 - abc - bc^2 - c^2a$$

$$= a^3 + b^3 + c^3 - a^2b + a^2b - ca^2 + a^2c + bc^2 - bc^2 - ab^2 + ab^2 - abc - abc - abc + ac^2 - ac^2 + b^2c - b^2c$$

$$= a^3 + b^3 + c^3 - 3abc$$

Question 4.

Evaluate:

(i) $(a + b)(a - b)$

(ii) $(a^2 + b^2)(a + b)(a - b)$; using the result of (i).

(iii) $(a^4 + b^4)(a^2 + b^2)(a + b)(a - b)$; using the result of (ii).

Solution:

(i) $(a + b)(a - b)$

$$= a(a - b) + b(a - b)$$

$$= a^2 - ab + ab - b^2 = a^2 - b^2$$

(ii) $(a^2 + b^2)(a + b)(a - b)$

$$= (a^2 + b^2)(a^2 - b^2) \quad \{\text{from } (i)\}$$

$$= a^2(a^2 - b^2) + b^2(a^2 - b^2)$$

$$= a^4 - a^2b^2 + a^2b^2 - b^4 = a^4 - b^4$$

(iii) $(a^4 + b^4)(a^2 + b^2)(a + b)(a - b)$

$$= (a^4 + b^4)(a^4 - b^4) \quad \{\text{from } (ii)\}$$

$$= a^4(a^4 - b^4) + b^4(a^4 - b^4)$$

$$= a^8 - a^4b^4 + a^4b^4 - b^8 = a^8 - b^8$$

Question 5.
Evaluate :

(i) $(3x - 2y)(4x + 3y)$
(ii) $(3x - 2y)(4x + 3y)(8x - 5y)$
(iii) $(a + 5)(3a - 2)(5a + 1)$
(iv) $(a + 1)(a^2 - a + 1)$ and $(a - 1)(a^2 + a + 1)$;
and then : $(a + 1)(a^2 - a + 1) + (a - 1)(a^2 + a + 1)$
(v) $(5m - 2n)(5m + 2n)(25m^2 + 4n^2)$

Solution:

(i) $(3x - 2y)(4x + 3y)$
$= 3x(4x + 3y) - 2y(4x + 3y)$
$= 12x^2 + 9xy - 8xy - 6y^2$
$= 12x^2 + xy - 6y^2$

(ii) $(3x - 2y)(4x + 3y)(8x - 5y)$
$= (12x^2 + xy - 6y^2)(8x - 5y)$ {from (i)}
$= 8x(12x^2 + xy - 6y^2) - 5y(12x^2 + xy - 6y^2)$
$= 96x^3 + 8x^2y - 48xy^2 - 60x^2y - 5xy^2 + 30y^3$
$= 96x^3 + 8x^2y - 60x^2y - 48xy^2 - 5xy^2 + 30y^3$
$= 96x^3 - 52x^2y - 53\,xy^2 + 30y^3$

(iii) $(a + 5)(3a - 2)(5a + 1)$
$= \{a(3a - 2) + 5(3a - 2)(5a + 1)$

$= (3a^2 - 2a + 15a - 10)(5a + 1)$
$= (3a^2 + 13a - 10)(5a + 1)$
$= 5a(3a^2 + 13a - 10) + 1(3a^2 + 13a - 10)$
$= 15a^3 + 65a^2 - 50a + 3a^2 + 13a - 10$
$= 15a^3 + 68a^2 - 37a - 10$

(iv) $(a+1)(a^2 - a + 1)$ and $(a-1)(a^2 + a + 1)$;
$= a(a^2 - a + 1) + 1(a^2 - a + 1)$
$= a^3 - a^2 + a + a^2 - a + 1 = a^3 + 1$
$(a - 1)(a^2 + a + 1)$
$= a(a^2 + a + 1) - 1(a^2 + a + 1)$
$= a^3 + a^2 + a - a^2 - a - 1 = a^3 - 1$
Now, $(a + 1)(a^2 - a + 1) + (a - 1)(a^2 + a + 1)$
$= a^3 + 1 + a^3 - 1 = 2a^3$

(v) $(5m - 2n)(5m + 2n)(25m^2 + 4n^2)$
$= \{5m(5m + 2n) - 2n(5m + 2n)\}(25m^2 + 4n^2)$
$= (25m^2 + 10mn - 10mn - 4n^2)(25m^2 + 4n^2)$
$= (25m^2 - 4n^2)(25m^2 + 4n^2)$
$= 25m^2(25m^2 + 4n^2) - 4n^2(25m^2 + 4n^2)$
$= 625m^4 + 100m^2n^2 - 100m^2n^2 - 16n^4$
$= 625m^4 - 16n^4$

Question 6.

Multiply:

(i) mn^4, m^3n and $5m^2n^3$
(ii) $2mnpq$, $4mnpq$ and $5mnpq$
(iii) $pq - pm$ and p^2m
(iv) $x^3 - 3y^3$ and $4x^2y^2$
(v) $a^3 - 4ab$ and $2a^2b$
(vi) $x^2 + 5yx - 3y^2$ and $2x^2y$.

Solution:

(i) mn^4, m^3n and $5m^2n^3$

$\Rightarrow 5m^2n^3 \times mn^4 \times m^3n)$

$\Rightarrow 5m^{(2+1+3)}n^{(3+4+1)}$

$= 5m^6n^8$

(ii) $2mnpq$, $4mnpq$ and $5mnpq$

$\Rightarrow 5mnpq \times 2mnpq \times 4mnpq)$

$\Rightarrow 5 \times 2 \times 4\ m^{(1+1+1)}n^{(1+1+1)}p^{(1+1+1)}$
$q^{(1+1+1)}$

$\Rightarrow 40m^3n^3p^3q^3$

(iii) $pq - pm$ and $p^2m \Rightarrow p^2m \times (pq - pm)$

$\Rightarrow p^3qm - p^3m^2$

(iv) $x^3 - 3y^3$ and $4x^2y^2$

$\Rightarrow 4x^2y^2 \times (x^3 - 3y^3)$

$\Rightarrow 4x^5y^2 - 12x^2y^5$

(v) $a^3 - 4ab$ and $2a^2b$

$\Rightarrow 2a^2b \times (a^3 - 4ab)$

$\Rightarrow 2a^5b - 8a^3b^2$

(vi) $x^2 + 5yx - 3y^2$ and $2x^2y$

$\Rightarrow 2x^2y \times (x^2 + 5yx - 3y^2)$

$\Rightarrow 2x^4y + 10x^3y^2 - 6x^2y^3$

Question 7.

Multiply:

(i) $(2x + 3y)(2x + 3y)$ (ii) $(2x - 3y)(2x + 3y)$

(iii) $(2x + 3y)(2x - 3y)$ (iv) $(2x - 3y)(2x - 3y)$

(v) $(-2x + 3y)(2x - 3y)$ (vi) $(xy + 2b)(xy - 2b)$

(vii) $(x - a)(x + 3b)$

(viii) $(2x + 5y + 6)(3x + y - 8)$

(ix) $(3x - 5y + 2)(5x - 4y - 3)$

(x) $(6x - 2y)(3x - y)$

(xi) $(1 + 6x^2 - 4x^3)(-1 + 3x - 3x^2)$

Solution:

(*i*) $(2x + 3y)(2x + 3y)$

$\Rightarrow 2x(2x + 3y) + 3y(2x + 3y)$

$\Rightarrow 4x^2 + 6xy + 6xy + 9y^2$

$\Rightarrow 4x^2 + 12xy + 9y^2$

(*ii*) $(2x - 3y)(2x + 3y)$

$\Rightarrow 2x(2x + 3y) - 3y(2x + 3y)$

$\Rightarrow 2x \times 2x + 2x \times 3y - 3y \times 2x - 3y \times 3y$

$\Rightarrow 4x^2 + 6xy - 6xy - 9y^2$

$\Rightarrow 4x^2 + 0 - 9y^2$

$\Rightarrow 4x^2 - 9y$

(*iii*) $(2x + 3y)(2x - 3y)$

$\Rightarrow 2x(2x - 3y) + 3y(2x - 3y)$

$\Rightarrow 2x \times 2x - 2x \times 3y + 3y \times 2x - 3y \times 3y$

$\Rightarrow 4x^2 - 6xy + 6xy - 9y^2$

$\Rightarrow 4x^2 - 0 - 9y^2$

$\Rightarrow 4x^2 - 9y^2$

(*iv*) $(2x - 3y)(2x - 3y)$

$\Rightarrow 2x(2x - 3y) - 3y(2x - 3y)$

$\Rightarrow 2x \times 2x - 2x \times 3y - 3y \times 2x + 3y \times 3y$

$\Rightarrow 4x^2 - 6xy - 6xy + 9y^2$

$\Rightarrow 4x^2 - 12xy + 9y^2$

(*v*) $(-2x + 3y)(2x - 3y)$

$\Rightarrow -2x(2x - 3y) + 3y(2x - 3y)$

$\Rightarrow -4x^2 + 6xy + 6xy - 9y^2$

$\Rightarrow -4x^2 + 12xy - 9y^2$

(*vi*) $(xy + 2b)(xy - 2b)$

$\Rightarrow xy(xy - 2b) + 2b(xy - 2b)$

$\Rightarrow x^2y^2 - 2bxy + 2bxy - 4b^2$

$\Rightarrow x^2y^2 - 4b^2$

(*vii*) $(x - a)(x + 3b)$

$\Rightarrow x(x + 3b) - a(x + 3b)$

$\Rightarrow x^2 + 3bx - ax - 3ab$

(*viii*) $(2x + 5y + 6)(3x + y - 8)$

$\Rightarrow 2x(3x + y - 8) + 5y(3x + y - 8) + 6(3x + y - 8)$

$\Rightarrow 6x^2 + 2xy - 16x + 15xy + 5y^2 - 40y + 18x + 6y - 48$

$\Rightarrow 6x^2 + 2xy + 15xy - 16x + 18x + 5y^2 - 40y + 6y - 48$

$\Rightarrow 6x^2 + 17xy + 2x + 5y^2 - 34y - 48$

(*ix*) $(3x - 5y + 2)(5x - 4y - 3)$

$\Rightarrow 3x(5x - 4y - 3) - 5y(5x - 4y - 3) + 2(5x - 4y - 3)$

$\Rightarrow 15x^2 - 12xy - 9x - 25xy + 20y^2 + 15y + 10x - 8y - 6$

$\Rightarrow 15x^2 - 12xy - 25xy - 9x + 10x + 20y^2 + 15y - 8y - 6$

$\Rightarrow 15x^2 - 37xy + x + 20y^2 + 7y - 6$

(*x*) $(6x - 2y)(3x - y)$

$\Rightarrow 6x(3x - y) - 2y(3x - 2y)$

$\Rightarrow 18x^2 - 6xy - 6xy + 2y^2$

$\Rightarrow 18x^2 - 12xy + 2y^2$

(*xi*) $(1 + 6x^2 - 4x^3)(-1 + 3x - 3x^2)$

$1(-1 + 3x - 3x^2) + 6x^2(-1 + 3x - 3x^2) - 4x^3(-1 + 3x - 3x^2)$

$-1 + 3x - 3x^2 - 6x^2 + 18x^3 - 18x^4 + 4x^3 - 12x^4 + 12x^5$

$-1 + 3x - 9x^2 + 22x^3 - 30x^4 + 12x^5$

EXERCISE 11 (D)

Question 1.

Divide:

(*i*) $-16ab^2c$ by $6abc$ (*ii*) $25x^2y$ by $-5y^2$

(*iii*) $8x + 24$ by 4 (*iv*) $4a^2 - a$ by $-a$

(*v*) $8m - 16$ by -8 (*vi*) $-50 + 40p$ by $10p$

(*vii*) $4x^3 - 2x^2$ by $-x$

(*viii*) $10a^3 - 15a^2b$ by $-5a^2$

(*ix*) $12x^3y - 8x^2y^2 + 4x^2y^3$ by $4xy$

(*x*) $9a^4b - 15a^3b^2 + 12a^2b^3$ by $-3a^2b$

Solution:

(*i*) $-16ab^2c$ by $6abc$

$$= -\frac{16ab^2c}{6abc} = -\frac{8}{3}b$$

(*ii*) $25x^2y$ by $-5y^2$

$$= \frac{25x^2y}{-5y^2} = -5\frac{x^2}{y}$$

(*iii*) $8x + 24$ by 4

$$= \frac{8x+24}{4} = \frac{8x}{4} + \frac{24}{4} = 2x + 6$$

(*iv*) $4a^2 - a$ by $-a$

$$= \frac{4a^2 - a}{-a} = \frac{4a^2}{-a} - \frac{a}{-a}$$

$= -4a + 1$ **Ans.**

(*v*) $8m - 16$ by -8

$$= \frac{8m-16}{-8} = \frac{8m}{-8} - \frac{16}{-8} = -m + 2$$

(*vi*) $-50 + 40p$ by $10p$

$$= \frac{-50+40p}{10p} = \frac{-50}{10p} + \frac{40p}{10p}$$

$$= -\frac{5}{p} + 4$$

(*vii*) $4x^3 - 2x^2$ by $-x$

$$= \frac{4x^3 - 2x^2}{-x} = \frac{4x^3}{-x} - \frac{2x^2}{-x}$$

$= -4x^2 + 2x$

(*viii*) $10a^3 - 15a^2b$ by $-5a^2$

$$= \frac{10a^3 - 15a^2b}{-5a^2} = \frac{10a^3}{-5a^2} - \frac{15a^2b}{-5a^2}$$

$= -2a + 3b$ **Ans.**

(*ix*) $12x^3y - 8x^2y^2 + 4x^2y^3$ by $4xy$

$$= \frac{12x^3y - 8x^2y^2 + 4x^2y^3}{4xy}$$

$$= \frac{12x^3y}{4xy} - \frac{8x^2y^2}{4xy} + \frac{4x^2y^3}{4xy}$$

$= 3x^2 - 2xy + xy^2$

(*x*) $9a^4b - 15a^3b^2 + 12a^2b^3$ by $-3a^2b$

$$= \frac{9a^4b - 15a^3b^2 + 12a^2b^3}{-3a^2b}$$

$$= \frac{9a^4b}{-3a^2b} - \frac{15a^3b^2}{-3a^2b} + \frac{12a^2b^3}{-3a^2b}$$

$= -3a^2 + 5ab - 4b^2$

Question 2.
Divide :

(i) $n^2 - 2n + 1$ by $n - 1$

(ii) $m^2 - 2mn + n^2$ by $m - n$

(iii) $4a^2 + 4a + 1$ by $2a + 1$

(iv) $p^2 + 4p + 4$ by $p + 2$

(v) $x^2 + 4xy + 4y^2$ by $x + 2y$

(vi) $2a^2 - 11a + 12$ by $a - 4$

(vii) $6x^2 + 5x - 6$ by $2x + 3$

(viii) $8a^2 + 4a - 60$ by $2a - 5$

(ix) $9x^2 - 24xy + 16y^2$ by $3x - 4y$

(x) $15x^2 + 31xy + 14y^2$ by $5x + 7y$

(xi) $35a^3 + 3a^2b - 2ab^2$ by $5a - b$

(xii) $6x^3 + 5x^2 - 21x + 10$ by $3x - 2$

Solution:

(i) $n^2 - 2n + 1$ by $n - 1$

$$
\begin{array}{r|l}
 & n - 1 \\
\hline
n - 1 & n^2 - 2n + 1 \\
 & n^2 - n \\
 & - \quad + \\
\hline
 & -n + 1 \\
 & -n + 1 \\
 & + \quad - \\
\hline
 & \times \\
\hline
\end{array}
$$

$= n - 1$

(*ii*) $m^2 - 2mn + n^2$ by $m - n$

$$
\begin{array}{r|l}
 & m - n \\
\hline
m - n & m^2 - 2mn + n^2 \; (\\
 & m^2 - mn \\
 & - \quad + \\
\hline
 & - mn + n^2 \\
 & - mn + n^2 \\
 & + \quad - \\
\hline
 & \times \\
\hline
\end{array}
$$

$= m - n$

(*iii*) $4a^2 + 4a + 1$ by $2a + 1$

$$
\begin{array}{r|l}
 & 2a + 1 \\
\hline
2a + 1 & 4a^2 + 4a + 1 \; (\\
 & 4a^2 + 2a \\
 & - \quad - \\
\hline
 & 2a + 1 \\
 & 2a + 1 \\
 & - \quad - \\
\hline
 & \times \\
\hline
\end{array}
$$

$= 2a + 1$

(*iv*) $p^2 + 4p + 4$ by $p + 2$

$$
\begin{array}{r|l}
 & p + 2 \\
\hline
p + 2 & p^2 + 4p + 4 \;(\\
 & p^2 + 2p \\
 & - \quad - \\
\hline
 & 2p + 4 \\
 & 2p + 4 \\
 & - \quad - \\
\hline
 & \times \\
\hline
\end{array}
$$

$= p + 2$

(*v*) $x^2 + 4xy + 4y^2$ by $x + 2y$

$$
\begin{array}{r|l}
 & x + 2y \\
\hline
x + 2y & x^2 + 4xy + 4y^2 \;(\\
 & x^2 + 2xy \\
 & - \quad - \\
\hline
 & 2xy + 4y^2 \\
 & 2xy + 4y^2 \\
 & - \quad - \\
\hline
 & \times \\
\hline
\end{array}
$$

$= x + 2y$

(*vi*) $2a^2 - 11a + 12$ by $a - 4$

$$
\begin{array}{r|l}
 & 2a - 3 \\
\hline
a - 4 & 2a^2 - 11a + 12 \;(\\
 & 2a^2 - 8a \\
 & - \quad + \\
\hline
 & -3a + 12 \\
 & -3a + 12 \\
 & + \quad - \\
\hline
 & \times \\
\hline
\end{array}
$$

$= 2a - 3$

(*vii*) $6x^2 + 5x - 6$ by $2x + 3$

$$
\begin{array}{r l}
 & \quad 3x - 2 \\
\cline{2-2}
2x + 3\,) & 6x^2 + 5x - 6\,(\\
 & 6x^2 + 9x \\
 & - \quad\; - \\
\cline{2-2}
 & \qquad -4x - 6 \\
 & \qquad -4x - 6 \\
 & \qquad\; + \quad + \\
\cline{2-2}
 & \qquad\quad \times \\
\cline{2-2}
\end{array}
$$

$= 3x - 2$

(*viii*) $8a^2 + 4a - 60$ by $2a - 5$

$$
\begin{array}{r l}
 & \quad 4a + 12 \\
\cline{2-2}
2a - 5\,) & 8a^2 + 4a - 60\,(\\
 & 8a^2 - 20a \\
 & - \quad\; + \\
\cline{2-2}
 & \qquad 24a - 60 \\
 & \qquad 24a - 60 \\
 & \qquad - \quad + \\
\cline{2-2}
 & \qquad\quad \times \\
\cline{2-2}
\end{array}
$$

$= 4a + 12$

(*ix*) $9x^2 - 24xy + 16y^2$ by $3x - 4y$

$$
\begin{array}{r l}
 & \quad 3x - 4y \\
\cline{2-2}
3x - 4y\,) & 9x^2 - 24xy + 16y^2\,(\\
 & 9x^2 - 12xy \\
 & - \quad\; + \\
\cline{2-2}
 & \qquad -12xy + 16y^2 \\
 & \qquad -12xy + 16y^2 \\
 & \qquad\; + \qquad - \\
\cline{2-2}
 & \qquad\qquad \times \\
\cline{2-2}
\end{array}
$$

$= 3x - 4y$

(x) $15x^2 + 31xy + 14y^2$ by $5x + 7y$

$$
\begin{array}{r|l}
 & 3x + 2y \\
\hline
5x + 7y & 15x^2 + 31xy + 14y^2 \; (\\
 & 15x^2 + 21xy \\
 & - \quad\; - \\
\hline
 & \quad\quad 10xy + 14y^2 \\
 & \quad\quad 10xy + 14y^2 \\
 & \quad\quad - \quad\; - \\
\hline
 & \quad\quad\quad \times \\
\hline
\end{array}
$$

$= 3x + 2y$ **Ans.**

(xi) $35a^3 + 3a^2b - 2ab^2$ by $5a - b$

$$
\begin{array}{r|l}
 & 7a^2 + 2ab \\
\hline
5a - b & 35a^3 + 3a^2b - 2ab^2 \; (\\
 & 35a^3 - 7a^2b \\
 & - \quad\; + \\
\hline
 & \quad\quad 10a^2b - 2ab^2 \\
 & \quad\quad 10a^2b - 2ab^2 \\
 & \quad\quad - \quad\; + \\
\hline
 & \quad\quad\quad \times \\
\hline
\end{array}
$$

$= 7a^2 + 2ab$

(xii) $6x^3 + 5x^2 - 21x + 10$ by $3x - 2$

$$
\begin{array}{r|l}
 & 2x^2 + 3x - 5 \\
\hline
3x - 2 & 6x^3 + 5x^2 - 21x + 10 \; (\\
 & 6x^3 - 4x^2 \\
 & - \quad\; + \\
\hline
 & \quad\quad 9x^2 - 21x \\
 & \quad\quad 9x^2 - 6x \\
 & \quad\quad - \quad\; + \\
\hline
 & \quad\quad\quad - 15x + 10 \\
 & \quad\quad\quad - 15x + 10 \\
 & \quad\quad\quad + \quad\; - \\
\hline
 & \quad\quad\quad\quad \times \\
\hline
\end{array}
$$

$= 2x^2 + 3x - 5$

Question 3.
The area of a rectangle is $6x^2 – 4xy – 10y^2$ square unit and its length is $2x + 2y$ unit. Find its breadth

Solution:

Area of a rectangle

$$= 6x^2 - 4xy - 10y^2 \text{ sq. units}$$

Length $= 2x + 2y$ units

$$\therefore \text{ Breadth} = \frac{\text{Area}}{\text{Length}}$$

$$= \frac{6x^2 - 4xy - 10y^2}{2x + 2y}$$

$$\begin{array}{r|l} & 3x - 5y \\ \hline 2x + 2y & 6x^2 - 4xy - 10y^2 \; (\\ & 6x^2 + 6xy \\ & - \quad - \\ \hline & \quad -10xy - 10y^2 \\ & \quad -10xy - 10y^2 \\ & \quad + \quad\quad + \\ \hline & \quad\quad \times \\ \hline \end{array}$$

$$= 3x - 5y \text{ units}$$

Hence breadth $= 3x - 5y$ units

Question 4.
The area of a rectangular field is $25x^2 + 20xy + 3y^2$ square unit. If its length is 5x + 3y unit, find its breadth, Hence find its perimeter.

Solution:
Area of a rectangular field

$$= 25x^2 + 20xy + 3y^2$$

Length = $(5x + 3y)$ unit

$$\therefore \quad \text{Breadth} = \frac{\text{Area}}{\text{Length}} = \frac{25x^2 + 20xy + 3y^2}{5x + 3y}$$

$$\begin{array}{r|l}
 & 5x + y \\ \hline
5x + 3y & 25x^2 + 20xy + 3y^2 \\
 & 25x^2 + 15xy \\
 & - \quad\quad - \\ \hline
 & 5xy + 3y^2 \\
 & 5xy + 3y^2 \\
 & - \quad\quad - \\ \hline
 & \times \\ \hline
\end{array}$$

Hence Breadth = $5x + y$

Hence perimeter of rectangular field

$$= 2\,(l + b)$$
$$= 2\,(5x + 3y + 5x + y)$$
$$= 2\,(10x + 4y)$$
$$= 20x + 8y$$

Question 5.

Divide:

(i) $2m^3n^5$ by $-mn$

(ii) $5x^2 - 3x$ by x

(iii) $10x^3y - 9xy^2 - 4x^2y^2$ by xy

(iv) $3y^3 - 9ay^2 - 6ab^2y$ by $-3y$

(v) $x^5 - 15x^4 - 10x^2$ by $-5x^2$

(vi) $12a^2 + ax - 6x^2$ by $3a - 2x$

(vii) $6x^2 - xy - 35y^2$ by $2x - 5y$

(viii) $x^3 - 6x^2 + 11x - 6$ by $x^2 - 4x + 3$

(ix) $m^3 - 4m^2 + m + 6$ by $m^2 - m - 2$

Solution:

(i) $2m^3n^5$ by $-mn$

$$= \frac{2m^3n^5}{-mn} = -2m^2n^4$$

(ii) $5x^2 - 3x$ by x

$$= \frac{5x^2 - 3x}{x} = \frac{5x^2}{x} - \frac{3x}{x} = 5x - 3$$

(iii) $10x^3y - 9xy^2 - 4x^2y^2$ by xy

$$= \frac{10x^3y - 9xy^2 - 4x^2y^2}{xy}$$

$$= \frac{10x^3y}{xy} - \frac{9xy^2}{xy} - \frac{4x^2y^2}{xy}$$

$$= 10x^2 - 9y - 4xy$$

(iv) $3y^3 - 9ay^2 - 6ab^2y$ by $-3y$

$$= \frac{3y^3 - 9ay^2 - 6aby^2}{-3y}$$

$$= \frac{3y^3}{-3y} - \frac{9ay^3}{-3y} - \frac{6ab^2y}{-3y}$$

$$= -y^2 + 3ay^2 + 2ab^2$$

(v) $x^5 - 15x^4 - 10x^2$ by $-5x^2$

$$= \frac{x^5 - 15x^4 - 10x^2}{-5x^2}$$

$$= \frac{x^5}{-5x^2} - \frac{15x^4}{-5x^2} - \frac{10x^2}{-5x^2}$$

$= \frac{-1}{5}x^3 + 3x^2 + 2$

(*vi*) $12a^2 + ax - 6x^2$ by $3a - 2x$

$$
\begin{array}{r l}
3a - 2x) & 12a^2 + ax - 6x^2 \,(4a + 3x \\
 & 12a^2 + 8ax \\
 & \underline{-\quad\;\; -\qquad\qquad} \\
 & \qquad\quad 9ax - 6x^2 \\
 & \qquad\quad 9ax - 6x^2 \\
 & \qquad\quad \underline{-\quad\;\; +\quad} \\
 & \qquad\quad \underline{\qquad \times \qquad}
\end{array}
$$

$= 4a + 3x$

(*vii*) $6x^2 - xy - 35y^2$ by $2x - 5y$

$$
\begin{array}{r l}
2x - 5y) & 6x^2 - xy - 35y^2 \,(3x + 7y \\
 & 6x^2 - 15xy \\
 & \underline{-\quad\; +\qquad\qquad} \\
 & \qquad 14xy - 35y^2 \\
 & \qquad 14xy - 35y^2 \\
 & \qquad \underline{-\qquad +\quad} \\
 & \qquad \underline{\qquad \times \qquad}
\end{array}
$$

$= 3x + 7y$

(*viii*) $x^3 - 6x^2 + 11x - 6$ by $x^2 - 4x + 3$

$$
\begin{array}{r l}
x^2 - 4x + 3) & x^3 - 6x^2 + 11x - 6 \,(x - 2 \\
 & x^3 - 4x^2 + 3x \\
 & \underline{-\quad +\quad -\qquad\qquad} \\
 & \qquad -2x^2 + 8x - 6 \\
 & \qquad -2x^2 + 8x - 6 \\
 & \qquad \underline{+\qquad -\qquad +} \\
 & \qquad \underline{\qquad\quad \times \qquad\quad}
\end{array}
$$

$= x - 2$

(ix) $m^3 - 4m^2 + m + 6$ by $m^2 - m - 2$

$$\begin{array}{r|l} m^2 - m - 2 & \overline{m^3 - 4m^2 + m + 6}\,(m - 3 \\ & m^3 - m^2 - 2m \\ & \underline{- \quad + \quad +} \\ & -3m^2 + 3m + 6 \\ & -3m^2 + 3m + 6 \\ & \underline{+ \quad - \quad -} \\ & \underline{\times} \end{array}$$

$= m - 3$

EXERCISE 11 (E)

Simplify

Question 1.

$\frac{x}{2} + \frac{x}{4}$

Solution:

$\frac{x}{2} + \frac{x}{4}$

$$\frac{2x + x}{4} = \frac{3x}{4}$$

Question 2.

$\frac{a}{10} + \frac{2a}{5}$

Solution:

$\frac{a}{10} + \frac{2a}{5}$

$$\frac{a + 4a}{10} = \frac{5a}{10} = \frac{a}{2}$$

Question 3.

. $\frac{y}{4}+\frac{3y}{5}$

Solution:

$\frac{y}{4}+\frac{3y}{5}$

$$\frac{5y+12y}{20}=\frac{17y}{20}$$

Question 4.

$\frac{x}{2}-\frac{x}{8}$

Solution:

$\frac{x}{2}-\frac{x}{8}$

$$\frac{4x-x}{8}=\frac{3x}{8}$$

Question 5.

$\frac{3y}{4}-\frac{y}{5}$

Solution:

$\frac{3y}{4}-\frac{y}{5}$

$$\frac{15y-4y}{20}=\frac{11y}{20}$$

Question 6.

$\frac{2p}{3}-\frac{3p}{5}$

Solution:

$$\frac{2p}{3}-\frac{3p}{5}$$

$$\frac{10p-9p}{15}=\frac{p}{15}$$

Question 7.

$$\frac{k}{2}+\frac{k}{3}+\frac{2k}{5}$$

Solution:

$$\frac{k}{2}+\frac{k}{3}+\frac{2k}{5}$$

$$\frac{15k+10k+12k}{30}=\frac{37k}{30}$$

(L.C.M. of 2, 3, 5 = 30)

Question 8.

$$\frac{2x}{5}+\frac{3x}{4}-\frac{3x}{5}$$

Solution:

$$\frac{2x}{5}+\frac{3x}{4}-\frac{3x}{5} \qquad \text{(LCM of 5, 4 = 20)}$$

$$\frac{8x+15x-12x}{20}=\frac{23x-12x}{20}=\frac{11x}{20}$$

Question 9.

$$\frac{4a}{7}-\frac{2a}{3}+\frac{a}{7}$$

Solution:

$$\frac{4a}{7}-\frac{2a}{3}+\frac{a}{7}$$

$$\frac{12a-14a+3a}{21}=\frac{15a-14a}{21}=\frac{a}{21}$$

(LCM of 7, 3 = 21)

Question 10.

$$\frac{2b}{5}-\frac{7b}{15}+\frac{13b}{3}$$

Solution:

$$\frac{2b}{5}-\frac{7b}{15}+\frac{13b}{3}$$ (L.C.M. of 3, 5, 15 = 15)

$$\frac{6b-7b+65b}{15}=\frac{71b-7b}{15}=\frac{64b}{15}$$

Question 11.

$$\frac{6k}{7}-\left(\frac{8k}{9}-\frac{k}{3}\right)$$

Solution:

$$\frac{6k}{7}-\left(\frac{8k}{9}-\frac{k}{3}\right)=\frac{54k-(56k-21k)}{63}$$

(L.C.M. of 7, 9, 3 = 63)

$$=\frac{54k-(35k)}{63}$$

$$=\frac{54k-35k}{63}=\frac{19k}{63}$$

Question 12.

$$\frac{3a}{8}+\frac{4a}{5}-\left(\frac{a}{2}+\frac{2a}{5}\right)$$

Solution:

$$\frac{3a}{8}+\frac{4a}{5}-\left(\frac{a}{2}+\frac{2a}{5}\right)$$

$$=\frac{15a+32a-(20a+16a)}{40}$$

(L.C.M. of 8, 5, 2 = 40)

$$=\frac{47a-36a}{40}=\frac{11a}{40}$$

Question 13.

$$x+\frac{x}{2}+\frac{x}{3}$$

Solution:

$$\frac{x}{1}+\frac{x}{2}+\frac{x}{3}$$

$$=\frac{6x+3x+2x}{6}=\frac{11x}{6}$$

Question 14.

$$\frac{y}{5}+y-\frac{19y}{15}$$

Solution:

$$\frac{y}{5}+\frac{y}{1}-\frac{19y}{15}$$

$$=\frac{3y+15y-19y}{15}=\frac{18y-19y}{15}$$

$$=\frac{-y}{15}$$

Question 15.

$$\frac{x}{5}+\frac{x+1}{2}$$

Solution:

$$\frac{x}{5}+\frac{x+1}{2}=\frac{2x+5x+5}{10}=\frac{7x+5}{10}.$$

(L.C.M. of 5, 2 = 10)

Question 16.

$$x+\frac{x+2}{3}$$

Solution:

$$\frac{x}{1}+\frac{x+2}{3}$$

$$=\frac{3x+x+2}{3}=\frac{4x+2}{3}$$

Question 17.

$$\frac{3y}{5}-\frac{y+2}{2}.$$

Solution:

$$\frac{3y}{5}-\frac{y+2}{2}$$

$$=\frac{6y-(5y+10)}{10}$$

$$=\frac{6y-5y-10}{10}=\frac{y-10}{10}.$$

Question 18.

$$\frac{2a+1}{3}+\frac{3a-1}{2}$$

Solution:

$$\frac{2a+1}{3}+\frac{3a-1}{2}$$

$$\frac{4a+2+9a-3}{6}=\frac{13a-1}{6}$$

(L.C.M. of 3, 2 = 6)

Question 19.

$$\frac{k+1}{2}+\frac{2k-1}{3}-\frac{k+3}{4}$$

Solution:

$$\frac{k+1}{2}+\frac{2k-1}{3}-\frac{k+3}{4}$$

$$\frac{6k+6+8k-4-3k-9}{12}$$

(L.C.M. of 2, 3, 4 = 12)

$$=\frac{14k-3k+6-13}{12}=\frac{11k-7}{12}$$

Question 20.

$$\frac{m}{5}-\frac{m-2}{3}+m$$

Solution:

$$\frac{m}{5}-\frac{m-2}{3}+\frac{m}{1}$$

$$\frac{3m-5(m-2)+15m}{15}$$

$$=\frac{3m-5m+10+15m}{15}$$

$$=\frac{18m-5m+10}{15}=\frac{13m+10}{15}$$

Question 21.

$$\frac{5(x-4)}{3}+\frac{2(5x-3)}{5}+\frac{6(x-4)}{7}$$

Solution:

$$\frac{5(x-4)}{3}+\frac{2(5x-3)}{5}+\frac{6(x-4)}{7}$$

$$\frac{175(x-4)+42(5x-3)+90(x-4)}{105}$$

(L.C.M. of 3, 5, 7 = 105)

$$=\frac{175x-700+210x-126+90x-360}{105}$$

$$=\frac{175x+210x+90x-700-126-360}{105}$$

$$=\frac{475x-1186}{105}$$

Question 22.

$$\left(p+\frac{p}{3}\right)\left(2p+\frac{p}{2}\right)\left(3p-\frac{2p}{3}\right)$$

Solution:

$$\left(p+\frac{p}{3}\right)\left(2p+\frac{p}{2}\right)\left(3p-\frac{2p}{3}\right)$$

$$=p\left(1+\frac{1}{3}\right)p\left(2+\frac{1}{2}\right)p\left(3-\frac{2}{3}\right)$$

$$=p^3\left(\frac{3+1}{3}\right)\left(\frac{4+1}{2}\right)\left(\frac{9-2}{3}\right)$$

$$=p^3\times\frac{4}{3}\times\frac{5}{2}\times\frac{7}{3}=p^3\times\frac{70}{9}=\frac{70p^3}{9}$$

Question 23.

$$\frac{7}{30} \text{ of } \left(\frac{p}{3}+\frac{7p}{15}\right)$$

Solution:

$$\frac{7}{30} \text{ of } \left(\frac{p}{3}+\frac{7p}{15}\right)$$

$$=\frac{7}{30} \text{ of } \left(\frac{5p+7p}{15}\right)=\frac{7}{30}\times\frac{12}{15}p=\frac{14p}{75}$$

Question 24.

$$\left(2p+\frac{p}{7}\right)\div\left(\frac{9p}{10}+4p\right)$$

Solution:

$$\left(2p+\frac{p}{7}\right)\div\left(\frac{9p}{10}+4p\right)$$

$$=\frac{14p+p}{7}\div\frac{9p+40p}{10}=\frac{15}{7}p\div\frac{49}{10}p$$

$$=\frac{15}{7}p\times\frac{10}{49p}=\frac{150}{343}$$

Question 25.

$$\left(\frac{5k}{8}-\frac{3k}{5}\right)\div\frac{k}{4}$$

Solution:

$$\left(\frac{5k}{8}-\frac{3k}{5}\right)\div\frac{k}{4}$$

$$=\frac{25k-24k}{40}\div\frac{k}{4}=\frac{k}{40}\div\frac{k}{4}$$

$$=\frac{k}{40}\times\frac{4}{k}=\frac{1}{10}$$

Question 26.

$$\left(\frac{y}{6}+\frac{2y}{3}\right)\div\left(y+\frac{2y-1}{3}\right)$$

Solution:

$$\left(\frac{y}{6}+\frac{2y}{3}\right)\div\left(y+\frac{2y-1}{3}\right)$$

$$=\left(\frac{y+4y}{6}\right)\div\left(\frac{3y+2y-1}{3}\right)$$

$$=\frac{5y}{6}\div\frac{5y-1}{3}=\frac{5y}{6}\times\frac{3}{5y-1}$$

$$=\frac{5y}{2(5y-1)}=\frac{5y}{10y-2}$$

EXERCISE 11 (F)

Enclose the given terms in brackets as required :

Question 1.
x – y – z = x-{…….)

Solution:
$x - y - z = x - (y + z)$

Question 2.
$x^2 - xy^2 - 2xy - y^2 = x^2 - (\ldots\ldots\ldots)$

Solution:
$x^2 - xy^2 - 2xy - y^2$
$= x^2 - (xy^2 + 2xy + y^2)$

Question 3.
$4a - 9 + 2b - 6 = 4a - (\ldots\ldots\ldots)$

Solution:
$4a - 9 + 2b - 6$
$= 4a - (9 - 2b + 6)$

Question 4.
$x^2 - y^2 + z^2 + 3x - 2y = x^2 - (\ldots\ldots\ldots\ )$

Solution:
$x^2 - y^2 + z^2 + 3x - 2y$
$= x^2 - (y^2 - z^2 - 3x + 2y)$

Question 5.
$- 2a^2 + 4ab - 6a^2b^2 + 8ab^2 = - 2a\ (\ldots\ldots\ldots\ )$

Solution:
$- 2a^2 + 4ab - 6a^2b^2 + 8ab^2$
$= - 2a\ (a - 2b + 3ab^2 - 4b^2)$

Simplify :

Question 6.
$2x - (x + 2y - z)$

Solution:
$2x - (x + 2y - z) = 2x - x - 2y + z$
$= x - 2y + z$

Question 7.
$p + q - (p - q) + (2p - 3q)$

Solution:
$p + q - (p - q) + (2p - 3q)$
$= p + q - p + q + 2p - 3q = 2p - q$

Question 8.
$9x - (-4x + 5)$

Solution:
$9x - (-4x + 5) = 9x + 4x - 5$
$= 13x - 5$

Question 9.
$6a - (- 5a - 8b) + (3a + b)$

Solution:
$6a - (- 5a - 8b) + (3a + b)$

= 6a + 5a + 8b + 3a + b
= 6a + 5a + 3a + 8b + b
= 14a + 9b

Question 10.
(p – 2q) – (3q – r)

Solution:
(p-2q) – (3q – r) =p – 2q – 3q + r =p – 5q + r

Question 11.
9a (2b – 3a + 7c)

Solution:
9a (2b – 3a + 7c)
= 18ab – 27a^2 + 63ca

Question 12.
-5m (-2m + 3n – 7p)

Solution:
-5m (-2m + 3n- 7p)
= – 5m x (-2m) + (-5m) (3n) – (-5m) (7p)
= 10m^2 – 15mn + 35 mp.

Question 13.
-2x (x + y) + x^2

Solution:
– 2x (x + y) + x^2
= -2x x x + (-2x)y + x^2
= – 2x^2 – 2xy + x^2
= – 2x^2 + x^2 – 2xy = – x^2 – 2xy

Question 14.

$$b\left(2b-\frac{1}{b}\right)-2b\left(b-\frac{1}{b}\right)$$

Solution:

$$b\left(2b-\frac{1}{b}\right)-2b\left(b-\frac{1}{b}\right)$$
$$=b\times 2b-b\times\frac{1}{b}-2b\times b+2b\times\frac{1}{b}$$
$$=2b^2-1-2b^2+2$$
$$=2b^2-2b^2-1+2=1$$

Question 15.
8 (2a + 3b – c) – 10 (a + 2b + 3c)

Solution:
8 (2a + 3b -c)- 10 (a + 2b + 3c)
= 16a + 24b – 8c – 10a – 20b- 30c
= 16a – 10a + 24b – 20b – 8c – 30c
= 6a + 4b – 38c

Question 16.

$$a\left(a+\frac{1}{a}\right)-b\left(b-\frac{1}{b}\right)-c\left(c+\frac{1}{c}\right)$$

Solution:

$$a\left(a+\frac{1}{a}\right)-b\left(b-\frac{1}{b}\right)-c\left(c+\frac{1}{c}\right)$$
$$=a^2+1-b^2+1-c^2-1$$
$$=a^2-b^2-c^2+1$$

Question 17.
5 x (2x + 3y) – 2x (x – 9y)

Solution:
5x (2x + 3y) – 2x (x – 9y)
= $10x^2$ + 15xy – $2x^2$ + 18xy
= $10x^2$ – $2x^2$ + 15xy+ 18xy
= $8x^2$ + 33 xy

Question 18.
a + (b + c – d)

Solution:
a + (b + c – d) = a + (b + c – d)
= a + b + c – d

Question 19.
5 – 8x – 6 – x

Solution:
5 – 8x – 6 – x
= 5 – 6 – 8x – x
= -1 -7x

Question 20.
2a + (6- $\overline{a-b}$)

Solution:
2a + (6 – $\overline{a-b}$)
= 2a + (b – a + b)
= 2a + b – a + b
= a + 2b

Question 21.
3x + [4x – (6x – 3)]

Solution:
3x + [4x – (6x – 3)]
= 3x + [4x – 6x + 3]
= 3x + 4x – 6x + 3
= 3x + 4x – 6x + 3
= 7x – 6x + 3= x + 3

Question 22.
5b – {6a + (8 – b – a)}

Solution:
5b- {6a + 8- 6-a}
= 5b – 6a – 8 + b + a
= -6a + a + 5b +b – 8
= -5a + 6b-8

Question 23.
2x-[5y- (3x -y) + x]

Solution:
2x – [5y- (3x – y) + x]
= 2x – {5y – 3x +y + x}
= 2x – 5y + 3x -y – x
= 2x + 3x – x – 5y – y
= 4x – 6y

Question 24.
6a – 3 (a + b – 2)

Solution:
6a – *3* (a + b – *2)*
= 6a – 3a – 3b + 6
= 3a -3b + 6

Question 25.
8 [m + 2n-p – 7 (2m -n + 3p)]

Solution:
8 [m + 2n-p -1 (2m – n + 3p)]
8 [m + 2n-p- 14m + 7n-21p]
= 8m+ 16n -8p- 112m + 56n – 168p
= 8m – 112m + 16n + 56n -8p – 168p
= -104m + 72n – 176p

Question 26.
{9 – (4p – 6q)} – {3q – (5p – 10)}

Solution:
{9 – {4p – 6q)} – {3q – (5p – 10)}
{9 – 4p + 6q} – {3q -5p+ 10}
= *9* – 4p + 6q – 3q + 5p – *10*
= *9* – *4p* + 5p + 6q – 3q – *10*
= p + 3q – 1

Question 27.
2 [a – 3 {a + 5 {a – 2) + 7}]

Solution:
2 [a – 3 {a + 5 {a – 2) + 7}]

= 2 [a- 3 {a + 5a- 10 + 7}]
= 2 [a -3a- 15a + 30 -21]
= 2a-6a- 30a + 60-42
= 2a- 36a + 60-42
= -34a + 18

Question 28.
5a – [6a – {9a – (10a – $\overline{4a - 3a}$)}]

Solution:
5a – [6a – {9a – (10a – 4a + 3a)}]
= 5a – [6a – {9a – (10a – 4a + 3a)}]
= 5a – [6a – {9a – 10a + 4a – 3a}]
= 5a- [6a – 9a + 10a – 4a + 3a]
= 5a – 6a + 9a – 10a + 4a – 3a
= 5a + 9a + 4a – 6a – 10a – 3a
= 18a – 19a = – a

Question 29.
9x + 5 – [4x – {3x – 2 (4x – 3)}]

Solution:
9x + 5 – [4x – {3x – 2 (4x – 3)}]
= 9x + 5 – [4x – {3x – 8x + 6}]
= 9x + 5 – [4x – 3x + 8x – 6]
= 9x + 5-4x + 3x-8x + 6
= 9x + 3x-4x-8x + 5 + 6
= 12x- 12x+ 11 = 11

Question 30.
(x + y – z)x + (z + x – y)y – (x + y – z)z

Solution:
(x + y – z)x + (z + x -y)y – (x + y -z)z
= x^2 + xy – zx + yz + xy -y^2 – zx – yz + z^2
= x^2 -y^2 + z^2 + *2*xy – *2*zx

Question 31.
-1 [a-3 {b -4 (a-b-8) + 4a} + 10]

Solution:
– 1 [a – 3 {b – 4(a – b – 8) + 4a} + 10]
= -1 [a-3 {b-4{a-b-8) + 4a} + 10]

= -1[a-3 {b-4a + Ab +32 + 4a} + 10]
= -1 [a-3b+ 12a- 126-96- 12a + 10]
= -a + 3b – 12a + 12b + 96 + 12a – 10
= -a-12a + 12a+ 3b+ 12b-96-10
= – a + 15b – 106

Question 32.

$p^2 - [x^2 - \{x^2 - (q^2 - \overline{x^2 - q^2}) - 2y^2\}]$

Solution:

$$p^2 - [x^2 - \{x^2 - (q^2 - \overline{x^2 - q^2}) - 2y^2\}]$$
$$= p^2 - [x^2 - \{x^2 - (q^2 - x^2 + q^2) - 2y^2\}]$$
$$= p^2 - [x^2 - \{x^2 - (2q^2 - x^2) - 2y^2\}]$$
$$= p^2 - [x^2 - \{x^2 - 2q^2 + x^2 - 2y^2\}]$$
$$= p^2 - x^2 + 2x^2 - 2q^2 - 2y^2$$
$$= p^2 + x^2 - 2q^2 - 2y^2$$

Question 33.

10 – {4a – (7 – $\overline{a-5}$) – (5a – $\overline{1+a}$)}

Solution:

10 – {4a – (7 – $\overline{a-5}$) – (5a – $\overline{1+a}$)}
= 10 – {4a – (7 – a + 5) – (5a – 1 – a)}
= 10- {4a -(12 -a) -(4a- 1)}
= 10 – {4a – 12 + a- 4a + 1}
= 10 – 4a + 12 – a + 4a- 1
= 10 + 12 – 1 – 4a – a + 4a
= 21 -a

Question 34.

7a- [8a- (11a-(12a- $\overline{6a-5a}$)}]

Solution:

7a – [8a – {1 la – (12a –$\overline{6a-5a}$)}]
= 7a-[8a-{11a-(12a-6a + 5a)}]
= 7a -[8a -{11a -(17a -6a)}]
= 7a- [8a- {11a-(11a)}]

= 7a- [8a- {11a- 11a}]
= 7a – 8a = -a

Question 35.

$$8x-[4y-\{4x+(2x-\overline{2y-2x})\}]$$

Solution:

$$8x-[4y-\{4x+(2x-\overline{2y-2x})\}]$$
$$=8x-[4y-\{4x+(2x-2y+2x)\}]$$
$$=8x-[4y-\{4x+(4x-2y)\}]$$
$$=8x-[4y-\{4x+4x-2y\}]$$
$$=8x-[4y-4x-4x+2y]$$
$$=8x-[-8x+6y]$$
$$=8x+8x-6y$$
$$=16x-6y$$

Question 36.
x-(3y- $\overline{4z-3x}$ +2z- $\overline{5y-7x}$)

Solution:
x-(3y- $\overline{4z-3x}$ +2z- $\overline{5y-7x}$)
= x – (3y – 4z + 3x + 2z -5y + 7x)
= x-(-2y-2z+10x)
= x + 2y + 2z- 10x
= -9x + 2y + 2z

CHAPTER - 12
SIMPLE LINEAR EQUATIONS

POINTS TO REMEMBER

1. **Equation:** An equation is a statement which states that two expressions are equal.
2. To solve an equation means to find the value of the variable (unknown quantity) used in it.
   Note : An equation remains unchanged if
   (i) the same number is added to each side of the equation. .
   (ii) the same number is subtracted from each side of the equation.
   (iii) the same number is multiplied to each side of the equation.
   (iv) Each side of the equation is divided by the same non-zero number.
   (v) In transposing any term of an equation from one side to another, then its sign is reversed is
   (a) from positive to negative and from negative to positive
   (b) from multiplication to division and from division to multiplication.
3. **In equation :**
   It is a statement of inequality between two expressions involving a single variable with the highest power one.
4. **Replacement set**
   For a given inequation, the set from which the values of its variable are taken is called the replacement set or domain of the variable.
5. **Solution set**
   It is the subset of the replacement set, consisting of those values of the variable which satisfy the given inequation
6. **Properties of inequations**
   Adding, subtracting, multiplying or dividing by the same positive number to each side of an inequation does not change the inequality but multiplying or dividing by a negative number to each side of an inequation, it changes the inequality.

EXERCISE 12 (A)

Solve the following equations :

Question 1.
x + 5 = 10

Solution:
$x + 5 = 10$
$\Rightarrow x = 10 - 5 = 5$

Question 2.
2 + y=7

Solution:
2 + y = 7
⇒ = 7- 2 = 5

Question 3.
a – 2 = 6

Solution:
a -2 =6
⇒a = 6 + 2 = 8

Question 4.
x – 5 = 8

Solution:
x-5 =8
⇒ x = 8 +5 = 13

Question 5.
5 – d= 12

Solution:
5-d = 12
⇒ -d = 12-5 =7
⇒ d = – 7

Question 6.
3p = 12

Solution:
3p = 12
⇒ P = $\frac{12}{3}$ = 4 Ans.

Question 7.
14 = 7m

Solution:
14 = 7m
⇒ m = $\frac{14}{7}$ = 2

Question 8.
2x = 0

Solution:
2x = 0 ⇒ x = $\frac{0}{2}$ = 0

Question 9.
$\frac{x}{9}$ = 2

Solution:
$\frac{x}{9}$ = 2
⇒x = 2 ×9 = 18
∴ x = 18

Question 10.
$\frac{y}{-12}$ = -4

Solution:
$\frac{y}{-12}$ = -4
⇒ $\frac{y}{-12}$ = -4
⇒ y = (-4) × (-12)
∴ y= 48

Question 11.
8x-2 =38

Solution:
8x-2 =38
8x = 38 + 2 = 40
⇒ x = $\frac{40}{8}$ = 5
∴ x = 5

Question 12.
2x + 5 = 5

Solution:
2x + 5 = 5
⇒ 2x = 5 – 5 = 0
x = $\frac{0}{2}$ = 0
∴x = 0

Question 13.
5x – 1 = 74

Solution:
5x- 1 = 74
⇒ 5x = 74 + 1 = 75
⇒ x =$\frac{75}{5}$ = 15

Question 14.
14 = 27-x

Solution:
14 = 27 -x
⇒ x = 27- 14
⇒ x = 13
∴ x= 13

Question 15.
10 + 6a = 40

Solution:
10 + 6a = 40
⇒ 6a = 40 -10 = 30
⇒ a = $\frac{30}{6}$ = 5
∴ a= 5

Question 16.
$$c - \frac{1}{2} = \frac{1}{3}$$

Solution:

$$c - \frac{1}{2} = \frac{1}{3}$$

$$c = \frac{1}{3} + \frac{1}{2} = \frac{2+3}{6} = \frac{5}{6}$$

$$\therefore \quad c = \frac{5}{6}$$

Question 17.

$$\frac{a}{15} - 2 = 0$$

Solution:

$$\frac{a}{15} - 2 = 0 \quad \Rightarrow \quad \frac{a}{15} = 2$$

$$\Rightarrow \quad a = 2 \times 15 = 30$$

$$\therefore \quad a = 30$$

Question 18.
12 = c – 2

Solution:
12 = c – 2
⇒ 12 + 2 =c
⇒ 14 = c
∴c = 14

Question 19.
4 = x- 2.5

Solution:
4 = x – 2.5
⇒4 + 2.5=x
⇒ 6.5 =x
∴ x = 6.5

Question 20.
$$y + 5 = 8\frac{1}{4}$$

Solution:
$$y + 5 = 8\frac{1}{4} \quad \Rightarrow \quad y + 5 = \frac{33}{4}$$
$$\Rightarrow y = \frac{33}{4} - \frac{5}{1} = \frac{33 - 20}{4} = \frac{13}{4}$$
$$\therefore\ y = \frac{13}{4} = 3\frac{1}{4}$$

Question 21.
$$x + \frac{1}{4} = -\frac{3}{8}$$

Solution:
$$x + \frac{1}{4} = -\frac{3}{8}$$
$$\Rightarrow\ x = -\frac{3}{8} - \frac{1}{4} = \frac{-3 - 2}{8} = \frac{-5}{8}$$
$$\therefore\quad x = \frac{-5}{8}$$

Question 22.
p + 0.02 = 0.08

Solution:
p + 0.02 = 0.08
⇒ p = 0.08 – 0.02 = 0.06
∴ p = 0 06

Question 23.
$$p - 12 = 2\frac{2}{3}$$

Solution:

$$p - 12 = 2\frac{2}{3} \quad \Rightarrow p - 12 = \frac{8}{3}$$

$$\Rightarrow \quad p = \frac{8}{3} + \frac{12}{1} = \frac{8+36}{3} = \frac{44}{3}$$

$$\therefore \quad p = \frac{44}{3} = 14\frac{2}{3}$$

Question 24.

$-3x = 15$

Solution:

$$-3x = 15 \quad \Rightarrow x = \frac{15}{-3} = -5$$

$$\therefore x = -5$$

Question 25.

Solution:

$$1 \cdot 3b = 39 \Rightarrow b = \frac{39}{1 \cdot 3} = \frac{39 \times 10}{13} = 30$$

$$\therefore b = 30$$

Question 26.

$\frac{5}{8}n = 20$

Solution:

$$\frac{5}{8}n = 20 \quad \Rightarrow 5n = 20 \times 8 = 160$$

$$\Rightarrow n = \frac{160}{5} = 32$$

$$\therefore n = 32$$

Question 27.
$$\frac{3}{16}m = 21$$

Solution:
$$\frac{3}{16}m = 21 \quad \Rightarrow 3m = 21 \times 16 = 336$$
$$\Rightarrow \; m = \frac{336}{3} = 112 \; \therefore \; m = 112$$

Question 28.
2a – 3 =5

Solution:
2a – 3 = 5
⇒2a = 5 +3
⇒ 2a = 8
⇒ a = $\frac{8}{2}$ = 4
∴a = 4

Question 29.
3p – 1 = 8

Solution:
3p – 1 = 8
⇒3p = 8 + 1 = 9
⇒ p = $\frac{9}{3}$ = 3
∴p = 3

Question 30.
9y -7 = 20

Solution:
$$9y - 7 = 20 \qquad \Rightarrow 9y = 20 + 7 = 27$$
$$\Rightarrow \; y = \frac{27}{9} = 3$$
$$\therefore \; y = 3$$

Question 31.
$2b - 14 = 8$

Solution:
$2b - 14 = 8 \quad \Rightarrow \quad 2b = 8 + 14 = 22$

$$\Rightarrow \quad b = \frac{22}{2} = 11$$

$$\therefore \quad b = 11$$

Question 32.
$$\frac{7}{10}x + 6 = 41$$

Solution:
$$\frac{7}{10}x + 6 = 41$$
$$\frac{7}{10}x + 6 = 41 \quad \Rightarrow \quad \frac{7}{10}x = 41 - 6 = 35$$
$$\Rightarrow \quad 7x = 35 \times 10 = 350$$
$$\Rightarrow \quad x = \frac{350}{7} = 50$$
$$\therefore \quad x = 50$$

Question 33.
$$\frac{5}{12}m - 12 = 48$$

Solution:
$$\frac{5}{12}m - 12 = 48 \quad \Rightarrow \frac{5}{12}m = 48 + 12 = 60$$
$$\Rightarrow 5m = 60 \times 12 = 720$$
$$\Rightarrow \quad m = \frac{720}{5} = 144$$
$$\therefore \quad m = 144$$

EXERCISE 12 (B)

Question 1.
8y – 4y = 20

Solution:

$$8y - 4y = 20 \Rightarrow 4y = 20$$

$$\Rightarrow \quad y = \frac{20}{4} = 5$$

$$\therefore \quad y = 5$$

Question 2.
9b – 4b + 3b = 16

Solution:

$$9b - 4b + 3b = 16$$

$$\Rightarrow (9 - 4 + 3)\, b = 16 \quad \Rightarrow 8b = 16$$

$$\Rightarrow \quad b = \frac{16}{8} = 2$$

$$\therefore \quad b = 2$$

Question 3.
5y + 8 = 8y – 18

Solution:

$$5y + 8 = 8y - 18$$

$$\Rightarrow 5y - 8y = -18 - 8$$

$$\Rightarrow \quad -3y = -26 \quad \Rightarrow y = \frac{-26}{-3} = \frac{26}{3}$$

$$\therefore \quad y = 8\frac{2}{3}$$

Question 4.
6 = 7 + 2p -5

Solution:

$$6 = 7 + 2p - 5 \quad \Rightarrow -2p = 7 - 5 - 6$$

$$\Rightarrow -2p = -4 \quad \Rightarrow \quad p = \frac{-4}{-2} = 2$$

$$\therefore \quad p = 2$$

Question 5.
8 – 7x = 13x + 8

Solution:

$$8 - 7x = 13x + 8 \quad \Rightarrow -7x - 13x = 8 - 8$$

$$\Rightarrow -20x = 0 \quad \Rightarrow x = \frac{0}{-20} = 0$$

$$\therefore \quad x = 0$$

Question 6.
4x – 5x + 2x = 28 + 3x

Solution:

$$4x - 5x + 2x = 28 + 3x$$

$$\Rightarrow 4x - 5x + 2x - 3x = 28$$

$$\Rightarrow 6x - 8x = 28$$

$$\Rightarrow \quad x = \frac{28}{-2} = -14$$

$$\therefore \quad x = -14$$

Question 7.
9 + m = 6m + 8 – m

Solution:

$$9 + m = 6m + 8 - m$$

$$\Rightarrow m - 6m + m = 8 - 9$$

$$\Rightarrow \quad 2m - 6m = -1 \quad \Rightarrow -4m = -1$$

$$\therefore m = \frac{-1}{-4} = \frac{1}{4}$$

Question 8.
24 = y + 2y + 3 + 4y

Solution:

$$24 = y + 2y + 3 + 4y$$

$$\Rightarrow 24 - 3 = y + 2y + 4y \quad \Rightarrow 21 = 7y$$

$$\Rightarrow \quad 7y = 21 \qquad \Rightarrow \quad y = \frac{21}{7} = 3$$

$$\therefore \quad y = 3$$

Question 9.
19x -+ 13 -12x + 3 = 23

Solution:

$$19x + 13 - 12x + 3 = 23$$

$$\Rightarrow 19x - 12x = 23 - 13 - 3$$

$$\Rightarrow \quad 7x = 23 - 16 = 7 \quad \Rightarrow x = \frac{7}{7} = 1$$

$$\therefore \quad x = 1$$

Question 10.
6b + 40 = – 100 – b

Solution:

$$6b + 40 = -100 - b$$

$$\Rightarrow 6b + b = -100 - 40 \quad \Rightarrow 7b = -140$$

$$\Rightarrow \quad b = \frac{-140}{7} = -20$$

$$\therefore \quad b = -20$$

Question 11.
6 – 5m – 1 + 3m = 0

Solution:

$$6 - 5m - 1 + 3m = 0$$

$$\Rightarrow -5m + 3m = -6 + 1 \quad \Rightarrow -2m = -5$$

$$\Rightarrow \quad m = \frac{-5}{-2} = \frac{5}{2}$$

$$\therefore \quad m = \frac{5}{2} = 2\frac{1}{2}$$

Question 12.
0.4x – 1.2 = 0.3x + 0.6

Solution:

$$0{\cdot}4x - 1{\cdot}2 = 0{\cdot}3x + 0{\cdot}6$$

$$\Rightarrow 0{\cdot}4x - 0{\cdot}3x = 0{\cdot}6 + 1{\cdot}2$$

$$\Rightarrow 0{\cdot}1x = 1{\cdot}8$$

$$\Rightarrow \frac{1}{10}x = \frac{18}{10} \quad \Rightarrow x = \frac{18}{10} \times \frac{10}{1} = 18$$

$$\therefore \quad x = 18$$

Question 13.
6(x+4) = 36

Solution:

$$6(x+4)=36 \quad \Rightarrow \quad 6x+24=36$$

$$\Rightarrow 6x = 36-24 = 12 \quad \Rightarrow x = \frac{12}{6} = 2$$

$$\therefore \quad x = 2$$

Question 14.
9 (a+ 5) + 2 = 11

Solution:

$$9(a+5)+2=11 \Rightarrow 9a+45+2=11$$

$$\Rightarrow 9a = 11-45-2 \Rightarrow 9a = 11-47 = -36$$

$$\Rightarrow a = \frac{-36}{9} = -4$$

$$\therefore \quad a = -4$$

Question 15.
4 (x- 2) = 12

Solution:

$$4(x-2)=12 \qquad \Rightarrow 4x-8=12$$

$$\Rightarrow 4x = 12+8 = 20 \quad \Rightarrow x = \frac{20}{4} = 5$$

$$\therefore x = 5$$

Question 16.
-3 (a- 6) = 24

Solution:

$$-3(a-6)=24 \qquad \Rightarrow -3a+18=24$$

$$\Rightarrow -3a = 24-18 = 6 \Rightarrow a = \frac{6}{-3} = -2$$

$$\therefore a = -2$$

Question 17.
7 (x-2) = 2 (2x -4)

Solution:
$7(x-2) = 2(2x-4) \Rightarrow 7x - 14 = 4x - 8$
$\Rightarrow 7x - 4x = -8 + 14 \Rightarrow 3x = 6$
$\Rightarrow \quad x = \frac{6}{3} = 2$
$\therefore \quad x = 2$

Question 18.
(x-4) (2x +3) = $2x^2$

Solution:
$(x-4)(2x+3) = 2x^2$
$\Rightarrow x(2x+3) - 4(2x+3) = 2x^2$
$\Rightarrow \quad 2x^2 + 3x - 8x - 12 = 2x^2$
$\Rightarrow \quad 2x^2 + 3x - 8x - 2x^2 = 12$
$\Rightarrow -5x = 12 \quad \Rightarrow x = \frac{12}{-5} = \frac{-12}{5}$
$\therefore \quad x = -2\frac{2}{5}$

Question 19.
21 – 3 (b-7) = b+ 20

Solution:

$$21 - 3(b - 7) = b + 20$$
$$\Rightarrow 21 - 3b + 21 = b + 20$$
$$\Rightarrow -3b + 42 = b + 20$$
$$\Rightarrow \quad -3b - b = 20 - 42 \quad \Rightarrow -4b = -22$$
$$\Rightarrow b = \frac{-22}{-4} = \frac{11}{2}$$
$$\therefore b = \frac{11}{2} = 5\frac{1}{2}$$

Question 20.

$x(x + 5) = x^2 + x + 32$

Solution:

$$x(x + 5) = x^2 + x + 32$$
$$\Rightarrow x^2 + 5x = x^2 + x + 32$$
$$\Rightarrow x^2 + 5x - x^2 - x = 32 \quad \Rightarrow 4x = 32$$
$$\Rightarrow x = \frac{32}{4} = 8$$
$$\therefore x = 8$$

EXERCISE 12 (C)

Solve

Question 1.

$$\frac{x}{2} + x = 9$$

Solution:

$$\frac{x}{2}+\frac{x}{1}=9$$

$$\frac{x+2x}{2}=9 \Rightarrow x+2x=2\times 9 \Rightarrow 3x=18$$

$$\Rightarrow x=\frac{18}{3}=6$$

$$\therefore \quad x=6$$

Question 2.

$$\frac{x}{5}+2x=33$$

Solution:

$$\frac{x}{5}+\frac{2x}{1}=33$$

$$\frac{x+10x}{5}=33 \quad \Rightarrow \frac{11x}{5}=33$$

$$\Rightarrow 11x=33\times 5=165$$

$$\Rightarrow x=\frac{165}{11}=15$$

$$\therefore \quad x=15$$

Question 3.

$$\frac{3x}{4}+4x=38$$

Solution:

$$\frac{3x}{4} + 4x = 38 \Rightarrow \frac{3x}{4} + \frac{4x}{1} = 38$$

$$\frac{3x + 16x}{4} = 38 \Rightarrow 3x + 16x = 38 \times 4$$

$$3x + 16x = 152 \Rightarrow 19x = 152 \Rightarrow x = \frac{152}{19} = 8$$

$$\therefore x = 8$$

Question 4.

$$\frac{x}{2} + \frac{x}{5} = 14$$

Solution:

$$\frac{x}{2} + \frac{x}{5} = 14 \Rightarrow \frac{x}{2} + \frac{x}{5} = 14$$

$$\frac{5x + 2x}{10} = 14 \Rightarrow 5x + 2x = 14 \times 10$$

$$\Rightarrow 5x + 2x = 140 \Rightarrow 7x = 140$$

$$\Rightarrow x = \frac{140}{7} = 20$$

$$\therefore x = 20$$

Question 5.

$$\frac{x}{3} - \frac{x}{4} = 2$$

Solution:

$$\frac{x}{3} - \frac{x}{4} = 2 \Rightarrow \frac{4x - 3x}{12} = 2$$

$$\Rightarrow 4x - 3x = 2 \times 12 \Rightarrow x = 24$$

$$\therefore x = 24$$

Question 6.

$$y+\frac{y}{2}=\frac{7}{4}-\frac{y}{4}$$

Solution:

$$\frac{y}{1}+\frac{y}{2}=\frac{7}{4}-\frac{y}{4} \Rightarrow \frac{4y+2y=7-y}{4}$$

$$\Rightarrow 4y+2y=7-y \Rightarrow 4y+2y \Rightarrow 7y=7$$

$$\Rightarrow y=\frac{7}{7}=1$$

$$\therefore \; y=1$$

Question 7.

$$\frac{4x}{3}-\frac{7x}{3}=1$$

Solution:

$$\frac{4x}{3}-\frac{7x}{3}=1 \Rightarrow \frac{4x-7x}{3}=1$$

$$\Rightarrow \frac{-3x}{3}=1 \Rightarrow -3x=3 \Rightarrow x=\frac{3}{-3}=-1$$

$$\therefore \quad x=-1$$

Question 8.

$$\frac{1}{2}m+\frac{3}{4}m-m=2\cdot5$$

Solution:

$$\frac{1}{2}m + \frac{3}{4}m - m = 2\cdot5$$

$$\frac{1}{2}m + \frac{3}{4}m - \frac{m}{1} = 2\cdot5$$

$$\frac{2m+3m-4m}{4} = 2\cdot5 \Rightarrow \frac{2m+3m-4m}{4} = 4 \times 2\cdot5$$

$$\Rightarrow 2m + 3m - 4m = 10 \Rightarrow 5m - 4m = 10 \Rightarrow m = 10$$

$$\therefore \ m = 10$$

Question 9.

$$\frac{2x}{3} + \frac{x}{2} - \frac{3x}{4} = 1$$

Solution:

$$\frac{2x}{3} + \frac{x}{2} - \frac{3x}{4} = 1 \Rightarrow \frac{8x+6x-9x}{12} = 1$$

$$\Rightarrow 8x + 6x - 9x = 12 \times 1 \Rightarrow 8x + 6x - 9x = 12$$

$$\Rightarrow 14x - 9x = 12 \Rightarrow 5x = 12 \Rightarrow x = \frac{12}{5} = 2\frac{2}{5}$$

$$\therefore \ x = 2\frac{2}{5}$$

Question 10.

$$\frac{3a}{4} + \frac{a}{6} = 66$$

Solution:

$$\frac{3a}{4}+\frac{a}{6}=66 \Rightarrow \frac{9a+2a}{12}=66$$

$$\Rightarrow 9a+2a=66\times 12 \Rightarrow 9a+2a=792$$

$$\Rightarrow 11a=792 \Rightarrow a=\frac{792}{11}=72$$

$$\therefore \quad a=72$$

Question 11.

$$\frac{2p}{3}-\frac{p}{5}=35$$

Solution:

$$\frac{2p}{3}-\frac{p}{5}=35 \Rightarrow \frac{10p-3p}{15}=35$$

$$\Rightarrow 10p-3p=35\times 15 \Rightarrow 10p-3p=525$$

$$\Rightarrow 7p=525$$

$$\Rightarrow p=\frac{525}{7}=75$$

$$\therefore \quad p=75$$

Question 12.

0.6a +0.2a = 0.4 a +8

Solution:

$$0{\cdot}6a+0{\cdot}2a=0{\cdot}4a+8$$

$$\frac{6}{10}a+\frac{2}{10}a=\frac{4}{10}a+\frac{8}{1}$$

$$\frac{6a+2a}{10}=4a+80 \Rightarrow 6a+2a=4a+80$$

$$\Rightarrow 6a+2a-4a=80 \Rightarrow 4a=80$$

$$\Rightarrow a=\frac{80}{4}=20 \quad \therefore \quad a=20$$

Question 13.
p + 104p= 48

Solution:

$$p + 1{\cdot}4p = 48$$

$$p + \frac{14}{10}p = 48 \Rightarrow \frac{10p + 14p}{10} = 48$$

$$\Rightarrow 10\text{ P} + 14\text{ P} = 48 \times 10 \Rightarrow 10p + 14p = 480$$

$$\Rightarrow 24p = 480 \quad \Rightarrow p = \frac{480}{24} = 20$$

$$\therefore \ p = 20$$

Question 14.
10% of x = 20

Solution:

$$10\% \text{ of } x = 20$$

$$\Rightarrow \frac{10}{100} \times x = 20 \Rightarrow \frac{x}{10} = 20 \Rightarrow x = 20 \times 10 = 200$$

$$\therefore x = 200$$

Question 15.
y + 20% of y = 18

Solution:

$$y + 20\% \text{ of } y = 18$$

$$\Rightarrow y + \frac{20}{100} \times y = 18 \Rightarrow \frac{100y + 20y}{100} = 18$$

$$\Rightarrow 100y + 20y = 18 \times 100$$

$$\Rightarrow 100y + 20y = 1800 \quad \Rightarrow 120y = 1800$$

$$\Rightarrow \ y = \frac{1800}{120} = 15$$

$$\therefore \ y = 15$$

Question 16.
x – 13% of x = 35

Solution:

$$x - 30\% \text{ of } x = 35$$

$$x - \frac{30}{100} \times x = 35 \Rightarrow \frac{100x - 30x}{100} = 35$$

$$\Rightarrow 100x - 30x = 35 \times 100$$

$$100x - 30x = 3500 \Rightarrow 70x = 3500$$

$$\Rightarrow x = \frac{3500}{70} = 50$$

$$\therefore x = 50$$

Question 17.

$$\frac{x+4}{2} + \frac{x}{3} = 7$$

Solution:

$$\frac{x+4}{2} + \frac{x}{3} = 7 \Rightarrow \frac{3x + 12 + 2x}{6} = 7$$

$$\Rightarrow 3x + 12 + 2x = 7 \times 6 \Rightarrow 3x + 12 + 2x = 42$$

$$\Rightarrow 5x = 42 - 12 = 30 \Rightarrow x = \frac{30}{5} = 6$$

$$\therefore x = 6$$

Question 18.

$$\frac{y+2}{3} + \frac{y+5}{4} = 6$$

Solution:

$$\frac{y+2}{3}+\frac{y+5}{4}=6 \Rightarrow \frac{4y+8+3y+15}{12}=6$$

$$\Rightarrow 4y+8+3y+15=6\times 12$$

$$\Rightarrow 4y+8+3y+15=72 \Rightarrow 7y+23=72$$

$$\Rightarrow 7y=72-23=49 \Rightarrow y=\frac{49}{7}=7$$

$$\therefore y=7$$

Question 19.

$$\frac{3a-2}{7}-\frac{a-2}{4}=2$$

Solution:

$$\frac{3a-2}{7}-\frac{a-2}{4}=2$$

$$\frac{12a-8-7a+14}{28}=2$$

$$\Rightarrow 12a-8-7a+14=2\times 28$$

$$\Rightarrow 12a-8-7a+14=56$$

$$\Rightarrow 12a-7a+14-8=56 \Rightarrow 5a+6=56$$

$$\Rightarrow 5a=56-6=50 \Rightarrow a=\frac{50}{5}=10$$

$$\therefore a=10$$

Question 20.

$$\frac{1}{2}(x+5)-\frac{1}{3}(x-2)=4$$

Solution:

$$\frac{1}{2}(x+5)-\frac{1}{3}(x-2)=4$$

$$\frac{3(x+5)-2(x-2)}{6}=4$$

$$\Rightarrow 3x+15-2x+4=4\times 6$$

$$3x+15-2x+4=24$$

$$\Rightarrow 3x-2x=24-15-4 \Rightarrow x=24-19=5$$

$$\therefore x=5$$

Question 21.

$$\frac{x-1}{2}-\frac{x-2}{3}-\frac{x-3}{4}=0$$

Solution:

$$\frac{x-1}{2}-\frac{x-2}{3}-\frac{x-3}{4}=0$$

$$\frac{6(x-1)-4(x-2)-3(x-3)}{12}=0$$

$$6(x-1)-4(x-2)-3(x-3)=0$$

$$\therefore 6x-6-4x+8-3x+9=0$$

$$\therefore 6x-4x-3x-6+8+9=0$$

$$\therefore 6x-7x=6-8-9$$

$$\Rightarrow -x=6-17=-11$$

$$\therefore \quad x=11$$

Question 22.

$$\frac{x+1}{3}+\frac{x+4}{5}=\frac{x-4}{7}$$

Solution:

$$\frac{x+1}{3}+\frac{x+4}{5}=\frac{x-4}{7}$$

$$\frac{35(x+1)+21(x+4)=15(x-4)}{105}$$

(L.C.M. of 3, 5, 7 = 105)

$$35\ (x+1)+21\ (x+4)=15\ (x-4)$$
$$35x+35+21x+84=15x-60$$
$$\Rightarrow 35x+21x-15x=-60-35-84$$
$$\Rightarrow \quad 56x-15x=-(60+35+84)$$
$$\Rightarrow \quad 41x=-179$$
$$\therefore \quad x=\frac{-179}{41}=-4\frac{15}{41}$$

Question 23.

15 – 2 (5-3x) = 4 (x-3) + 13

Solution:

$$15-2\ (5-3x)=4\ (x-3)+13$$
$$\Rightarrow 15-10+6x=4x-12+13$$
$$6x-4x=-12+13-15+10$$
$$\Rightarrow 2x=23-27=-4 \quad \Rightarrow x=\frac{-4}{2}=-2$$

Hence $x=-2$

Question 24.

$$\frac{2x+1}{3x-2}=1\frac{1}{4}$$

Solution:

$$\frac{2x+1}{3x-2}=1\frac{1}{4} \Rightarrow \frac{2x+1}{3x-2}=\frac{5}{4}$$

By cross multiplication

$$(3x - 2) \times 5 = 4\,(2x + 1)$$

$$\Rightarrow 15x - 10 = 8x + 4$$

$$\Rightarrow 15x - 8x = 4 + 10 \Rightarrow 7x = 14 \Rightarrow x = \frac{14}{7} = 2$$

$$\therefore x = 2$$

Question 25.

21 – 3 (x – 7) = x + 20

Solution:

$$21 - 3\,(x - 7) = x + 20$$

$$\Rightarrow 21 - 3x + 21 = x + 20 \Rightarrow 42 - 3x = x + 20$$

$$\Rightarrow 42 - 20 = x + 3x \Rightarrow 4x = 22$$

$$\Rightarrow x = \frac{22}{4} = \frac{11}{2} = 5\frac{1}{2} \qquad \therefore x = 5\frac{1}{2}$$

Question 26.

$$\frac{3x-2}{7} - \frac{x-2}{4} = 2$$

Solution:

$$\frac{3x-2}{7} - \frac{x-2}{4} = 2$$

$$\Rightarrow \frac{12x - 8 - 7x + 14}{28} = 2$$

$$\Rightarrow \frac{5x+6}{28} = 2 \Rightarrow 5x + 6 = 28 \times 2$$

$$\Rightarrow 5x + 6 = 56 \Rightarrow 5x = 56 - 6 = 50 \Rightarrow x = \frac{50}{5} = 10$$

$$\therefore x = 10$$

Question 27.

$$\frac{2x-3}{3}-(x-5)=\frac{x}{3}$$

Solution:

$$\frac{2x-3}{3}-\frac{(x-5)}{1}=\frac{x}{3}$$

$$=\frac{2x-3-3x+15=x}{3} -x+12=x$$

$$\Rightarrow 12=x+x \Rightarrow 2x=12$$

$$\therefore\ x=\frac{12}{2}=6$$

Question 28.

$$\frac{x-4}{7}=\frac{x+3}{7}+\frac{x+4}{5}$$

Solution:

$$\frac{x-4}{7}=\frac{x+3}{7}+\frac{x+4}{5}$$

$$=\frac{5x-20=5x+15+7x+28}{35}$$

$$\Rightarrow 5x-5x-7x=15+28+20$$

$$\Rightarrow -7x=63 \quad \Rightarrow x=\frac{63}{-7}=-9$$

$$\therefore\ =x=-9$$

Question 29.

$$\frac{x-1}{5}-\frac{x}{3}=1-\frac{x-2}{2}$$

Solution:

$$\frac{x-1}{5}-\frac{x}{3}=1-\frac{x-2}{2}$$

$$=\frac{6x-6-10x=30-15x+30}{30}$$

$$\Rightarrow -4x-6=60-15x$$

$$\Rightarrow 15x-4x=60+6 \Rightarrow 11x=66 \quad \Rightarrow x=\frac{66}{11}=6$$

$$\therefore\ x=6$$

Question 30.
2x + 20% of x = 12.1

Solution:

$$2x + 20\% \text{ of } x = 12.1$$

$$\Rightarrow 2x + \frac{x \times (20)}{100} = 12.1 \Rightarrow 2x + \frac{20x}{100} = 12.1$$

$$\Rightarrow 2x + \frac{2x}{10} = 12.1$$

$$\frac{20x + 2x = 121}{10} \Rightarrow 22x = 121 \Rightarrow x = \frac{121}{22} = \frac{11}{2}$$

$$\therefore x = \frac{11}{2} \text{ or } 5\frac{1}{2}$$

EXERCISE 12 (D)

Question 1.
One-fifth of a number is 5, find the number.

Solution:
Let the number = x
According to the condition
$\frac{1}{5}x = 5 \Rightarrow x = 5 \times 5$
$\Rightarrow x = 25$
$\therefore$ Number = 25

Question 2.
Six times a number is 72, find the number.

Solution:
Let the number = x
According to the condition
$6x = 72$
$\Rightarrow x = \frac{72}{6}$
$\Rightarrow x = 12$
$\therefore$ Number = 12

Question 3.
If 15 is added to a number, the result is 69, find the number.

Solution:
Let the number = x
According to the condition
x+ 15 = 69
⇒ x = 69 – 15 x = 54
∴Number = 54

Question 4.
The sum of twice a number and 4 is 80, find the number.

Solution:
Let the number = x
According to the condition
2x + 4 = 80
⇒2x = 80 – 4
⇒ 2x = 76
⇒ x = $\frac{76}{2}$ = 38
Number = 38

Question 5.
The difference between a number and one- fourth of itself is 24, find the number.

Solution:
Let the number = x
According to the condition

$$x - \frac{1}{4}x = 24$$

$$\Rightarrow \frac{4x-x}{x} = 24 \quad \Rightarrow \frac{3x}{4} = 24$$

$$\Rightarrow x = 24 \times \frac{4}{3} \quad \Rightarrow x = 8 \times 4$$

$$\Rightarrow x = 32$$

$\therefore$ Number = 32

Question 6.
Find a number whose one-third part exceeds its one-fifth part by 20.

Solution:
Let the number = x
According to the condition

$$\frac{1}{3}x - \frac{1}{5}x = 20$$

$$\Rightarrow \frac{5x - 3x}{15} = 20$$

[$\because$ LCM of 3 and 5 = 15]

$$\Rightarrow \frac{2x}{15} = 20 \quad \Rightarrow x = \frac{20 \times 15}{2}$$

$$\Rightarrow x = 150$$

$\therefore$ Number = 150

Question 7.
A number is as much greater than 35 as is less than 53. Find the number.

Solution:
Let the number = x
According to the condition
x – 35 = 53 – x
⇒ x + x = 53 + 35
88
⇒2x = 88
⇒ x = $\frac{88}{2}$ = 44
∴Number = 44

Question 8.
The sum of two numbers is 18. If one is twice the other, find the numbers.

Solution:
Let the first number = x
and the second number = y
According to the condition
x + y= 18 …(i)
and x = 27 ….(ii)
Substitute the eq. (ii) in eq. (i), we get
2y + y= 18
x= 2y = 18
⇒ 3y= 18 ⇒y= $\frac{18}{3}$ = 6

Now, substitute the value of y in eq. (ii), we get
x = 2 x 6= 12
∴ The two numbers are 12, 6

Question 9.
A number is 15 more than the other. The sum of of the two numbers is 195. Find the numbers.

Solution:
Let the First number = x
and the Second number = y
According to the condition
x = y+ 15 …(i)
x + 7=195 …(ii)
Substitute the eq. (i) in eq. (ii), we get
y+15+7=195
⇒2y= 195- 15
⇒ y = $\frac{180}{2}$ = 90
Now, substitute the value of y in eq. (i), we get
x = 90+ 15 = 105
∴ The two numbers are 105 and 90

Question 10.
The sum of three consecutive even numbers is 54. Find the numbers.

Solution:
Let the first even number = x
second even number = x + 2
and third even number = x + 4
According to the condition,
x + x + 2 + x + 4 = 54
⇒ 3x + 6 = 54
⇒ 3x = 54 – 6
⇒ x = $\frac{48}{3}$ = 16
∴ First even number = 16
Second even number = 16 + 2 = 18
and third even number = 16 + 4 = 20

Question 11.
The sum of three consecutive odd numbers is 63. Find the numbers.

Solution:
Let the first odd number = x

second odd number = x + 2
and third odd number = x + 4
According to the condition,
x+ x + 2 + x+4 = 63
3x + 6 = 63 ⇒ 3x = 63 – 6
⇒3x = 57 ⇒ x = $\frac{57}{3}$ =19
∴ First odd number = 19
Second odd number = 19 + 2 = 21
third odd number = 19 + 4 = 23

Question 12.
A man has ₹ x from which he spends ₹6. If twice of the money left with him is ₹86, find x.

Solution:
Let the total amount be x
According to the condition
2x = 86
⇒x = $\frac{86}{2}$
⇒ x = 43
Amount spent by him = 6
∴Total money he have = ₹43 + ₹6 = ₹49

Question 13.
A man is four times as old as his son. After 20 years, he will be twice as old as his son at that time. Find their present ages.

Solution:
Let the present age of the son = x years
Present age of the father = 4x years
After 20 years,
Son's age will be (x + 20) years
and Father's age will be (4x + 20) years
According to the condition,
4x + 20 = 2 (x + 20)
4x + 20 = 2x + 40
4x – 2x = 40 – 20
2x = 20
⇒ x = 10
∴Present age of the son = 10 years and Present age of the father = 4×10 years = 40 years

Question 14.
If 5 is subtracted from three times a number, the result is 16. Find the number.

Solution:
Let the number = x
According to the condition,
$3x - 5 = 16$
$\Rightarrow 3x = 16 + 5$
$\Rightarrow 3x = 21$
$\Rightarrow x = \frac{21}{3}$
$\Rightarrow x = 7$
$\therefore$The number = 7

Question 15.
Find three consecutive natural numbers such that the sum of the first and the second is 15 more than the third.

Solution:
Let the first conscutive number = x,
Second consecutive number = x + 1
and Third consecutive number = x + 2
According to the condition,
$x + x + 1 = 15 + x + 2$
$\Rightarrow 2x + 1 = 17 + x$
$\Rightarrow 2x - x = 17 - 1$
$\Rightarrow x = 16$
$\therefore$ The first consecutive number = 16
Second consecutive number =16+1 = 17
Third consecutive number =16 + 2=18

Question 16.
The difference between two numbers is 7. Six times the smaller plus the larger is 77. Find the numbers.

Solution:
Let the smallest number = x
and the largest number = y
According to the condition,
$y - x = 7$ …(i)
and $6x + y = 77$ ….(ii)
From eq. (i)
$y = 7 + x$ …(iii)
Substitute the eq. (iii) in eq. (ii)
$6x + 7 + x = 77$

$\Rightarrow$ 7x = 77-7

$\Rightarrow$ x = $\frac{70}{7}$ = 10

Now, substitute the value of x in eq. (iii)

y = 7+ 10= 17

$\therefore$The smallest number 10 and the largest number is 17.

Question 17.

The length of a rectangular plot exceeds its breadth by 5 metre. If the perimeter of the plot is 142 metres, find the length and the breadth of the plot.

Solution:

Let the length of a rectangular plot = x

and the breadth of a rectangular plot = y

According to the condition,

$$x = y + 5 \qquad \ldots(i)$$

and $2(x + y) = 142$

$$\Rightarrow x + y = \frac{142}{2} = 71$$

$$\Rightarrow x + y = 71 \qquad \ldots(ii)$$

Now, substitute the value of eq. (*i*) in eq. (*ii*)

$$y + 5 + y = 71$$

$$\Rightarrow 2y = 71 - 5$$

$$\Rightarrow y = \frac{66}{2} = 33$$

Now, put the value of y in eq. (i)

$$x = 33 + 5 = 38$$

$\therefore$ The length of rectangular plot is 38 m and breadth is 33 m

Question 18.
The numerator of a fraction is four less than its denominator. If 1 is added to both, is numerator and denominator, the fraction becomes $\frac{1}{2}$ Find the fraction.

Solution:

Let the numerator of a fraction = x
and the denominator of a fraction = y
According to the condition,

$x = y - 4$ ...(i)

and $\frac{(x+1)}{(y+1)} = \frac{1}{2}$

$\Rightarrow 2(x+1) = y + 1 \Rightarrow 2x + 2 = y + 1$

$\Rightarrow 2x - y = -1$ ..(ii)

Substitute the eq. (i) in eq. (ii)

$2(y - 4) - y = -1$

$2y - 8 - y = -1$

$y = -1 + 8$

$y = 7$

Now, put the value of y in eq. (i), we get

$x = 7 - 4$

$x = 3$

$\therefore$ The numerator of a fraction is 3

and denominator is 7 and the fraction is $\frac{3}{7}$

Question 19.
A man is thrice as old as his son. After 12 years, he will be twice as old as his son at that time. Find their present ages.

Solution:
Let the present age of the son = x years
and the present age of the father = 3x years
After 12 years,
Son’s age will be (x + 12) years

and father’s age will be (3x + 12) years
According to the condition,
3x + 12 = 2 (x + 12)
3x + 12 = 2x+ 24
3x – 2x = 24 – 12
x= 12
∴Present age of the son = 12 years
and Present age of the father = 3×12 years
= 36 years

Question 20.
A sum of ₹ 500 is in the form of notes of denominations of ₹ 5 and₹ 10. If the total number of notes is 90, find the number of notes of each type.

Solution:
Let the number of ₹ 5 notes = x
∴ The number of ₹10 notes = 90 – x
Value of ₹10 notes = x ×₹ 5 = ₹3x
and value of ₹10 notes = (90 – x) x ₹ 10 =₹(900 – 10x)
∴Total value of all the notes = ₹500
∴5x+ (900- 10x) = 500
⇒ 5x + 900 – 10x = 500
⇒ -5x = 500 – 900
⇒ x = $\frac{400}{5}$
⇒ x = 80
∴ The number of ₹5 notes = x = 80
and the number of ₹10 notes = 90 – x
= 90 – 80= 10

CHAPTER - 13
SET CONCEPTS

POINTS TO REMEMBER

1. Definition of a Set : In our day to day life, different collective nouns are used to describe collection of objects ; such as : a group of students playing cricket, a pack of cards, a bunch of flowers, etc

In mathematics, such collections of objects are named as sets.

A set is a collection of well-defined objects, things or symbols, etc.

The phrase 'well-defined' means ; it must be possible to know, without any doubt, whether a given object (thing or symbol) belongs to the set under consideration or not.

For example :

"The set of tall boys of Class 10" is not well-defined ; since it is not possible to know that which boys are to be included and exactly what is the limit.

But when we say, "The set of boys of Class 10, which are taller than Peter", now we can compare the heights of different boys with the height of Peter and can know exactly, that which boys are to be included in the required set. Thus, the objects are well-defined.

2. Elements of a set: The objects (things, symbols, etc.) used to form a set are called elements or members of the set.

In general, a set is denoted by a capital letter of English alphabet with its elements written inside curly braces and separated by commas.

e.g., Set A = {5, 10, 12, 15}

3. Use of Symbol '∈' or Symbol '∉ ' : The symbol '∈' stands for 'belongs to' or ' is an element of' or 'is a member of'; whereas the symbol '∉' stands for 'does not belong to' or 'is not an element of or 'is not a member of'.

e.g., For set P = {3, 6, 8, 13, 18} ; 3 ∈ P, 5∉ P and so on.

(i) The elements in a set can be written in any order.

Thus, {a, b, c, d] is the same set as {b, d, a, c} or {c, b, d, a}, etc.

(ii) The elements in a set should not be repeated, i.e. if any element occurs many times, it should be written only once.

Thus, set of letters of the word 'crook' = {c, r, o, k}.

There are two os in the given word "crook" ; but in the set, it is written only once.

4. Representation of A Set: A set, in general, is represented in :

(i) Description method (form)

(ii) Tabular or Roster method (form)

(iii) Set-builder or Rule method

For example :

N is the set of natural numbers [Description method]

N = {1, 2, 3, 4, 5, ……. } [Roster or Tabular method]

N = {x : x is a natural number}, or {x : x ∈ N} [Set-builder or Rule method]

[The symbol ' : ' stands for such that and the set {x : x ∈ N} is read as, "the set of x such that x is a natural number"].

It is clear from the example given above that:

(i) in description method a well-defined description about the set is given.

(ii) in roster or tabular method the elements of the set are written inside a pair of curly

braces and are separated by commas.
(iii) in set-builder or ruler method the actual elements of the set are not written, but a rule or a statement or a formula is written in the briefest possible way.
5. Cardinal Number : The cardinal number of a set is the number of elements in it.
Thus, if a set A has 5 elements ; its cardinal number is 5 and we represent it by writing n (A) = 5 .
Similarly, if set B = Set of even natural numbers less than 10 then, B = {2, 4, 6, 8} and n (B) = 4.
If B = {0}, then n (B) = 1. Since, 0 is an element of set B.
6. Types of Sets :
(i) Finite Set: A set is said to be a finite set, if it has a limited (countable) number of elements in it.
For example :
(a) S = Set of natural numbers between 10 and 15 = {11, 12, 13, 14}
(b) P = {0, 1,2, , 20} = {x : x ∈ W and x ≤ 20} and so on.
(ii) Infinite set: A set is said to be an infinite set, if it has an unlimited (uncountable) number of elements in it.
For example :
(a) P = Set of prime numbers = {2, 3, 5,…. }
(b) B = (x : x ∈ N and x ≥ 21} = (21, 22, 23, } ans so on.
(iii) Empty set or Null set: The set, with no element in it, is called the empty set or the null set.
The empty set is represented by a pair of braces with no element in it or by the Danish letter Φ, which is pronounced as 'oe'
Thus, the empty set = { } = Φ
Note : For empty set, it is wrong to call '**an empty set**' or '**a null set**' as there is one and only one empty set though it may have many descriptions.
Therefore, it is always called **"the empty set or the null set".**
Some examples of the empty set :
(a) Let A = {a man of age more than 400 years}.
Since there can not be any man with the age more than 400 years; the set A will have no element in it i.e. It is the empty set. And we write : A = { } or Φ
(b) If B = (Triangles with 4 sides} ; it is clear that B = Φ
Note :
1. Φ ≠{0}, since {0} is a set with 0 as its element whereas Φ has no element.
2. {Φ} ≠ {0}, since both the sets have different elements.
3. The cardinal number of the empty set is 0 i.e. n (Φ) = 0.
(iv) Disjoint sets : Sets having no element in common are called disjoint sets.
For example :
Sets P = (5, 7, 9} and O = (4, 6, 10, 12} are disjoint; as they do not have any element in common.
(v) Joint (overlapping) sets : Sets having atleast one element in common are called joint or overlapping sets.
For example:
Set B = (4,6, 8,10,12} and set C = (3,6,9,12,15} are joint sets; as they have elements 6 and 12 common.

(vi) Equal sets : Two sets are said to be equal, if the elements of both the sets are the same.
For example :
If set A = {x, y, z} and set B = {last three letters of English alphabet}.
Clearly, sets A and B have the same elements and so **set A = set B.**
(vii) Equivalent sets : Two sets are said to be equivalent, if they have equal number of elements in them, i.e. the cardinal numbers of both the sets are equal.
For example :
Let A = (3, 6, 9} and B = {a, b, c}.
Since, set A has 3 elements and set B also has 3 elements i.e., n (A) = n (B); therefore, sets A and B are equivalent and for this, we write : A ↔ B.
Note:
1. Equal sets are always equivalent; but the converse is not always true (i.e. it is not necessary that equivalent sets are equal also).
2. In equivalent sets, the number of elements (cardinal number) are equal, whereas in equal sets the element are the same.
3. Two infinite sets are always equivalent;

EXERCISE 13 (A)

Question 1.
Find, whether or not, each of the following collections represent a set:
(i) The collection of good students in your school.
(ii) The collection of the numbers between 30 and 45.
(iii) The collection of fat-people in your colony.
(iv) The collection of interesting books in your school library.
(v) The collection of books in the library and are of your interest.
Solution:
(i) It is not a set as it is not well defined.
(ii) It is a set
(iii) It is not a set as it is not well defined.
(iv) It is not a set as it is not well defined.
(v) It is a set.

Question 2.
State whether true or false :
(i) Set {4, 5, 8} is same as the set {5, 4, 8} and the set {8, 4, 5}
(ii) Sets {a, b, m, n} and {a, a, m, b, n, n) are same.
(iii) Set of letters in the word ‘suchismita’ is {s, u, c, h, i, m, t, a}
(iv) Set of letters in the word ‘MAHMOOD’ is {M, A, H, O, D}.
Solution:
(i) True
(ii) True
(iii) True as it has the same elements
(iv) True as it has the same elements.

Question 3.
Let set A = {6, 8, 10, 12} and set B = {3, 9, 15, 18}.
Insert the symbol ‘ ∈ ’ or ‘ ∉ ’ to make each of the following true :
(i) 6 …. A
(ii) 10 …. B
(iii) 18 …. B
(iv) (6 + 3) …. B
(v) (15 – 9) …. B
(vi) 12 …. A
(vii) (6 + 8) …. A
(viii) 6 and 8 …. A
Solution:
(i) 6 ∈ A
(ii) 10 ∉ B
(iii) 18 ∈B
(iv) (6 + 3) or 9 ∈ B
(v) 15 – 9 or 6 g B
(vi) 12 ∈ A
(vii) 6 + 8 or 14 ∉ A
(viii) 6 and 8 ∈ A

Question 4.
Express each of the following sets in roster form :
(i) Set of odd whole numbers between 15 and 27.
(ii) A = Set of letters in the word “CHITAMBARAM”
(iii) B = {All even numbers from 15 to 26}
(iv) P = {x : x is a vowel used in the word ‘ARITHMETIC’}
(v) S = {Squares of first eight whole numbers}
(vi) Set of all integers between 7 and 94; which are divisible by 6.
(vii) C = {All composite numbers between 2 and 20}
(viii) D = Set of Prime numbers from 2 to 23.
(ix) E = Set of natural numbers below 30 which are divisible by 2 or 5.
(x) F = Set of factors of 24.
(xi) G = Set of names of three closed figures in Geometry.
(xii) H = {x : x eW and x < 10}
(xiii) J = {x: x e N and 2x – 3 ≤17}
(xiv) K = {x : x is an integer and – 3 < x < 5}
Solution:
(i) {17, 19, 21, 23, 25}
(ii) A = (C, H, I, T, A, M, B, R}
(iii) B = {16, 18, 20, 22, 24, 26}
(iv) P = {a, e, i}
(v) S = {0, 1, 4, 9, 16, 25, 36, 49}
(vi) {12, 18, 24; 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90}
(vii) C = {4, 6, 8, 9, 10, 12, 14, 15, 16, 18}

(viii) D = {2, 3, 5, 7, 11, 13, 17, 19,23}
(ix) E = {2, 4, 5, 6, 8, 10, 12, 14, 15, 16, 18, 20, 22, 24, 25, 26, 28}
(x) F={l,2, 3, 4, 6, 8, 12, 24}
(xi) G = {Triangle, quadrilateral, circle}
(xii) H = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}
(xiii) $2x - 3 \leq 17$
$\Rightarrow 2x \leq 17 + 3$ $2x \leq 20$
$\Rightarrow x \leq \frac{20}{2}$
$x \leq 10$
$\therefore$ J = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
(xiv) $\because -3 < x < 5$
$\therefore$x lies between – 3 and 5
$\therefore$K = {- 2, – 1, 0, 1, 2, 3, 4}

Question 5.

Express each of the following sets in set- builder notation (form) :
(i) {3, 6, 9, 12, 15}
(ii) {2, 3, 5, 7, 11, 13 …. }
(iii) {1, 4, 9, 16, 25, 36}
(iv) {0, 2, 4, 6, 8, 10, 12, …. }
(v) {Monday, Tuesday, Wednesday}
(vi) {23, 25, 27, 29, … }
(vii) $\{\frac{1}{3},\frac{1}{4},\frac{1}{5},\frac{1}{6},\frac{1}{7},\frac{1}{8}\}$
(viii) {42, 49, 56, 63, 70, 77}
Solution:
(i) {3, 6, 9, 12, 15}
= {x: x is a natural number divisible by 3 ;x< 18}
(ii) {2, 3, 5, 7, 11, 13, }
= {x : x is a prime number}
(iii) {1,4,9, 16,25,36}
= {x : x is a perfect square ; x < 36}
(iv) {0, 2, 4, 6, 8, 10, 12, }
= {x : x is a whole number divisible by 2}
(v) {Monday, Tuesday, Wednesday}
= {x : x is one of the first three days of 3 week}
(vi) {23, 25, 27, 29, }
= {x : x is an odd natural number; x ≥ 23}
(vii) $\{\frac{1}{3},\frac{1}{4},\frac{1}{5},\frac{1}{6},\frac{1}{7},\frac{1}{8}\}$
= {x: x = $\frac{1}{n}$ when n is a natural number: $3 \leq n \leq 8$}
(viii) {42, 49, 56, 63, 70, 77}
= (x: x is a natural number divisible by 7 ; $42 \leq x \leq 77$}

Question 6.
Given : A = {x : x is a multiple of 2 and is less than 25}
B = {x : x is a square of a natural number and is less than 25}
C = {x : x is a multiple of 3 and is less than 25}
D = {x: x is a prime number less than 25}
Write the sets A, B, C and D in roster form.
Solution:
A = {x: x is a multiple of 2 and is less than 25}
= {2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24}
B = {x : x is a square of natural number and is less than 25}
= {1,4,9,16}
C = {x : x is a multiple of 3 and is less than 25}
= {3, 6, 9, 12, 15, 18,21,24}
D = {x : x is a prime number less than 25}
= {2, 3, 5, 7, 11, 13, 17, 19, 23}

EXERCISE 13 (B)

Question 1.
Write the cardinal number of each of the following sets:
(i) A = Set of days in a leap year.
(ii) B = Set of numbers on a clopk-face.
(iii) C = {x : x ∈ N and x ≤ 7}
(iv) D = Set of letters in the word “PANIPAT”.
(v) E = Set of prime numbers between 5 and 15.
(vi) F = {x : x ∈ Z and – 2 < x ≤ 5}
(vii) G = {x : x is a perfect square number, x ∈N and x ≤ 30}.
Solution:
(i) n A = 366
(ii) n B = 12
(iii) n C =7
(iv) n D = 5
(v) n E = 3
(vi) n F =7
(vii) n G = 5

Question 2.
For each set, given below, state whether it is finite set, infinite set or the null set :
(i) {natural numbers more than 100}
(ii) A = {x : x is an integer between 1 and 2}
(iii) B = {x : x ∈ W ; x is less than 100}.
(iv) Set of mountains in the world.
(v) {multiples of 8}.
(vi) {even numbers not divisible by 2}.
(vii) {squares of natural numbers}.

(viii) {coins used in India}
(ix) C = {x | x is a prime number between 7 and 10}.
(x) Planets of the Solar system.
Solution:
(i) {Natural numbers more than 100}
= It is an infinite set
(ii) A = {x : x is an integer between 1 and 2}
It is a null set
(iii) B = {x : x ∈ W, x is less than 100}
It is finite set as it has 100 elements i.e. from 0 to 99.
(iv) Set of mountains in the world.
∴ It is an infinite set
(v) {Multiples of 8}
It is an infinite set
(vi) {Even numbers not divisible by 2}
It is a null set
(vii) {Squares of natural numbers}
∴ It is an infinite set
(viii) {Coins used in India}
∴It is a finite set as these are countable
(ix) {x | x is a prime number between 7 and 10}
As there is not such prime number between 7 and 10.
Hence it is null set
(x) Planets of two Solar system.
It is finite set as there are countable.

Question 3.
State, which of the following pairs of sets are disjoint :
(i) {0, 1, 2, 6, 8} and {odd numbers less than 10.
(ii) {birds} and {tress}
(iii) {x : x is a fan of cricket} and {x : x is a fan of football}.
(iv) A = {natural numbers less than 10} and B = {x : x is a multiple of 5}.
(v) {people living in Calcutta} and {people living in West Bengal}.
Solution:
(i) {0, 1, 2, 6, 8} and {odd numbers less than 10}
⇒ {0, 1,2, 6, 8} and {1,3, 5, 7, 9}
∴There sets are not disjoint sets as there is one element (1) is common.
(ii) {Birds} and {trees}
These are disjoint sets as there is no common element in term
(iii) {x : x is a fan of cricket} and {x : x is a fan of football}
These are not disjoint sets as there can be a person who is fan of both the games.
(iv) A = {Natural numbers less than 10} and B = {x : x is a multiple of 5}
⇒ A = {1, 2, 3, 4, 5, 6, 7, 8, 9} and B = {5, 10, 15 }
These are hot disjoint sets as there is one element 5, which is common.
(v) {People living in Calcutta} and {People living in West Bengal}.
These are not disjoint sets as people of Calcutta are the people of West Bengal as

Calcutta is a city of West Bengal.
So, only (ii) is a pair of disjoint sets.

Question 4.
State whether the given pairs of sets are equal or equivalent.
(i) A = {first four natural numbers} and B = {first four whole numbers}.
(ii) A = Set of letters of the word "FOLLOW" and B = Set of letters of the word "WOLF".
(iii) E = {even natural numbers less than 10} and O = {odd natural numbers less than 9}
(iv) A = {days of the week starting with letter S} and B = {days of the week starting with letter T}.
(v) M = {multiples of 2 and 3 between 10 and 20} and N = {multiples of 2 and 5 between 10 and 20}.
(vi) P = {prime numbers which divide 70 exactly} and Q = {prime numbers which divide 105 exactly}
(vii) A = $\{0^2, 1^2, 2^2, 3^2, 4^2\}$ and = {16, 9,4, 1, 0}.
(viii) E = {8,JO, 12, 14, 16} and F = {even natural numbers between 6 and 18}.
(ix) A = {letters of the word SUPERSTITION} and B = {letters of the word JURISDICTION}.
Solution:
(i) A = {first four natural numbers}
= {1,2, 3, 4}
B = {first first whole number)
= {0, 1,2,3}
These are equivalent sets as both have equal number of elements but not same.
(ii) A = Set of letters of the word 'FOLLOW'
= {F, O, L, W}
B = Set of letters of the word 'WOLF'
= {W, O, L, F}
These are equal sets as these have same and equal elements.
(iii) E = {even natural numbers less than 10}
= {2, 4, 6, 8}
O = {odd natural numbers less than 9}
= {L3, 5, 7}
These are equivalent sets as both have equal number of elements but not the same.
(iv) A = {Days of the week starting with letter S}
= {Sunday, Saturday}
B = {Days of the week starting with letter T}
= {Tuesday, Thursday}
These are equivalent sets as both have equal number of elements.
(v) M = {Multiples of 2 and 3 between 10 and 20}
= {12, 14, 15, 16, 18}
N = {Multiples of 2 and 5 between 10 and 20}
= {12, 14, 15, 16, 18}
These are equal sets as these have same and equal number of elements.

(vi) P = {Prime numbers which divide 70 exactly}
= {2, 5, 7}
Q = {Prime numbers which divide 105 exactly}
= {3, 5, 7}
These are equivalent sets as these have equal number of elements.
(vii) A = {02, l2, 22, 32, 42} = {0, 1, 4, 9, 16} B = {16, 9, 4, 1,0}
These are equal sets as these have same and equal number of elements.
(viii) E = {8, 10, 12, 14, 16}
F = {even natural numbers between 6 and 18}
= {8, 10, 12, 14, 16}
These sets are equal as these have same and equal number of elements
(ix) A = {Letters of the word SUPERSTITION}
= {S, U, P, E, R, T, I, O, N}
B = Letters of the word JURISDICTION.
= (J, U, R, I, S, D, C, T, O, N}
These are neither equal nor equivalent sets as these have different and unequal elements.

Question 5.
Examine which of the following sets are the empty sets :
(i) The set of triangles having three equal sides.
(ii) The set of lions in your class.

(iii) {x + 3 = 2 and xeN}
(iv) P = {x : 3x = 0}
Solution:
(i) The set of triangle having three equal sides. This is not an empty set
(ii) The set of lions in your class This is an empty set
(iii) {x : x +3 = 2 and x ∈N}
x ≠ 3 = 2 ⇒ x = 2-3= -l
which is not a natural number.
∴ It is an empty set.
(iv) P = {x : 3x = 0} = {0} which is not an empty set.
Hence (ii) and (iii) are empty sets.

Question 6.
State true or false :
(i) All examples of the empty set are equal.
(ii) All examples of the empty set are equivalent.
(iii) If two sets have the same cardinal number, they are equal sets.
(iv) If n (A) = n (B) then A and B are equivalent sets.
(v) If B = {x : x + 4 = 4}, then B is the empty set.
(vi) The set of all points in a line is a finite set.
(vii) The set of letters in your Mathematics book is an infinite set.
(viii) If M = {1, 2, 4, 6} and N = {x : x is a factor of 12} ; then M = N.
(ix) The set of whole numbers greater than 50 is an infinite set.

(x) If A and B are two different infinite sets, then n (A) = n (B).
Solution:
(i) True
(ii) True
(iii) False
(iv) True
(v) False
(vi) False
(vii) False
(viii) False
(ix) True
(x) False

Question 7.
Which of the following represent the null set ?
φ, {0}, 0, { }, {φ}
Solution:
φ and { } are the null sets other are not as there have same element.

EXERCISE 13 (C)

Question 1.
Fill in the blanks :
(i) If each element of set P is also an element of set Q, then P is said to be …… of Q and Q is said to be of P.
(ii) Every set is a ….. of itself.
(iii) The empty set is a …… of every set.
(iv) If A is proper subset of B, then n (A) …. n (B).
Solution:
(i) If each element of set P is also an element of set Q then P is said to be subject of Q ; and Q is said to be super set of P.
(ii) Every set is a subset of itself.
(iii) The empty set is subset of every set.
(iv) If A is proper subset of B, then n (A) is less than n (B)

Question 2.
If A = {5, 7, 8, 9} ; then which of the following are subsets of A ?
(i) B = {5, 8}
(ii) C = {0}
(iii) D = {7, 9, 10}
(iv) E = { }
(V) F = {8, 7, 9, 5}
Solution:
(i) B = {5, 8,}
∴B ⊂ A

(ii) C = {0}
∴ C φ A
(iii) D = {7, 9, 10}
∴D ⊄ A
(iv) E = { }
∴E ⊂A (An empty set is subset of every set)
(v) F = (8, 7, 9, 5}
∴F ⊂A
∵ Every set is subset of it self.
Hence (i), (iv)
and (v) are subsets of A.

Question 3.
If P = {2, 3, 4, 5} ; then which of the following are proper subsets of P ?
(i) A = {3, 4}
(ii) B = { }
(iii) C = {23, 45}
(iv) D = {6, 5, 4}
(v) E = {0}
Solution:
P = {2, 3, 4, 5}
(i) A = {3,4},
(ii) B = { }, C = {23, 45},
D = {6, 5, 4} and E = {0}.
We see that only A and B are the proper subset of P.

Question 4.
If A = {even numbers less than 12},
B = {2, 4},
C = {1, 2, 3},
D = {2, 6} and E = {4}
State which of the following statements are true :
(i) B⊂A
(ii) C⊆A
(iii) D⊂C
(iv) D ⊄ A
(v)E⊇B
(vi) A⊇B⊇E
Solution:
A = {Even number less than 12} = {2, 4, 6, 8, 10}
B = {2, 4}, C = {1, 2, 3},
D = {2, 6} and E = {4}
(i) B ⊂ A: It is true
(ii) C ⊆ A: It is false
(iii) D ⊂ C : It is false

(iv) D ⊄ A
(v) E ⊇ B : It is false
(vi) A ⊇ B ⊇ E : It is true

Question 5.
Given A = {a, c}, B = {p, q, r} and C = Set of digits used to form number 1351. Write all the subsets of sets A, B and C.
Solution:
(i) A = {a, c}
∴ Subsets are : { } or φ, {a}, {c} and {a, c}
(ii) B = {p,q, r)
∴ subsets are : { } or φ, {p}, {q}, {r}, {p, q}, ip, r}, {q, r} and {p, q, r}
(iii) C = Set of digits used in 135, = {1,3,5}
∴ Subsets are = { }
or φ, {1}, {3}, {5}, {1,3}, {1,5}, {2,5} and {1, 3, 5}

Question 6.
(i) If A = {p, q, r}, then number of subsets of A = ……
(ii) If B = {5, 4, 6, 8}, then number of proper subsets of B = ……
(iii) If C = {0}, then number of subsets of C = …..
(iv) If M = {x : x ∈ N and x < 3}, then M has …… proper subsets.
Solution:
(i) If A = {p, q, r},
then number of subsets of A = 2^3 = 2×2×2 = 8
(ii) If B = {5, 4, 6, 8},
then number of proper subsets of B = 2^4 – 1 = 2 × 2 × 2× 2 – 1 = 16 – 1 = 15
(iii) If C = {0},
then number of subsets of C = 2^1 = 2
(iv) If M = {x: x ∈ N and x < 3}, = {1, 2}
Then M has proper subsets = 2^2—1 = 4 — 1 = 3

Question 7.
For the universal set {4, 5, 6, 7, 8, 9, 10, 11,12,13} ; find its subsets A, B, C and D such that
(i) A = {even numbers}
(ii) B = {odd numbers greater than 8}
(iii) C = {prime numbers}
(iv) D = {even numbers less than 10}.
Also, find compliments of each set i.e., find A’, B’, C’ and D’.
Solution:
(i) A = {even numbers}
= {4, 6, 8, 10, 12}
(ii) B = {odd numbers greater then 8}
= {9, 11, 13}
(iii) C = {Prime numbers}

= {5, 7, 11, 13}
(iv) D = {even numbers less than 10}
= {4, 6, 8}
A’= {5, 7, 9, 11, 13},
B’ = {4, 5, 6, 7, 8, 10, 12}
C’ = { 4, 6, 8, 9, 10, 12}
and D’ = {5, 7, 9, 10, 11, 12, 13}

EXERCISE 13 (D)

Question 1.
If A = {4, 5, 6, 7, 8} and B = {6, 8, 10, 12}, find :
(i) A∪B
(ii) A∩B
(iii) A-B
(iv) B-A
Solution:
(i) A∪B
= [All the elements from set A and all the elements from set B]
= {4, 5, 6, 7, 8, 10, 12}
(ii) A∩B
= [elements common to both the sets A and B]
= {6, 8}
(iii) A-B
= [elements of set A which are not in set B]
= {4, 5, 7}
(iv) B-A
= [elements of set B which are not in set A]
= {10, 12}

Question 2.
If A = {3, 5, 7, 9, 11} and B = {4, 7, 10}, find:
(i) n(A)
(ii) n(B)
(iii) A∪B and n(A∪B)
(iv) A∩B and n(A∩B)
Solution:
(i) n(A) = (3, 5, 7, 9, 11) = 5
(ii) n(B) = (4, 7, 10) = 3
(iii) A ∪ B = {3, 4, 5, 7, 9, 10, 11} n(A ∪ B) = 7
(iv) A∩B = {6} n(A∩B)=l

Question 3.
If A = {2, 4, 6, 8} and B = {3, 6, 9, 12}, find:
(i) (A ∩ B) and n(A ∩ B)

(ii) (A – B) and n(A – B)
(iii) n(B)
Solution:
(i) (A ∩ B) = {2, 4, 8} n(A ∩ B) = 3
(ii) (A – B) and n(A – B)
⇒ (A – B) = (2, 4, 8)
⇒ n(A-B) = 3
(iii) n(B) = {3, 6, 9, 12} = 4

Question 4.
If P = {x : x is a factor of 12} and Q = {x: x is a factor of 16}, find :
(i) n(P)
(ii) n(Q)
(iii) Q – P and n(Q – P)
Solution:
(i) n(P) = Factors of 12 are
= 1, 2, 3, 4, 6, 12
∴ n(P) = 6
(ii) n(Q) = Factors of 16 are = 1. 2, 4, 8, 16
∴n(Q) = 5
(iii) Q – P and n(Q – P)
Elements of set P = {1, 2, 3, 4, 6, 12}
Elements of set Q = {1, 2, 4, 8, 16}
∴ Q – P = 8, 16
n(Q-P) = 2

Question 5.
M = {x : x is a natural number between 0 and 8) and N = {x : x is a natural number from 5 to 10}. Find :
(i) M – N and n(M – N)
(ii) N – M and n(N – M)
Solution:
Natural numbers between 0 and 8 M = {0, 1, 2, 3, 4, 5, 6, 7} and Natural numbers between 5 to 10 N = {6, 7, 8, 9, 10}
(i) M – N = {1, 2, 3, 4} and n(M – N) = 4
(ii) N – M = {8, 9, 10} and n (N – M) = 3

Question 6.
If A = {x: x is natural number divisible by 2 and x< 16} and B = {x:x is a whole number divisible by 3 and x < 18}, find :
(i) n(A)
(ii) n(B)
(iii) A∩B and n(A∩B)
(iv) n(A – B)
Solution:

(i) A = {x : x is natural number divisible-by 2 and x < 16}
A = {2, 4, 6, 8, 10, 12, 14}
n(A) = 7
(ii) B = {x: x is a whole number divisible by 3 and x < 18}
B = {3, 6, 9, 12, 15, 18}
n(B) = 6
(iii) A n B = {2, 4, 6, 8, 10, 12, 14} n {3, 6, 9, 12, 15, 18}
A∩B = {6,12} n(A ∩ B) = 2
(iv) A – B = {2, 4, 6, 8, 10, 12, 14} – {3, 6, 9, 12, 15, 18}
A-B = {2,4, 8, 10, 14} n(A – B) = 5

Question 7.
Let A and B be two sets such that n(A) = 75, M(B) = 65 and n(A ∩ B) = 45, find :
(i) n(A∪ B)
(ii) n(A – B)
(iii) n(B – A)
Solution:
n(A ∩ B)
n(A) = 75, n(B) = 65 and n(A ∩ B) = 45
(i) We know that,
n(A ∪B) = n(A) + n(B) – n(A ∩ B)
n(A ∪B) =75 + 65 – 45
n(A∪B) = 140-45 = 95
(ii) We know that,
n(A – B) = n(A) – n(A ∩ B)
n(A – B) = 75 – 45 = 30
(iii) We know that,
n(B – A) = n(B) – n(A ∩ B)
n(B – A) = 65 – 45 = 20

Question 8.
Let A and B be two sets such that n(A) = 45, n(B) = 38 and n(A ∪B) = 70, find :
(i) n(A∩B)
(ii) n(A-B)
(iii) n(B – A)
Solution:
n(A) = 45, n(B) = 38 and n(A∪ B) = 70
(i) We know that,
n(A ∩ B) = n(A) + M(B) – n(A ∪B)
n(A ∩ B) = 45 + 38 – 70 = 83 – 70 = 13
(ii) We know that,
n(A-B) = n(A ∪B)-n(B)
n(A – B) = 70 – 38 = 32
(iii) We know that,

$n(B – A) = n(A \cup B) – n(A)$
$n(B – A) = 70 – 45 = 25$

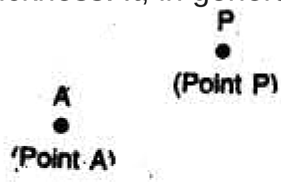
Question 9.
Let n(A) 30, n(B) = 27 and n(A∪B) = 45, find :
(i) n(A∩B)
(ii) n(A-B)
Solution:
$n(A) = 30$, $n(B) = 27$ and $n(A \cup B) = 45$
(i) We know that,
$n(A \cap B) = n(A) + n(B) – n(A\cup B)$
$n(A \cap B) = 30 + 27 – 45$
$n(A \cap B) = 57 – 45 = 12$
(ii) We know that,
$n(A-B) = n(A \cup B) – n(B)$
$n(A – B) = 45 – 27 = 18$

Question 10.
Let n(A) = 31, n(B) = 20 and n(A ∩ B) = 6, find:
(i) n(A-B)
(ii) n(B – A)
(iii) n(A ∪B)
Solution:
$n(A) = 31$, $n(B) = 20$ and $n(A \cap B) = 6$
(i) We know that,
$n(A – B) = n(A) – n(A \cap B)$
$n(A -B) = 31 -6 = 25$
(ii) We know that,
$n(B – A) = n(B) – n(A n B)$
$n(B – A) = 20 – 6 = 14$
(iii) We know that,
$n(A \cup B) = n(A) + n(B) – n(A \cup B)$ $n(A\cup B) = 31 +20-6 = 45$

CHAPTER - 14
LINES & ANGLES

POINTS TO REMEMBER

1. POINT : A point: is a mark of position; which has no length, no breadth and no thickness. It, in general, is represented by a capital letter as shown alongside.



2. LINE : A line has length, but no breadth or thickness.

"The given figure shows a line AB in which two arrow-heads in opposite directions show that can be extended infinitely in both the directions.
A line may be straight or curved but when we say a line' it means a straight line only.

(i)  **(ii)** 

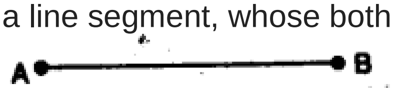
(A line or a straight line) **(Curved lines)**

(ii) Each line, whatever be its length, has an infinite number of points in it.

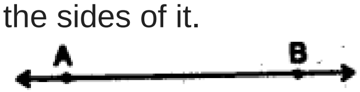
3. RAY : It is a straight line which stats from a fixed point and moves in the same direction.

The given figure shows a ray $\xrightarrow{AB}$ with fixed initial point A 'and moving in the direction AB.

4. LINE SEGMENT : It is a straight line with its both ends fixed. The given figure shows a line segment, whose both the ends A and B are fixed.



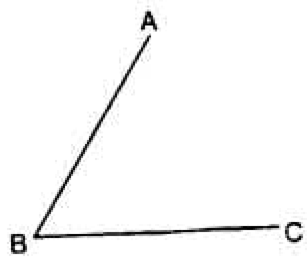
(i) The adjoining figure shows a line AB which can be extended upto infinitey on both the sides of it.



(ii) The adjoining figure shows a ray AB with fixed end as point A and which can be extended upto infinity through point B. It is clear from the figure, that a ray is a part of a line.

(iii) The adjoining figure shows a line-segment AB with fixed ends A and B. It is clear from the figure, that a line-segment is a part of a ray as well as of a line. Also, a line segment is the shortest distance between two fixed points.

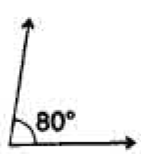
5. ANGLE : An angle is formed when two line segments or two rays have a common end-point.
The two line segments, forming an angle, are called the arms of the angle whereas their common end-point is called the vertex of the angle.

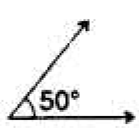


The adjacent figure represents an angle ABC or ∠ABC or simply ∠B. AB and BC are the arms of the angle and their common point B is the vertex.

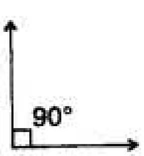
6. MEASUREMENT OF AN ANGLE : The unit of measuring an angle is degree. The symbol for degree is °.
Thus : 60 degree = 60°, 87 degree = 87’ and so on.
If one degree is divided into 60 equal parts, each part is called a minute (‘) and if one minute is further divided into 60 equal parts, each part is called a second (”).
Thus, **(i)** r = 60′ and l’ = 60″
(ii) 9 minutes 45 seconds = 9′ 45″
(iii) 85 degrees 30 minutes 15 seconds = 85° 30′ 15″ and so on.

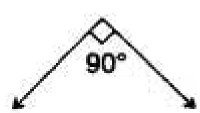
7. TYPES OF ANGLES :
1. Acute angle :
measures less than 90°

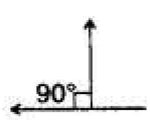




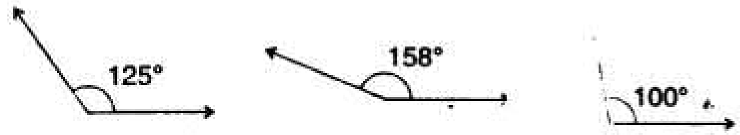
2. Right angle:
measures 90°



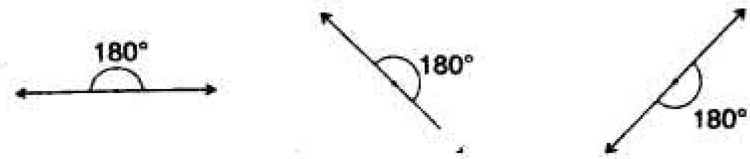




3. Obtuse angle :
measures between 90° and 180°

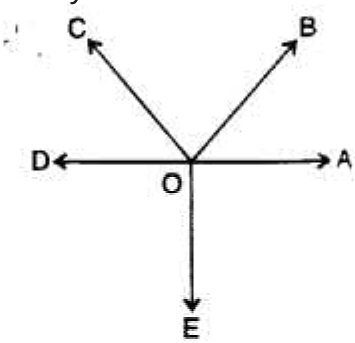


4. Straight angle :
measures 180°

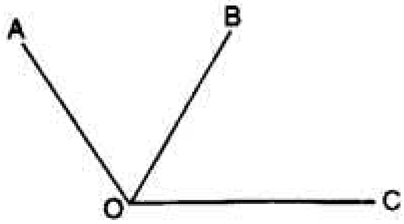


5. Reflex angle :
measures between 180° and 360°

8. MORE ABOUT ANGLES :
(A) Angles about a point: If a number of angles are formed about a point, their sum is always 360°.



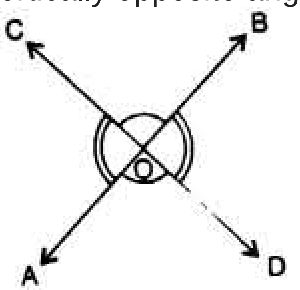
In the adjoining figure :
∠AOB + ∠BOC + ∠COD +∠DOE + ∠EOA = 360°
(B) Adjacent angles : Two angles are said to be adjacent angles, if:
(i) they have a common vertex,
(ii) they have a common arm and
(iii) the other arms of the two angles lie on opposite sides of the common arm.
The adjoining figure shows a pair of adjacent angles :
(i) they have a common vertex (O),
(ii) they have a common ann (OB) and



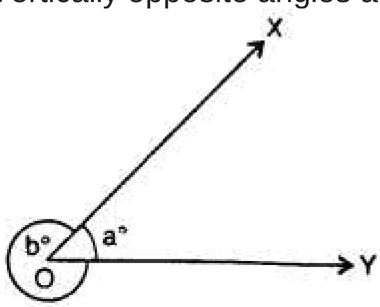
(iii) the other arms OA and OC of the two angles are on opposite sides of the common arm OB.

(C) Vertically opposite angles : When two straight lines intersect each other four angles are formed.

The pair of angles which lie on the opposite sides of the point of intersection are called vertically opposite angles.



In the adjoining figure, two straight lines AB and CD intersect each other at point 0.
Angles AOD and BOC form one pair of vertically opposite angles; whereas angles AOC and BOD form another pair of vertically opposite angles.
Vertically opposite angles are always equal.



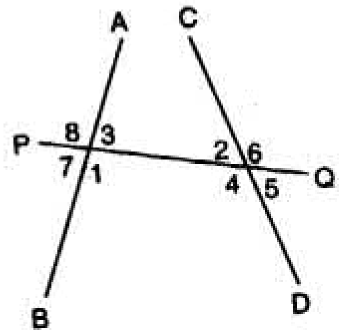
i. e. $\angle AOD = \angle BOC$ and $\angle AOC = \angle BOD$.
Important: In the adjoining figure, rays OX and OY meet a O to form $\angle XOY$ (i.e. $\angle a$) and reflex $\angle XOY$ (i. e. $\angle b$). It must be noted that $\angle XOY$ means the smaller angle only unless it is mentioned to take otherwise.

9. COMPLEMENTARY AND SUPPLEMENTARY ANGLES

1. Two angles are called **complementary angles,** if their sum is one'right angle i.e. 90° Each angle is called the **complement** of the other.
e.g., 20″ and 70″ are complementary angles, because 20° + 70° = 90°.
Clearly, 20″ is the complement of 70° and 70° is the complement of 20°.
Thus, the complement of angle 53° = 90° – 53° = 37°.
2. Two angles are called **supplementary angles**, if their sum is two right angles i.e. 180″. Each angle is called the **supplement of the other.**

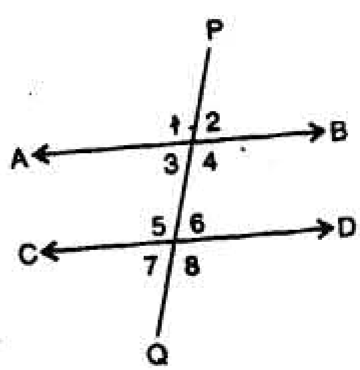
e.g., 30″ and 150° are supplementary angles because 30° + 150° = 180°.
Clearly, 30″ is the supplement of 150° and vice-versa.
Thus, the supplement of 105° = 180° – 105° = 75°.

10. Transversal : It is a straight line which cuts two or more given straight lines.



In the adjoining figure, PQ cuts straight lines AB and CD, and so it is a transversal.
When a transversal cuts two given straight lines (refer the adjoining figure), the following pairs of angles are formed.
1. Two pairs of interior alternate angles : Angles marked 1 and 2 form one pair of interior alternate angles, while angles marked 3 and 4 form another pair of interior alternate angles.
2. Two pairs of exterior alternate angles : Angles marked 5 and 8 form one pair, while angles marked 6 and 7 form the other pair of exterior alternate angles.
3. Four pairs of corresponding angles : Angles marked 3 and 6; 1 and 5; 8 and 2; 7 and 4 form the four pairs of corresponding angles.
4. Two pairs of allied or co-interior or conjoined angles : Angles marked 3 and 2 form one pair and angles marked 1 and 4 form another pair of allied angles.

11. PARALLEL LINES : Two straight lines are said to be parallel, if , they do not meet anywhere; no matter how long are they produced in any direction.



The adjacent figure shows two parallel straight lines AB and CD.
When two parallel lines AB and CD are cut by a transversal PQ :
(i) Interior and exterior alternate angles are equal:
i.e. ∠3 = ∠6 and ∠4 = ∠5 [Interior alternate angles]
∠1 = ∠8 and ∠2 = ∠7 [Exterior alternate angles]
(ii) Corresponding angles are equal:

i.e. $\angle 1 = \angle 5; \angle 2 = \angle 6; \angle 3 = \angle 7$ and $\angle 4 = \angle 8$
(iii) Co-interior or allied angles are supplementary :
i. e. $\angle 3 + \angle 5 = 180°$ and $\angle 4 + \angle 6 = 180°$

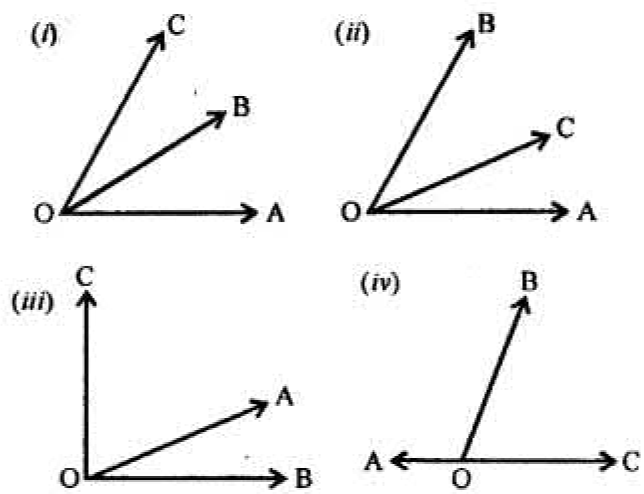
12. CONDITIONS OF PARALLELISM : If two straight lines are cut by a transversal such that:
(i) a pair of alternate angles are equal, or
(ii) a pair of corresponding angles are equal, or
(iii) the sum of the interior angles on the same side of the transversal is 180°, then the two straight lines are parallel to each other.
Therefore, in order to prove that the given lines are parallel, show either alternate angles are equal or, corresponding angles are equal or, the co-interior angles are supplementary.

EXERCISE 14 (A)

Question 1.
State, true or false :
(i) A line segment 4 cm long can have only 2000 points in it.
(ii) A ray has one end point and a line segment has two end-points.
(iii) A line segment is the shortest distance between any two given points.
(iv) An infinite number of straight lines can be drawn through a given point.
(v) Write the number of end points in
(a) a line segment AB (b) arayAB
(c) alineAB
(vi) Out of $\overleftrightarrow{AB}$, $\overrightarrow{AB}$, $\overleftarrow{AB}$ and $\overline{AB}$, which one has a fixed length?
(vii) How many rays can be drawn through a fixed point O?
(viii) How many lines can be drawn through three
(a) collinear points?
(b) non-collinear points?
(ix) Is 40° the complement of 60°?
(x) Is 45° the supplement of 45°?
Solution:
(i) False : It has infinite number of points.
(ii) True
(iii) True
(iv) True
(v) (a) 2 (b) 1 (c) 0
(vi) AB
(vii) Infinite
(viii) (a) 1 (b) 3
(ix) False : 40° is the complement of 50° as 40° + 50° = 90°
(x) False : 45° is the supplement of 135° not 45°.

Question 2.

In which of the following figures, are ∠AOB and ∠AOC adjacent angles? Give, in each case, reason for your answer.



Solution:

If ∠AOB and ∠AOC are adjacent angle if they have OA their common arm.

(i) In the figure, OB is their common arm

∴∠AOB and ∠AOC are not adjacent angles.

(ii) In the figure, OC is their common arm

∴∠AOB and ∠AOC also not adjacent angles.

(iii) In this figure, OA is their common arm

∴ ∠AOB and ∠AOC are adjacent angles.

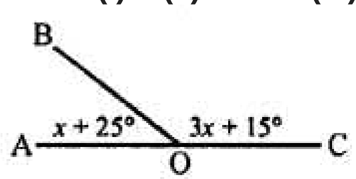
(iv) In this figure, OB is their common arm

∴ ∠AOB and ∠AOC are not adjacent angles.

Question 3.

In the given figure, B AC is a straight line.

Find : (i) x (ii) ∠AOB (iii) ∠BOC



Solution:

$\because \angle AOB$ and $\angle COB$ are linear pairs

$\therefore \angle AOB + \angle COB = 180°$

$\Rightarrow x + 25° + 3x + 15° = 180°$

$\Rightarrow 4x + 40° = 180°$

$\Rightarrow 4x = 180° - 40° = 140°$

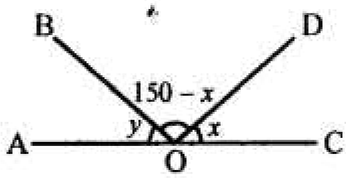
(i) $\Rightarrow x = \dfrac{140°}{4} = 35°$

Hence, $x = 35°$

(ii) $\angle AOB = x + 25° = 35° + 25° = 60°$

(iii) $\angle BOC = 3x + 15° = 3 \times 35° + 15°$
$= 105° + 15° = 120°$

Question 4.
Find yin the given figure.



Solution:

$\because$ AOC is a straight line

$\therefore \angle AOB + \angle BOD + \angle DOC = 180°$

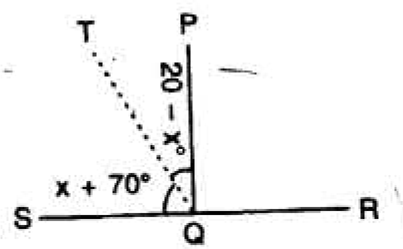
$\Rightarrow y + 150° - x + x = 180°$

$\Rightarrow y + 150° = 180°$

$\Rightarrow y = 180° - 150° = 30°$

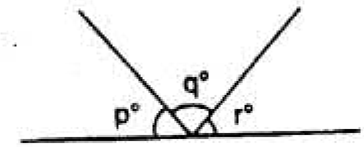
Hence, $y = 30°$

Question 5.
In the given figure, find ∠PQR.



Solution:
SQR is a straight line
∴∠SQT + ∠TQP + ∠PQR = 180°
⇒ x + 70° + 20° – x + ∠PQR = 180°
⇒ 90″ + ∠PQR = 180°
⇒ ∠PQR = 180°-90° = 90°
Hence ∠PQR = 90°

Question 6.
In the given figure. p° = q° = r°, find each.



Solution:

$p^\circ + q^\circ + r^\circ = 180^\circ$ (straight angle)

But $p^\circ = q^\circ = r^\circ$ (given)

$\therefore p^\circ + p^\circ + p^\circ = 180^\circ$

$\Rightarrow 3p^\circ = 180^\circ \quad \Rightarrow p^\circ = \dfrac{180^\circ}{3} = 60^\circ$

Hence $p^\circ = q^\circ = r^2 = 60^\circ$

Question 7.
In the given figure, if x = 2y, find x and y

Solution:

$x^\circ + y^\circ = 180^\circ$ (straight angle)

But $x = 2y$ (given)

$\therefore 2y + y = 180^\circ$

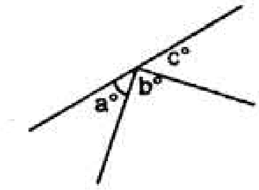
$\Rightarrow 3y = 180^\circ$

$\Rightarrow y = \dfrac{180^\circ}{3} = 60^\circ$

Hence $y = 60^\circ$

and $x = 2y = 2 \times 60^\circ = 120^\circ$

Question 8.
In the adjoining figure, if b° = a° + c°, find b.



Solution:

$a^\circ + b^\circ + c^\circ = 180^\circ$ (straight angle)

But $b^\circ = a^\circ + c^\circ$ (given)

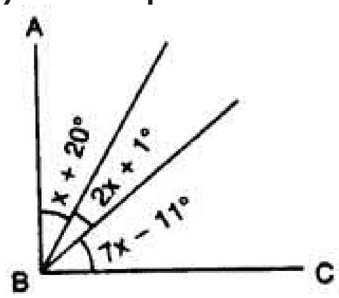
$\therefore a^\circ + c^\circ + b^\circ = 180^\circ$

$\Rightarrow b^\circ + b^\circ = 180^\circ \Rightarrow 2b^\circ = 180^\circ$

$$\Rightarrow b^\circ = \frac{180^\circ}{2} = 90^\circ$$

Hence $b^\circ = 90^\circ$

Question 9.
In the given figure, AB is perpendicular to BC at B.
Find : (i) the value of x.
(ii) the complement of angle x.



Solution:

(*i*) In the given figure,

AB + BC at B.

$\therefore \angle ABC = 90^\circ$

$\Rightarrow x + 20^\circ + 2x + 1^\circ + 7x - 11^\circ = 90^\circ$

$\Rightarrow 10x + 10^\circ = 90^\circ$

$\Rightarrow 10x = 90^\circ - 10^\circ = 80^\circ$

$$\Rightarrow x = \frac{80^\circ}{10} = 8^\circ$$

Hence $x = 8^\circ$

(*ii*) Complement of angle $x = 90^\circ - x$

$= 90^\circ - 8^\circ = 82^\circ$

Question 10.
Write the complement of:

(*i*) $25°$ (*ii*) $90°$ (*iii*) $a°$
(*iv*) $(x + 5)°$ (*v*) $(30 - a)°$

(*vi*) $\frac{1}{2}$ of a right angle

(*vii*) $\frac{1}{3}$ of $180°$ (*viii*) $21°\ 17'$

Solution:

(*i*) Complement of $25° = 90° - 25° = 65°$

(*ii*) Complement of $90° = 90° - 90° = 0°$

(*iii*) Complement of $a° = 90° - a°$

(*iv*) Complement of $(x + 5)° = 90° - (x + 5°)$
$= 90° - x - 5° = 85° - x$

(*v*) Complement of $(30 - a)°$
$= 90° - (30 - a)°$
$= 90° - 30° + a° = 60° + a°$

(*vi*) Complement of $\frac{1}{2}$ of a right angle

$= 90° - \frac{1}{2}$ right angle

$= 90° - \frac{1}{2} \times 90° = 90° - 45° = 45°$

(*vii*) Complement of $\frac{1}{3}$ of $180°$

$= 90° - \frac{1}{3}$ of $180° = 90° - 60° = 30°$

(*viii*) Complement of $21°\ 17' = 90° - 21°\ 17'$
$= 68°\ 43' = 68°\ 43'$ $(\because P = 60')$

Question 11.
Write the supplement of:
(i) 100°
(ii) 0°
(iii) x°
(iv) (x + 35)°
(v) (90 +a + b)° f
(vi) (110 – x – 2y)°
(vii) $\frac{1}{5}$ of a right angle
(viii) 80° 49′ 25″
Solution:

(i) Supplement of $100^\circ = 180^\circ - 100^\circ = 80^\circ$

(ii) Supplement of $0^\circ = 180^\circ - 0^\circ = 180^\circ$

(iii) Supplement of $x^\circ = 180^\circ - x^\circ$

(iv) Supplement of $(x + 35)^\circ$

$= 180^\circ - (x + 35)^\circ$

$= 180^\circ - x^\circ - 35^\circ = 145^\circ - x^\circ$

(v) Supplement of $(90 + a + b)^\circ$

$= 180^\circ - (90 + a + b)^\circ$

$= 180^\circ - 90^\circ - a^\circ - b^\circ$

$= 90^\circ - a^\circ - b^\circ$

$= (90 - a - b)^\circ$

(vi) Supplement of $(110 - x - 2y)^\circ$

$= 180^\circ - (110 - x - 2y)^\circ$

$= 180^\circ - 110^\circ + x^\circ + 2y^\circ = 70^\circ + x^\circ + 2y^\circ$

(vii) Supplement of $\frac{1}{5}$ of a right angle

$= 180^\circ - \frac{1}{5}$ of a right angle

$= 180^\circ - \frac{1}{5} \times 90^\circ$

$= 180^\circ - 18^\circ = 162^\circ$

(viii) Supplement of $80^\circ\ 49'\ 25''$

$= 180^\circ - 80^\circ\ 49'\ 25''$

$= 99^\circ\ 10'\ 35''$ **Ans.** $\left\{\begin{matrix} \because 1^\circ = 60' \\ 1' = 60'' \end{matrix}\right\}$

Question 12.
Are the following pairs of angles complementary ?
(i) 10° and 80°
(ii) 37° 28′ and 52° 33′
(iii) (x+ 16)°and(74-x)°
(iv) 54° and $\frac{2}{5}$ of a right angle.
Solution:

(i) 10° and 80° : Yes, these are complementary angles as their sum

$$= 10^\circ + 80^\circ = 90^\circ$$

(ii) 37° 28′ and 52° 33′ : No, these are not complementary angles as their sum is not 90° ($37^\circ 28' + 52^\circ 33' = 90^\circ 1'$)

(iii) $(x + 16)^\circ$ and $(74 - x)^\circ$: Yes these are complementary angles as their sum is 90° ($x^\circ + 16^\circ + 74^\circ - x^\circ = 90^\circ$)

(iv) 54° and $\frac{2}{5}$ of a right angle

$\Rightarrow 54^\circ$ and $\frac{2}{5} \times 90^\circ \quad \Rightarrow 54^\circ$ and 36° :

Yes, there are complementary angles as their sum is 90° ($54^\circ + 36^\circ = 90^\circ$)

Question 13.
Are the following pairs of angles supplementary?

(i) 139° and 39°.

(ii) 26° 59′ and 153° 1′.

(iii) $\frac{3}{10}$ of a right angle and $\frac{4}{15}$ of two right angles.

(iv) $2x^\circ + 65^\circ$ and $115^\circ - 2x^\circ$.

Solution:

(i) 139° and 39° : No, these are not supplementary angles as their sum is not 180° $(139^\circ + 39^\circ = 178^\circ)$

(ii) $26^\circ\ 59'$ and $153^\circ\ 1'$: Yes, there are supplementary angles as their sum is 180° $(26^\circ\ 59' + 153^\circ\ 1' = 180^\circ)$

(iii) $\frac{3}{10}$ of a right angle and $\frac{4}{15}$ of two right angles

$\Rightarrow \frac{3}{10}$ of 90° and $\frac{4}{15}$ of 180°

$\Rightarrow 27^\circ$ and 48° : No, there are not supplementary angles as their sum is not 180° $(27^\circ + 48^\circ = 75^\circ)$

(iv) $2x^\circ + 65^\circ$ and $115^\circ - 2x^\circ$: Yes there are supplementary angles as their sum is 180° $(2x^\circ + 65^\circ + 115^\circ - 2x^\circ = 65^\circ + 115^\circ = 180^\circ)$

Question 14.

If 3x + 18° and 2x + 25° are supplementary, find the value of x.

Solution:

$\because 3x + 18^\circ$ and $2x + 25^\circ$ are supplementary angles

$\therefore 3x + 18^\circ + 2x + 25^\circ = 180^\circ$

$\Rightarrow 5x + 43^\circ = 180^\circ$

$\Rightarrow \quad 5x = 180^\circ - 43^\circ = 137^\circ$

$\Rightarrow \quad x = \frac{137}{5} = 27{\cdot}4^\circ$ or $27^\circ\ 24'$

Question 15.
If two complementary angles are in the ratio 1:5, find them.
Solution:

Two complementary angles are in the ratio $= 1 : 5$

Let these angles be x and $5x$

$\therefore x + 5x = 90^\circ \quad \Rightarrow 6x = 90^\circ$

$\Rightarrow x = \frac{90^\circ}{6} = 15^\circ$

$\therefore$ Angles will be 15° and $15^\circ \times 5 = 75^\circ$

Question 16.
If two supplementary' angles are in the ratio 2 : 7, find them.
Solution:

Ratio between two supplementary angle $= 2 : 7$

Let the angles be $2x$ and $7x$

$\therefore 2x + 7x = 180^\circ$

$\Rightarrow 9x = 180^\circ \quad \Rightarrow x = \frac{180^\circ}{9} = 20^\circ$

$\therefore$ Angles are $2x = 2 \times 20^\circ = 40^\circ$

and $7x = 7 \times 20^\circ = 140^\circ$

Question 17.
Three angles which add upto 180° are in the ratio 2:3:7. Find them.
Solution:

Ratio of three angles $= 2 : 3 : 7$

Let first angle be $= 2x$

second angle $= 3x$

and third angle $= 7x$

$\therefore 2x + 3x + 7x = 180^\circ$

$\Rightarrow 12x = 180^\circ \quad \Rightarrow x = \frac{180^\circ}{12} = 15^\circ$

$\therefore$ First angle $= 2x = 2 \times 15^\circ = 30^\circ$

second angle $= 3x = 3 \times 15^\circ = 45^\circ$

and third angle $= 7x = 7 \times 15^\circ = 105^\circ$

Hence, angles are 30°, 45° and 105°

Question 18.
20% of an angle is the supplement of 60°. Find the angle.
Solution:

Let the given angle be x

then 20% of $x + 60^\circ = 180^\circ$

$\Rightarrow$ 20% of $x = 180^\circ - 60^\circ = 120^\circ$

$$\Rightarrow \frac{20}{100} \times x = 120^\circ \quad \Rightarrow x \frac{120 \times 100}{20}$$

$$\Rightarrow \qquad x = 600^\circ$$

Hence $x = 600^\circ$

Question 19.
10% of x° is the complement of 40% of 2x°. Find x
Solution:

$\because$ 10% of x° is the complement of 40% of $2x^\circ$

$\therefore$ 10% of x° + 40% of $2x = 90^\circ$

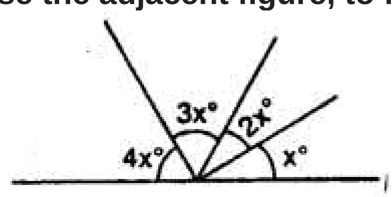
$$\Rightarrow \frac{10}{100} x + \frac{40}{100} \times 2x = 90^\circ$$

$$\Rightarrow \frac{10x}{100} + \frac{80x}{100} = 90^\circ \quad \Rightarrow \frac{90x}{100} = 90^\circ$$

$$\therefore \qquad x = \frac{90 \times 100}{90} = 100^\circ$$

Hence $x = 100^\circ$

Question 20.
Use the adjacent figure, to find angle x and its supplement.



Solution:

In the given fig.

$x + 2x + 3x + 4x = 180^\circ$ (Straight angle)

$\Rightarrow 10x = 180^\circ$

$\Rightarrow x = \frac{180^\circ}{10} = 18^\circ$

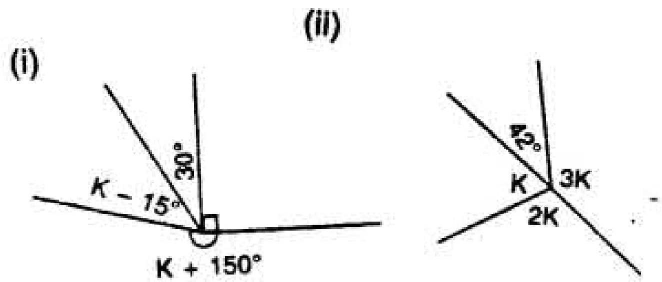
$\therefore x = 18^\circ$

Its supplementary angle $= 180^\circ - 18^\circ$

$= 162^\circ$

Question 21.

Find k in each of the given figures.



Solution:

(*i*) In the fig. (*i*) :

$k + 150^\circ + k - 15^\circ + 30^\circ + 90^\circ = 360^\circ$

(Angles at a point)

$\Rightarrow 2k + 150^\circ + 90^\circ + 30^\circ - 15^\circ = 360^\circ$

$\Rightarrow 2k + 270^\circ - 15^\circ = 360^\circ$

$\Rightarrow 2k = 360^\circ - 270^\circ + 15^\circ = 105^\circ$

$\Rightarrow k = \frac{105^\circ}{2} = 52{\cdot}5^\circ$ or $52^\circ\ 30'$

$\therefore k = 52{\cdot}5^\circ$ or $52^\circ\ 30'$

(*ii*) In the fig. (*ii*),

$k + 2k + 3k + 42^\circ = 360^\circ$

(Angles at a point)

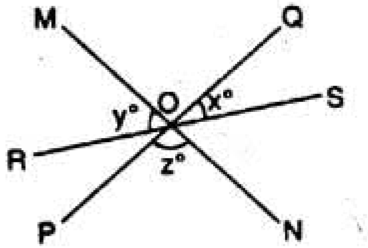
$\Rightarrow 6k + 42^\circ = 360^\circ$

$\Rightarrow \quad 6k = 360^\circ - 42^\circ = 318^\circ$

$\Rightarrow \quad k = \frac{318^\circ}{6} = 53^\circ$

$\therefore k = 53^\circ$

Question 22.
In the given figure, lines PQ, MN and RS intersect at O. If x : y = 1 : 2 and z = 90°, find ∠ROM and ∠POR.



Solution:

In the fig., lines PQ, MN and RS are intersecting each other at O

$x : y = 1 : 2, z = 90°$

$\angle MOQ = \angle PON = z$

(Vertically opposite angles)

Now, RS is a straight line

$\therefore x + z + y = 180°$

$\Rightarrow x + y + 90° = 180°$ $(\because z = 90°)$

$\Rightarrow x + y = 180° - 90° = 90°$

But $x : y = 1 : 2$

Let $x = a$ then $y = 2a$

$\therefore a + 2a = 90°$

$\Rightarrow 3a = 90°$

$\Rightarrow a = \frac{90°}{3} = 30°$

$\therefore x = 30°$ and $y = 2a = 2 \times 30° = 60°$

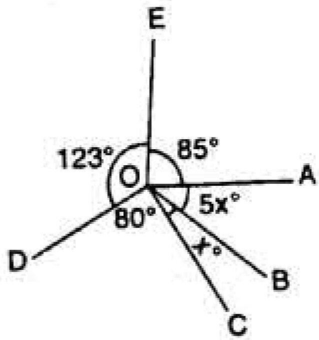
Now, $\angle ROM = y = 60°$

and $\angle POR = \angle SOQ$

(Vertically opposite angles)

$= x = 30°$

Question 23.
In the given figure, find ∠AOB and ∠BOC.



Solution:

In the figure,

$5x + x + 80° + 123° + 85° = 360°$

(Angles at a point)

$\Rightarrow 6x + 80° + 123° + 85° = 360°$

$\Rightarrow 6x + 288° = 360°$

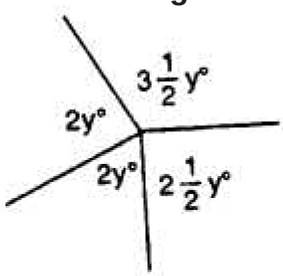
$\Rightarrow \quad 6x = 360° - 288° = 72°$

$\Rightarrow \quad x = \frac{72°}{6} = 12°$

Now, $\angle AOB = 5x = 5 \times 12° = 60°$

and $\angle BOC = x = 12°$

Question 24.
Find each angle shown in the diagram.



Solution:

In the figure,

$$3\frac{1}{2}y^\circ + 2y^\circ + 2y^\circ + 2\frac{1}{2}y^\circ = 360^\circ$$

(Angles at a point)

$$\Rightarrow \frac{7}{2}y^\circ + 2y^\circ + 2y^\circ + \frac{5}{2}y^\circ = 360^\circ$$

$$\Rightarrow \frac{7}{2}y^\circ + \frac{5}{2}y^\circ + 4y^\circ = 360^\circ$$

$$\Rightarrow \frac{12}{2}y^\circ + 4y^\circ = 360^\circ$$

$$\Rightarrow 6y^\circ + 4y^\circ = 360^\circ$$

$$\Rightarrow 10y^\circ = 360^\circ$$

$$\Rightarrow y = \frac{360^\circ}{10} = 36^\circ$$

$$\therefore \angle AOB = 3\frac{1}{2}y^\circ = \frac{7}{2}y^\circ = \frac{7}{2} \times 36^\circ$$

$$= 126^\circ$$

$$\angle BOC = 2y^\circ = 2 \times 36 = 72^\circ$$

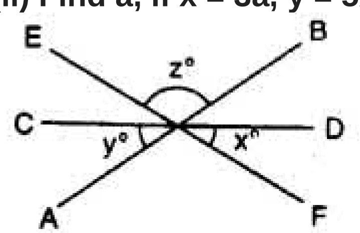
$$\angle COD = 2y^\circ = 72^\circ$$

$$\angle DOA = 2\frac{1}{2}y^\circ = \frac{5}{2}y^\circ$$

$$= \frac{5}{2} \times 36^\circ = 90^\circ$$

Question 25.

AB, CD and EF are three lines intersecting at the same point.
(i) Find x, if y = 45° and z = 90°.
(ii) Find a, if x = 3a, y = 5x and r = 6x.



Solution:

AB, CD and EF are intersecting each other at 0.

and $\angle DOF = x^o$, $\angle AOC = y^o$

and $\angle BOE = z^o$

But $\angle DOB = \angle AOC = y^o$

(Vertically opposite angles)

Similarly, $\angle COE = \angle DOF = x^o$

and $\angle AOF = \angle BOE = z^o$

$\because$ CD is a straight line

$\therefore \angle COE + \angle BOE + \angle DOB = 180^o$

$\Rightarrow x^o + z^o + y^o = 180^o$

$\Rightarrow x^o + y^o + z^o = 180^o$

(*i*) If $y = 45^o$ and $z = 90^o$, then

$\Rightarrow x^o + 45^o + 90^o = 180^o$

$\Rightarrow \quad x^o + 135^o = 180^o$

$\therefore x^o = 180^o - 135^o = 45^o$

(*ii*) If $x = 3a$, $y = 5x$, $z = 6x$,

then $x + y + z = 180^o$

$\Rightarrow x + 5x + 6x = 180^o \Rightarrow 12x = 180^o$

$$\Rightarrow \quad x = \frac{180^o}{12} = 15^o$$

But $x = 3a$

$\therefore 3a = 15^o \quad \Rightarrow a = \frac{15^o}{3} = 5^o$

Hence $a = 5^o$

EXERCISE 14 (B)

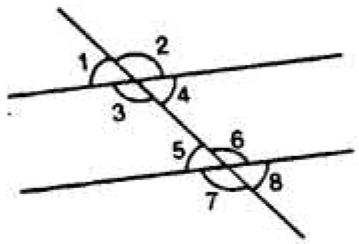
In questions 1 and 2, given below, identify the given pairs of angles as corresponding angles, interior alternate angles, exterior alternate angles, adjacent angles, vertically opposite angles or allied angles :

Question 1.

(i) ∠3 and ∠6

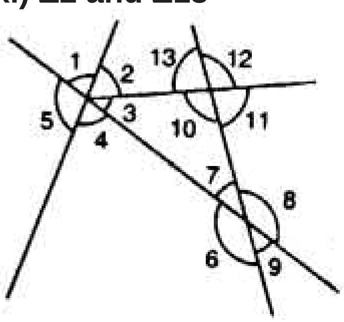
(ii) ∠2 and ∠4

(iii) ∠3 and ∠7
(iv) ∠2 and ∠7
(v) ∠4 and∠6
(vi) ∠1 and ∠8
(vii) ∠1 and ∠5
(viii) ∠1 and ∠4
(ix) ∠5 and ∠7



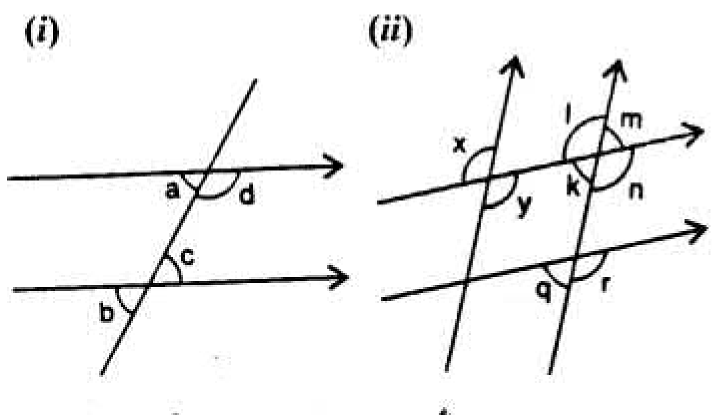
Solution:
(i) ∠3 and ∠6 are interior alternate angles.
(ii) ∠2 and ∠4 are adjacent angles.
(iii) ∠3 and ∠7 are corresponding angles.
(iv) ∠2 and ∠7 are exterior alternate angles,
(v) ∠4 and ∠6 are allied or co-interior angles,
(vi) ∠1 and ∠8 are exterior alternate angles.
(vii) ∠1 and ∠5 are corresponding angles.
(viii) ∠1 and ∠4 are vertically opposite angles.
(ix) ∠5 and ∠7 are adjacent angles.

Question 2.
(i) ∠1 and ∠4
(ii) ∠4 and ∠7
(iii) ∠10 and ∠12
(iv) ∠7 and ∠13
(v) ∠6 and ∠8
(vi) ∠11 and ∠8
(vii) ∠7 and ∠9
(viii) ∠4 and ∠5
(ix) ∠4 and ∠6
(x) ∠6 and ∠7
(xi) ∠2 and ∠13



Solution:
(i) ∠1 and ∠4 are vertically opposite angles.
(ii) ∠4 and ∠7 are alternate angles.
(iii) ∠10 and ∠12 are vertically opposite angles.
(iv) ∠7 and ∠13 are corresponding angles.
(v) ∠6 and ∠8 are vertically opposite angles.
(vi) ∠11 and ∠8 are allied or co-interior angles.
(vii) ∠7 and ∠9 are vertically opposite angles.
(viii) ∠4 and ∠5 are adjacent angles.
(ix) ∠4 and ∠6 are allied or co-interior angles.
(x) ∠6 and ∠7 are adjacent angles.
(xi) ∠2 and ∠13 are allied or co-interior angles.

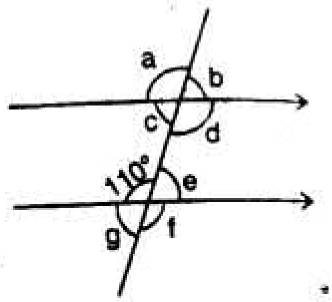
Question 3.
In the given figures, the arrows indicate parallel lines. State which angles are equal. Give reasons.



Solution:
In the figure (i),
a = b (corresponding angles)
b = c (vertically opposite angles)
a = c (alternate angles)
∴ a = b = c
(ii) In the figure (ii),
x =y (vertically opposite angles)
y=l (alternate angles)
x = l (corresponding angles)
1 = n (vertically opposite angles)
n = r (corresponding angles)
∴ x = y = l = n = r
Again m = k (vertically opposite angles)
k = q (corresponding angles)
∴ m = k = q

Question 4.

In the given figure, find the measure of the unknown angles :

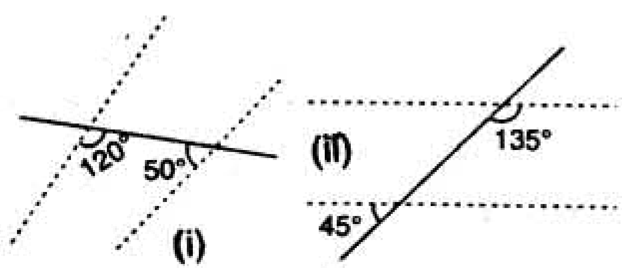


Solution:

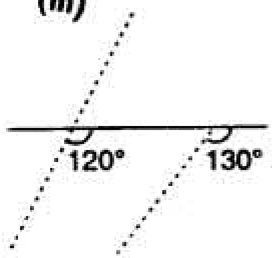
a = d (vertically opposite angles)
d=f (corresponding angles)
f= 110° (vertically opposite angles)
∴ a = d = f = 110°
e + 110° = 180° (co-interior angles)
∴ e = 180°- 110° = 70°
b = c (vertically opposite angles)
b = e (corresponding angles)
e = g (vertically opposite angles)
∴ b = c = e = g = 70° ”
Hence a = 110°, b = 70°, e = 70°, d = 110°, e = 70°,f= 110° and g = 70°

Question 5.

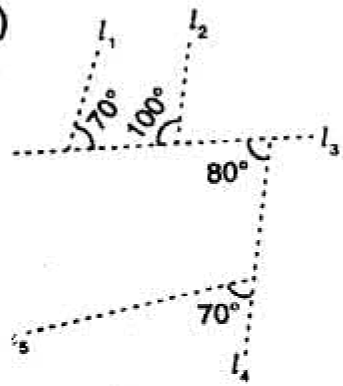
Which pair of the dotted line, segments, in the following figures, are parallel. Give reason:



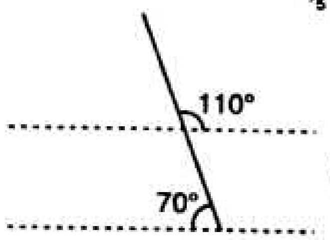
(iii)



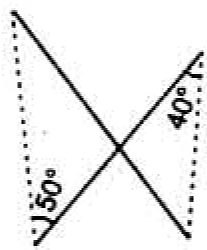
(v)



(iv)



(vi)



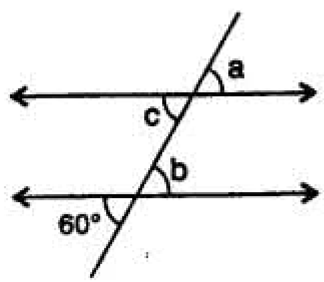
Solution:

(i) In figure (i), If lines are parallel, then 120°+ 50° =180°
But there are co-interior angles
⇒170° = 180°. But it not true Hence, there are not parallel lines
(ii) In figure (ii),
∠1 = 45° (vertically opposite angles)
Lines are parallel if
∠1 + 135° = 180° (co-interior angles)
⇒45°+ 135°= 180°
⇒ 180° = 180° which is true.
Hence, the lines are parallel.
(iiii) In figure (iii),
Lines are parallel if corresponding angles are equal
If 120° =130° which is not correct
∴ Lines are not parallel.
(iv) ∠1 = 110° (vertically opposite angles)
If lines are parallel then
∠1 + 70° = 180° (co-interior angles)
⇒110° + 70°= 180°
⇒180° =180°
Which is correct.
∴ Lines are parallel.
(v) ∠1 + 100°= 180°
⇒∠1 = 180°- 100°= 80 (linear pair)
Lines l1 and l2 will be parallel If ∠1 = 70°
⇒ 80° = 70° which is not true
∴ l1 and 12 are not parallel Again, A, l3and l5 will be parallel
If 80° = 70° (corresponding angle)
Which is not true.

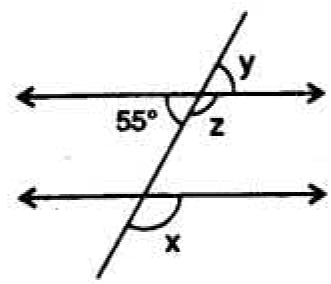
∴l3 and l5 are not parallel
But ∠1 = 80° (alternate angles)
⇒ 80° = 80°
Which is true
∴ l2 and l4 are parallel
(vi) Lines are parallel
If alternate angles are equal
⇒ 50° = 40°
Wliich is not ture lines are not parallel.

Question 6.
In the given figures, the directed lines are parallel to each other. Find the unknown angles.

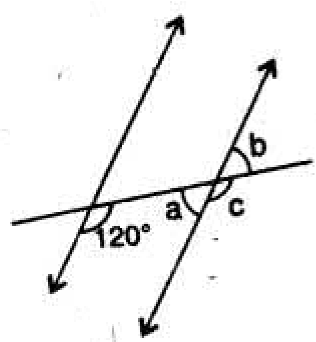
(i)



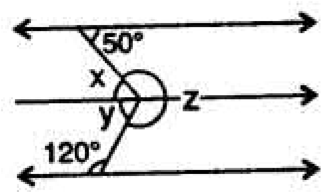
(ii)



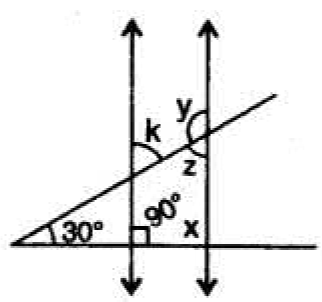
(iii)



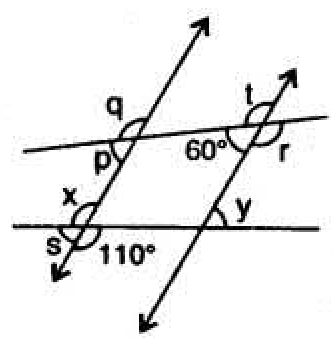
(iv)



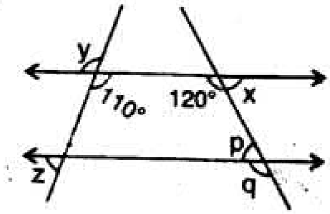
(v)



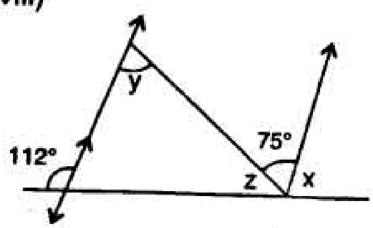
(vi)



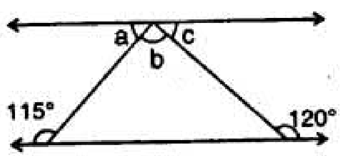
(vii)



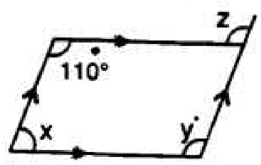
(viii)



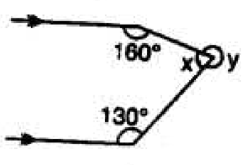
(ix)



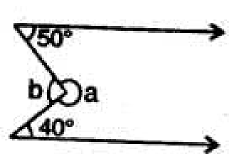
(x)



(xi)



(xii)



Solution:

(i) $\because$ Lines are parallel

$\therefore a = b$ (corresponding angles)

$a = c$ (vertically opposite angles)

$\therefore a = b = c$

But $b = 60^\circ$ (vertically opposite angles)

$\therefore a = b = c = 60^\circ$

(ii) $\because$ Lines are parallel

$\therefore x = z$ (corresponding angles)

But $z + y = 180^\circ$ (linear pair)

But $y = 55^\circ$ (vertically opposite angles)

$\therefore z + 55^\circ = 180^\circ$

$\Rightarrow z = 180^\circ - 55^\circ \Rightarrow z = 125^\circ$

But $x = z$

$\therefore x = 125^\circ$

Hence $x = 125^\circ, y = 55^\circ, z = 125^\circ$

(iii) $\because$ Lines are parallel

$\therefore c = 120^\circ$

$a + 120^\circ = 180^\circ$ (co-interior angles)

$\therefore a = 180^\circ - 120^\circ = 60^\circ$

But $a = b$ (vertically opposite angles)

$\therefore b = 60^\circ$

Hence $a = 60^\circ$, $b = 60^\circ$ and $c = 120^\circ$

(iv) $\because$ Lines are parallel

$\therefore x = 50^\circ$ (alternate angles)

and $y + 120^\circ = 180^\circ$ (co-interior angles)

$\therefore y = 180^\circ - 120^\circ = 60^\circ$

But $x + y + z = 360^\circ$ (angles at a point)

$\Rightarrow 50^\circ + 60^\circ + z = 360^\circ$

$\Rightarrow 110^\circ + z = 360^\circ$

$\Rightarrow z = 360^\circ - 110^\circ = 250^\circ$

Hence $x = 50^\circ, y = 60^\circ, z = 250^\circ$

(v) $\because$ Lines are parallel

$\therefore x + 90^\circ = 180^\circ$ (co-interior angles)

$\Rightarrow x = 180^\circ - 90^\circ = 90^\circ \Rightarrow \angle 2 = x$

$\Rightarrow \angle 2 = 90^\circ$

But $\angle 1 + \angle 2 + 30^\circ = 180^\circ$

(sum of angles of a triangle)

$\Rightarrow \angle 1 + 90^\circ + 30^\circ = 180^\circ$

$\Rightarrow \quad \angle 1 + 120^\circ = 180^\circ$

$\Rightarrow \quad \angle 1 = 180^\circ - 120^\circ = 60^\circ$

But $\angle 1 = k$ (vertically opposite angle)

$\therefore k = 60^\circ$

But $\angle 1 = z$ (alternate angles)

$\therefore z = 60^\circ$

But $k + y = 180^\circ$ (co-interior angles)

$\Rightarrow 60^\circ + y = 180^\circ \Rightarrow y = 180^\circ - 60^\circ = 120^\circ$

Hence $x = 90^\circ$, $y = 120^\circ$,

$z = 60^\circ$, $k = 60^\circ$

(*vi*) $\because$ Lines are parallel

$\therefore q = t$ and $p = 60^\circ$ (corresponding angles)

But $t + 60^\circ = 180^\circ$ (linear pair)

$\Rightarrow t = 180^\circ - 60^\circ = 120^\circ$

$t = r$ (vertically opposite angles)

$\therefore r = 120^\circ$

$q = t = 120^\circ$

$x = 110^\circ$ (vertically opposite angles)

$x + s = 180^\circ$ (linear pair)

$\Rightarrow 110^\circ + s = 180^\circ$

$\Rightarrow \quad s = 180^\circ - 110^\circ = 70^\circ$

$s = \angle 1$ (corresponding angles)

$\therefore \angle 1 = 70^\circ$

$y = \angle 1 = 70^\circ$ (vertically opposite angles)

Hence $x = 110^\circ$, $y = 70^\circ$, $p = 60^\circ$, $q = 120^\circ$,
$r = 120^\circ$, $s = 70^\circ$, $t = 120^\circ$

(*vii*) $\because$ Lines are parallel

$\therefore x = p$

and $p + 120^\circ = 180^\circ$ (alternate angles)

$\Rightarrow p = 180^\circ - 120^\circ = 60^\circ$

$\therefore x = 60^\circ$

$q = 120^\circ$ (corresponding angles)

$y = 110^\circ$ (vertically opposite angles)

and $\angle 1 + 110^\circ = 180^\circ$

(co-interior angles)

$\therefore \angle 1 = 180^\circ - 110^\circ = 70^\circ$

But $z = \angle 1$ (vertically opposite angles)

$\therefore \angle z = 70^\circ$

Hence $x = 60^\circ, y = 110^\circ, z = 70^\circ,$

$p = 60^\circ, q = 120^\circ$.

(*viii*) $\because$ Lines are parallel

$\therefore y = 75^\circ$ (alternate angles)

$\angle 1 + 112^\circ = 180^\circ$ (linear pair)

$\angle 1 = 180^\circ - 112^\circ = 68^\circ$

$\angle 1 = x$ (corresponding angles)

$\therefore x = 68^\circ$

But $x + 75 + z = 180^\circ$ (angles on a line)

$\Rightarrow 68^\circ + 75^\circ + z = 180^\circ$

$\Rightarrow \quad z + 143^\circ = 180^\circ$

$\Rightarrow \quad z = 180^\circ - 143^\circ = 37^\circ$

Hence $x = 68^\circ, y = 75^\circ, z = 37^\circ$

(*ix*) $\because$ Line are parallel

$\angle a = \angle 1$ and $\angle c = \angle 2$ (alternate angles)

But $\angle 1 + 115^\circ = 180^\circ$ (linear pair)

$\therefore \angle 1 = 180^\circ - 115^\circ = 65^\circ$

Similarly $\angle 2 + 120^\circ = 180^\circ$

$\therefore \angle 2 = 180^\circ - 120^\circ = 60^\circ$

$\therefore \angle a = \angle 1 = 65^\circ, \angle c = \angle 2 = 60^\circ$

But $a + b + c = 180^\circ$ (angles on a line)

$\Rightarrow 65^\circ + b + 60^\circ = 180^\circ$

$\Rightarrow \quad b + 125^\circ = 180^\circ$

$\Rightarrow \quad b = 180^\circ - 125^\circ = 55^\circ$

Hence $a = 65^\circ, b = 55^\circ, c = 60^\circ$.

(x) $\because$ Lines are parallel

$\therefore x + 110^\circ = 180^\circ$ (co-interior angles)

$x = 180^\circ - 110^\circ = 70^\circ$

and $x + y = 180^\circ$ (co-interior angles)

$\Rightarrow 70^\circ + y = 180^\circ$

$\Rightarrow \quad y = 180^\circ - 70^\circ = 110^\circ$

$z = y$ (corresponding angles)

$\therefore z = 110^\circ$

Hence $x = 70^\circ, y = 110^\circ, z = 110^\circ$

(xi) From 0, draw a line parallel to the given parallel lines

$\because$ Lines are parallel

$\therefore \angle 1 = 160^\circ$ and $\angle 2 = 130^\circ$
(alternate angles)

$\therefore y = \angle 1 + \angle 2 = 160^\circ + 130^\circ = 290^\circ$

But $x + y = 360^\circ$ (angles at a point)

$\Rightarrow 290^\circ + x = 360^\circ$

$\Rightarrow \quad x = 360^\circ - 290^\circ = 70^\circ$

Hence $x = 70^\circ, y = 290^\circ$

(xii) From 0, draw a line parallel to the given parallel lines

$\therefore \angle 1 = 50^\circ$ (alternate angles)

$\angle 2 = 40^\circ$

$\therefore b = \angle 1 + \angle 2 = 50^\circ + 40^\circ = 90^\circ$

But $a + b = 360^\circ$ (angles at a point)

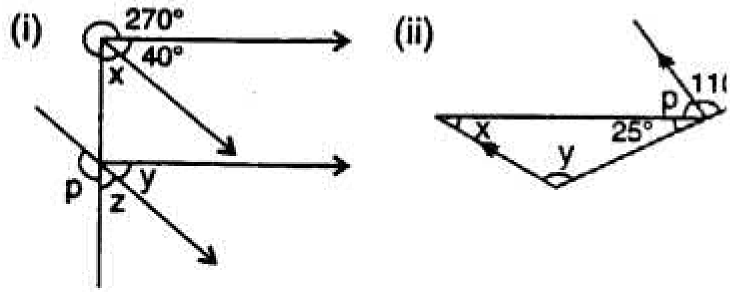
$\therefore a + 90^\circ = 360^\circ$

$\Rightarrow a = 360^\circ - 90^\circ = 270^\circ$

Hence $a = 270^\circ, b = 90^\circ$

Question 7.

Find x. y and p is the given figures



Solution:

In figure (i)

$\because$ Lines are parallel

$\therefore x = z$ (corresponding angles)

$y = 40°$ (corresponding angles)

But $x + 40° + 270° = 360°$

(angles at a point)

$\Rightarrow x + 310° = 360°$

$\Rightarrow x = 360° - 310° = 50°$

$\therefore z = x = 50°$

But $p + z = 180°$ (linear pair)

$\Rightarrow p + 50° = 180°$

$\Rightarrow p = 180° - 50° = 130°$

Hence $x = 50°, y = 40°, z = 50°$

and $p = 130°$

(ii) In figure (ii)

$\because$ Lines are parallel

$\therefore y = 110°$ (corresponding angles)

But $25° + p + 110° = 180°$

(angles on a line)

$\Rightarrow p + 135° = 180°$

$\Rightarrow p = 180° - 135° = 45°$

$\Rightarrow x + y + 25° = 180°$

(sum of angles of a triangle)

$\Rightarrow x + 110° + 25° = 180°$

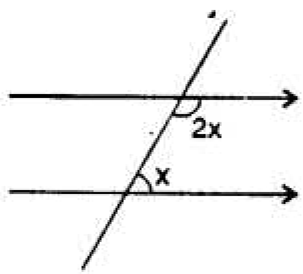
$\Rightarrow x + 135° = 180°$

$\Rightarrow x = 180° - 135° = 45°$

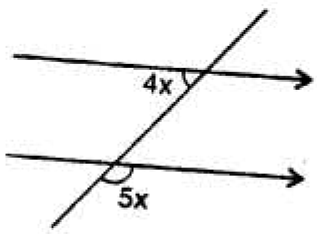
Hence $x = 45°, y = 110°$ and $p = 45°$

Question 8.
Find x in the following cases :

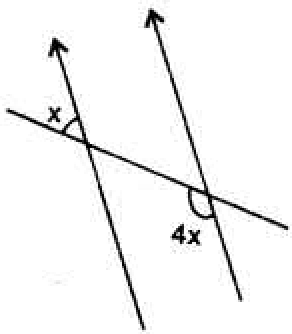
(i)



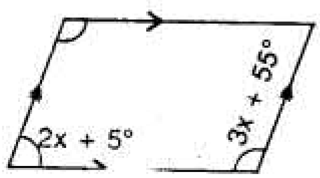
(ii)



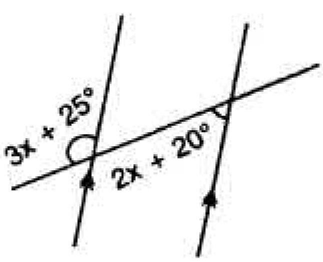
(iii)



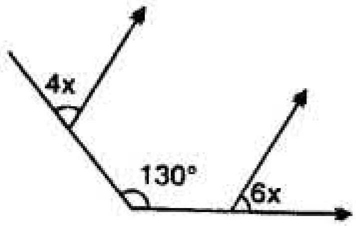
(iv)



(v)



(vi)



Solution:

(i) In figure (i),

$\because$ Lines are parallel

$\therefore 2x + x = 180^\circ$ (co-interior angles)

$\Rightarrow 3x = 180^\circ \quad \Rightarrow x = \dfrac{180^\circ}{3} = 60^\circ$

(ii) In figure (ii),

$\because$ Lines are parallel

$\therefore 4x + 1 = 180^\circ$ (co-interior angles)

But $\angle 1 = 5x$ (vertically opposite angles)

$\therefore 4x + 5x = 180^\circ \quad \Rightarrow 9x = 180^\circ$

Hence $x = \dfrac{180^\circ}{9} = 20^\circ$

(iii) In figure (iii),

$\because$ Lines are parallel

$\therefore \angle 1 + 4x = 180^\circ$ (co-interior angles)

But $\angle 1 = x$ (vertically opposite angles)

$\therefore x + 4x = 180^\circ$

$\Rightarrow \quad 5x = 180^\circ \quad \Rightarrow x = \dfrac{180^\circ}{5} = 36^\circ$

Hence $x = 36^\circ$

(iv) In figure (iv),

$\because$ Lines are parallel

$\therefore 2x + 5^\circ + 3x + 55^\circ = 180^\circ$

(co-inteior angles)

$\Rightarrow 5x + 60^\circ = 180^\circ$

$\Rightarrow \quad 5x = 180^\circ - 60^\circ = 120^\circ$

$\therefore x = \dfrac{120^\circ}{5} = 24^\circ$

Hence $x = 24°$

(v) In figure (v),

$\because$ Lines are parallel

$\therefore \angle 1 = 2x + 20°$ (alternate angles)

But $\angle 1 + 3x + 25° = 180°$ (linear pair)

$\Rightarrow 2x + 20° + 3x + 25° = 180°$

$\Rightarrow 5x + 45° = 180°$

$\Rightarrow 5x = 180° - 45° = 135°$

$\therefore x = \frac{135°}{5} = 27°$

(vi) In figure (vi),

From 0, draw a line parallel to the given parallel lines

$\therefore \angle 1 = 4x$ and $\angle 2 = 6x$

(corresponding angles)

But $\angle 1 + \angle 2 = 130°$

$\Rightarrow 4x + 6x = 130° \Rightarrow 10x = 130°$

$\therefore x = \frac{130°}{10} = 13°$

EXERCISE 14 (C)

Question 1.

Using ruler and compasses, construct the following angles :

(i)30°

(ii)15°

(iii) 75°

(iv) 180°

(v) 165°

(vi) 22.5°

(vii) 37.5°

(viii) 67.5°

Solution:

(i) 30°

Steps of Construction :

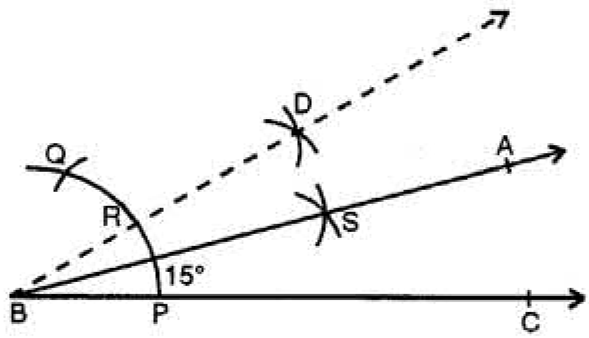
(i) Draw a line segment BC.

(ii) With centre B and a suitable radius draw an arc meeting BC at P.

(iii) With centre P and with same radius cut off the arc at Q.

(iv) Now with centre P and Q draw two arcs intersecting each other at R.

(v) Join BR and produce it to A, forming ZABC
= 30°



(ii) (15°)

Steps of Construction:

(i) Draw a line segment BC.

(ii) With centre B and a suitable radius draw an arc meeting BC at P.

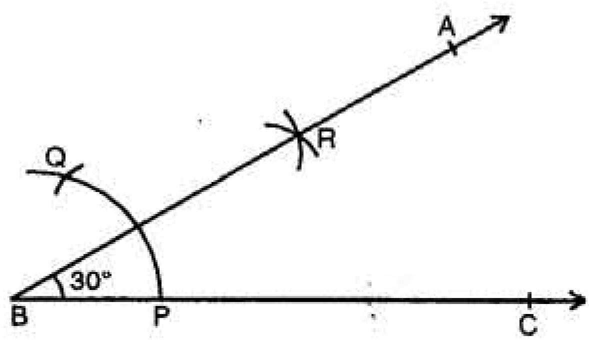
(iii) With centre P and with same radius cut off the arc at Q.

(iv) Taking P and Q as curves, draw two arcs intersecting each other at D nnd join BD.

(v) With centre P and R, draw two more arcs intersecting each other at S.

(vi) Join BS and produce it to A.

Then ∠ABC = 15°.



(iii) 75°

Steps of Construction :

(i) Draw a line segment BC.

(ii) With centre B and a suitable radius draw an arc and cut off PQ, then QR of the same radius.

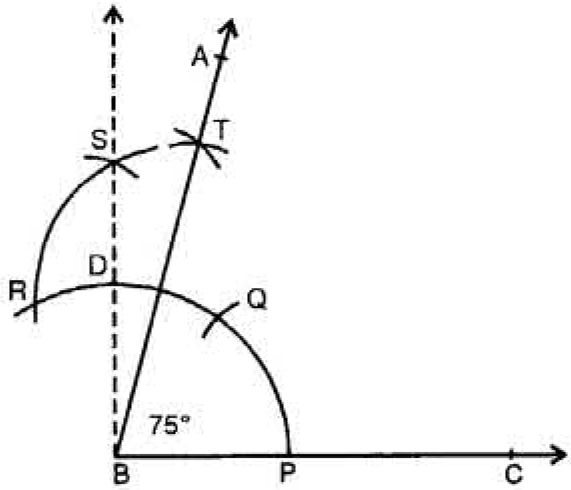
(iii) With centre Q and R, draw two arcs intersecting each other at S.

(iv) Join SB.

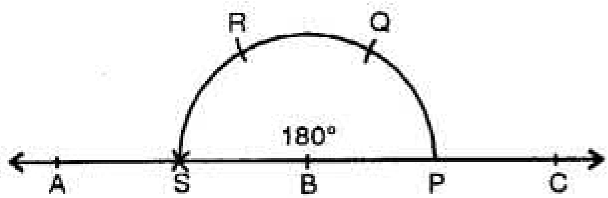
(v) With centre Q and D draw two arcs intersecting each other at T.

(vi) Join BT and produce it to A.

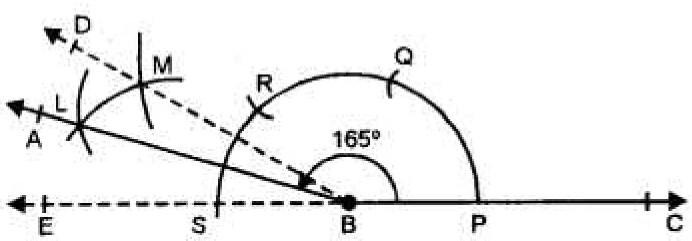
Then ∠ABC = 75°.



(iv) 180°
Steps of Construction :
(i) Draw a line segment BC.
(ii) With centre B and some suitable radius draw arc meeting BC at P.
(iii) With centre P and with same radius cut of arcs PQ, QR and then RS.
(iv) Join BS and produce it to A.
Then ∠ABC = 180°.



(v) 165°
Steps of Construction :
(i) Draw a line segment BC.
(ii) With centre B and some suitable radius draw an arc meeting BC at P.
(iii) With centre P and same radius cut off arcs PQ, QR and then RS.



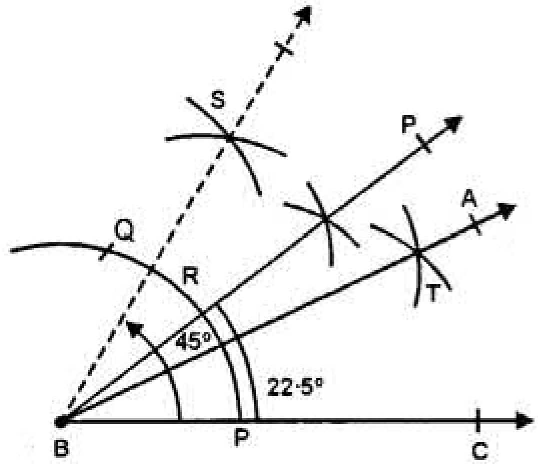
(iv) Join SB.
(v) With centres R and S, draw two arcs intersecting each other at M.
(vi) With centre T and S draw two arcs intersecting each other at L.
(vi) Join BL and produce it to A. Then ∠ABC = 165°

(vi) 22.5°

Steps of Construction :

(i) Draw a line segment BC.

(ii) With centre B and some suitable radius, draw an arc meeting BC at P.



(iii) With centre P and some radius, cut off arcs PQ.

(iv) Bisect arc PQ at R and join BR.

(v) Bisect arc QR at S and join BS.

(vii) Now bisect arc PR at T.

(viii) Join BT and produce it to A.

Then ∠ABC = 22 $\frac{1}{2}$ ° or 22.5°.

(vii) 37.5°

Steps of Construction :

(i) Draw a line segment BC.

(ii) With centre B and some suitable radius, draw an arc meeting BC at P.

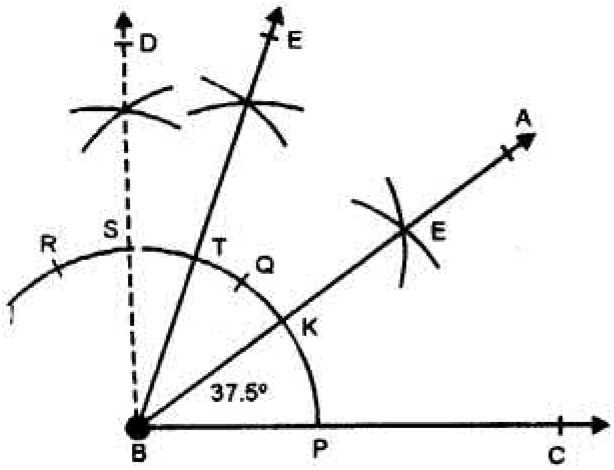
(iii) With centre P and same radius cut off arcs PQ and QR.

(iv) Now bisect arc QR at S and again bisect arc QS at T.

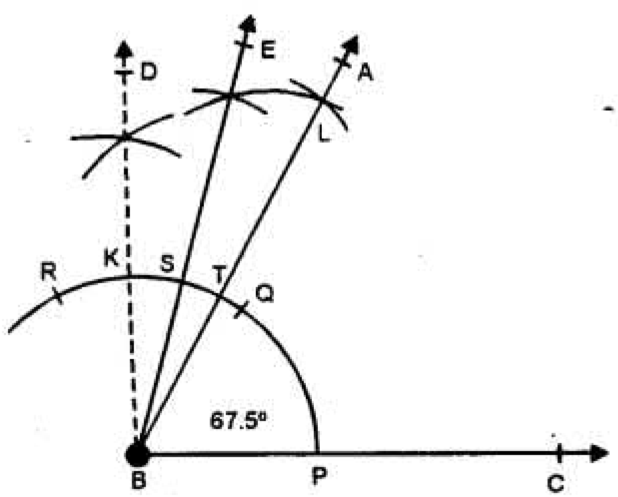
(v) Bisect arc PT at K.

(vi) Join BK and produce it to A.

Then, ∠ABC – 37 $\frac{1}{2}$ °or 37-5.

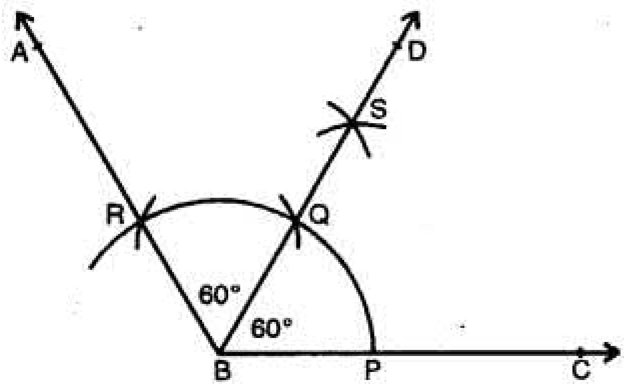


(viii) 67.5°
Steps of Construction :
(i) Draw a line segment BC.
(ii) With centre B and some suitable radius, draw an arc meeting BC at P.
(iii) With centre P and with same radius, cut arcs PQ and then QR.
(iv) Bisect arc QR at K and again bisect arc QK at S.
(v) Bisect again arc SQ at T.
(vi) Join BT and produce it to A.
Then ∠ABC = 67$\frac{1}{2}$ ° or 67.5°



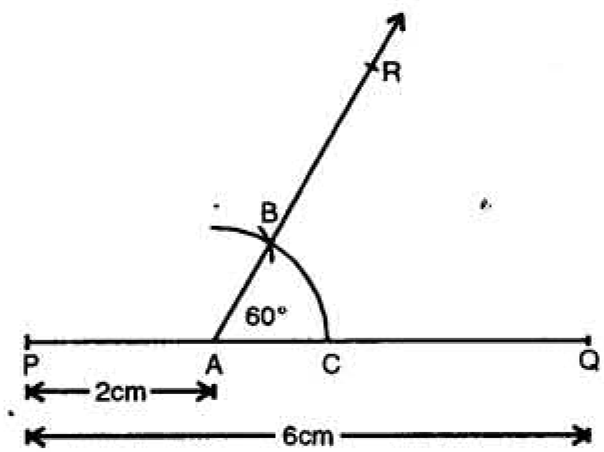
Question 2.
Draw ∠ABC = 120°. Bisect the angle using ruler and compasses only. Measure each 1 angle so obtained and check whether the angles obtained on bisecting ∠ABC are equal or not.
Solution:
Steps of Construction :

(i) Draw a line segment BC.
(ii) With centre B and some suitable radius, draw an arc meeting BC at P.



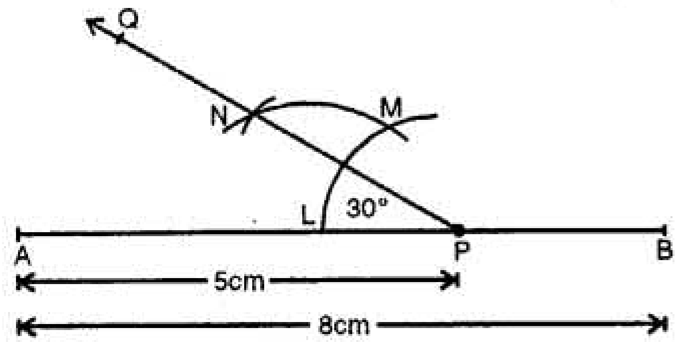
(iii) With centre P and with same radius, cut arcs PQ and QR.
(iv) Join BR and produce it to A.
Then ∠ABC = 120°
(v) With centres P and R, draw two arcs intersecting each other at S.
(vi) Join BS and produce it to D. BD is the bisector of ∠ABC.
On measuring each angle, it is of 60° each. Yes, both angles are equal in measure.

Question 3.
Draw a line segment PQ = 6 cm. Mark a point A in PQ so that AP = 2 cm. At point A, construct angle QAR = 60°.
Solution:
Steps of Construction :
(i) Draw a line segment PQ = 6 cm.

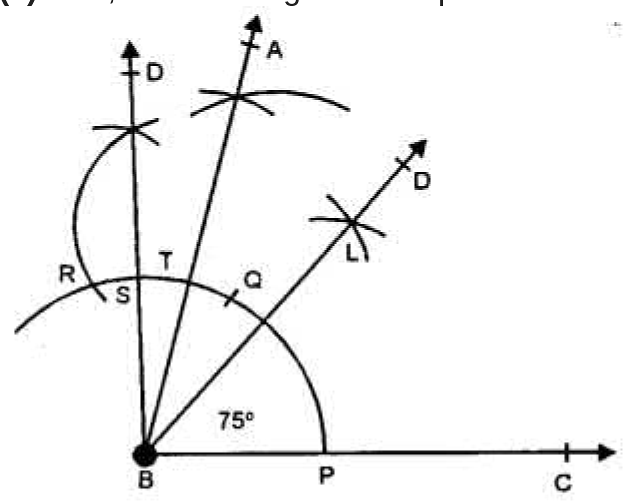


(ii) Mark a point A on PQ so that AP = 2 cm.
(iii) With centre A and some suitable radius draw an arc meeting AQ at C.
(iv) With centre C and with same radius, cut arc CB.
(v) Join AB and produce it to R.
Then ∠QAR = 60°

Question 4.
Draw a line segment AB = 8 cm. Mark a point P in AB so that AP = 5 cm. At P, construct angle APQ = 30°.
Solution:
Steps of Construction :
(i) Draw a line segment AB = 8 cm.
(ii) Mark a point P in AB such that AP = 5 cm.
(iii) With centre P and some suitable radius, draw an arc meeting AB in L.
(iv) With centre L and same radius cut arc LM.
(v) Bisect arc LM at N.
(vi) Join PN and produce it to Q.
Then ∠APQ = 30°

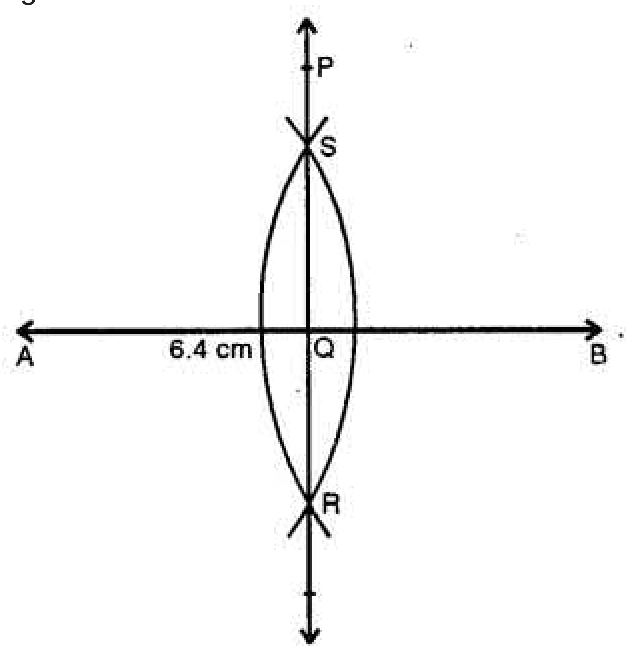


Question 5.
Construct an angle of 75° and then bisect it.
Solution:
Steps of Construction :
(i) Draw a line segment BC.
(ii) At B, draw an angle ABC equal to 75°.



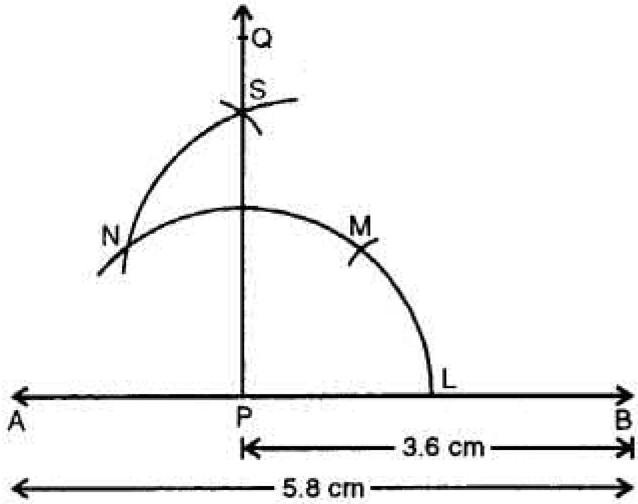
(iii) With centres P and T, draw arcs intersecting each other at L.
(iv) Join BL and produce it to D. Then BD bisects ∠ABC.

Question 6.
Draw a line segment of length 6 .4 cm. Draw its perpendicular bisector.
Solution:
Steps of Construction :
(i) Draw a line segment AB = 6.4 cm.
(ii) With centres A and B and with some suitable radius, draw arcs intersecting each other at S and R.
(iii) Join SR intersecting AB at Q. Then PQR is the perpendicular bisector of line segment AB



Question 7.
Draw a line segment AB = 5.8 cm. Mark a point P in AB such that PB = 3.6 cm. At P, draw perpendicular to AB.
Solution:
Steps of Construction :
(i) Draw a line segment AB = 5.8 cm.
(ii) Mark a point P in AB such that PB = 3.6 cm.
(iii) With centre P and some suitable radius draw an arc meeting AB in L.
(iv) With centre L and same radius cut arcs LM and then as N.
(v) Bisect arc MN at S.

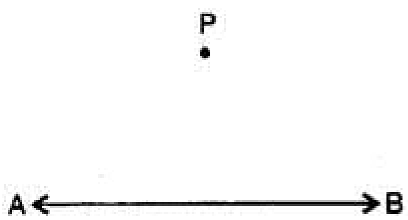
(vi) Join PS and produce it to Q. Then PQ is perpendicular to AB at P.



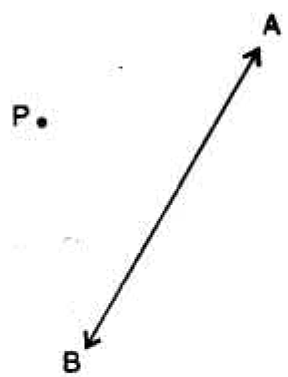
Question 8.

In each case, given below, draw a line through point P and parallel to AB :

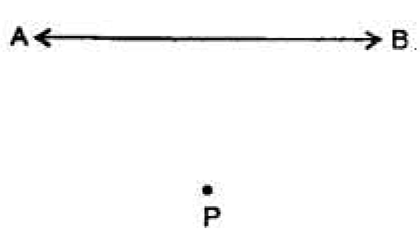
(i)



(ii)



(iii)



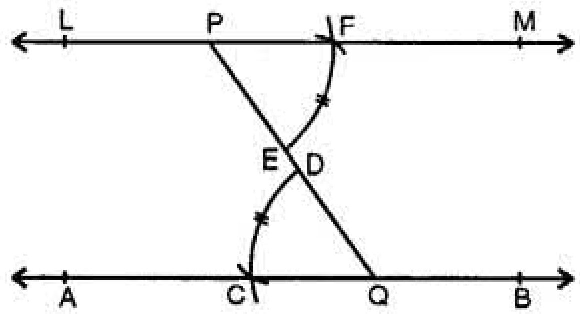
Solution:

Steps of construction :

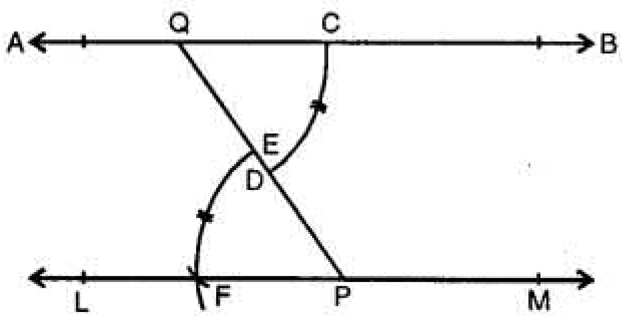
(i) From P. draw a line segment meeting AB at

(**ii)** With centre Q and some suitable radius draw an arc CD.

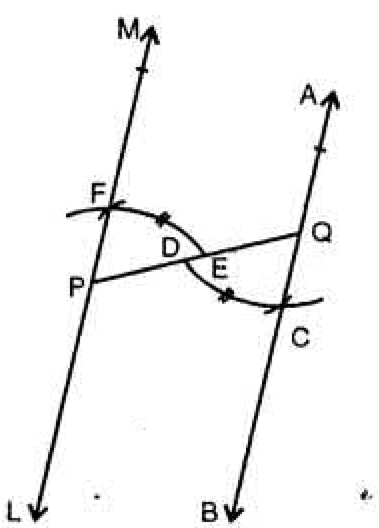
(iii) With centre P and same radius draw another arc meeting PQ at E.



(iv) With centre E and radius equal to CD, cut this arc at F



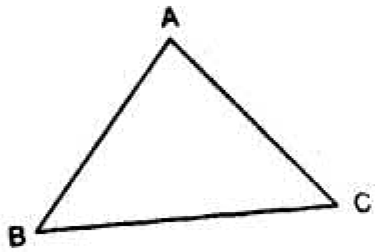
(v) Join PF and produce it to both sides to L and M. Then line LM is parallel to given line AB.



CHAPTER - 15
TRIANGLES

POINTS TO REMEMBER

1. Definition of a triangle : A closed figure, having 3 sides, is called a triangle and is usually denoted by the Greek letter Δ (delta).



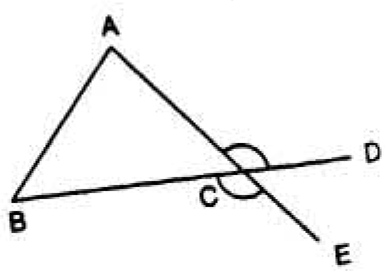
The figure, given alongside, shows a triangle ABC (ΔABC) bounded by three sides AB, BC and CA.

Hence it has six elements : 3 angles and 3 sides.

2. Vertex : The point, where any two sides of a triangle meet, is called a vertex. Clearly, the given triangle has three vertices; namely : A, B and C. [Vertices is the plural of vertex]

3. Interior angles : In ΔABC (given above), the angles BAC, ABC and ACB are called its interior angles as they lie inside the Δ ABC. The sum of interior angles of a triangle is always 180°.

4. Exterior angles : When any side of a triangle is produced the angle so formed, outside the triangle and at its vertex, is called its **exterior angle.**

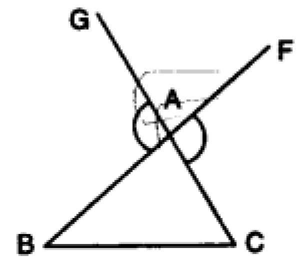


e.g. if side BC is produced to the point D; then $\angle$ACD is its exterior angle. And, if side AC is produced to the point E, then the exterior angle would be $\angle$BCE.

Thus. at every vertex, two exterior angles can be formed and that these two angles being vertically opposite angles, are always equal.

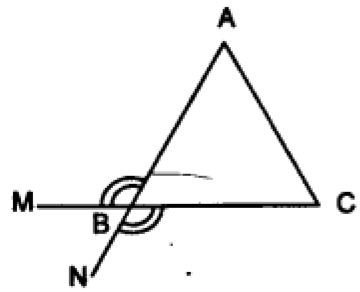
Make the following figures clear :

(i)



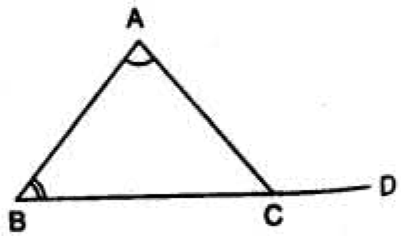
[Ext. ∠BAG = Ext. ∠CAF]

(ii)



[Ext. ∠ABM = Ext. ∠CBN]

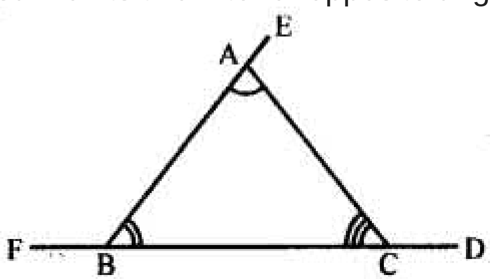
5. **Interior opposite angles :** When any side of a triangle is produced; an exterior angle is formed. The two interior angles of this triangle, that are opposite to the exterior angle formed; are called its **interior opposite angles.**



In the given figure, side BC of ∆ABC is produced to the point D, so that the exterior ∠ACD is formed. Then the two interior opposite angles are ∠B AC and ∠ABC.

6. **Relation between exterior angle and interior opposite angles :**

Exterior angle of a triangle is always equal to the sum of its two interior opposite angles.



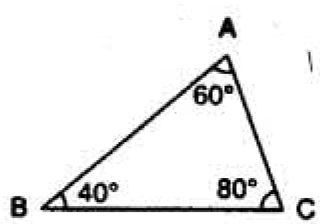
In ∆ABC,

Ext. ∠ACD = ∠A + ∠B

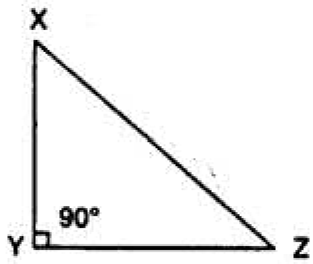
7. CLASSIFICATION OF TRIANGLES

(A) With regard to their angles :

1. Acute angled triangle : It is a triangle, whose each angle is acute i.c. each angle is less than 90°.

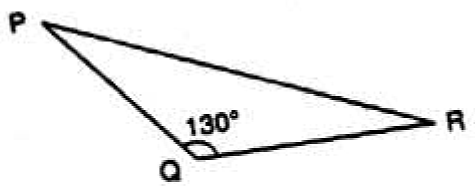


2. Right angled triangle : It is a triangle, whose one angle is a right angle i.e. equal to 90”.
The figure, given alongside, shows a right angled triangle XYZ as ∠XYZ = 90°
Note : (i) One angle of a right triangle is 90° and the other two angles of it are acute; such that their sum is always 90”.

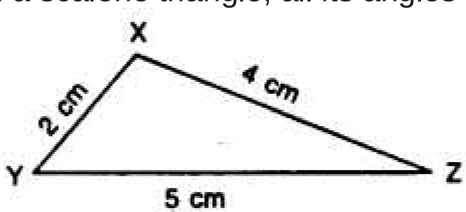


In ∆XYZ, given above, ∠Y = 90° and each of ∠X and ∠Z is acute such that ∠X + ∠Z = 90°. .
(ii)In a right triangle, the side opposite to the right angle is largest of all its sides and is called the**hypotenuse**. In given right angled ∆ XYZ side XZ is its hypotenuse

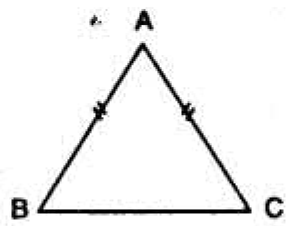
3.Obtuse angled triangle : If one angle of a triangle is 1
obtuse, it is called an obtuse angled triangle.
Note : In case of an obtuse angled triangle, each of the other two angles is always acute and their sum is less than 90”.



(B) With regard to their sides :
(1) Scalene triangle: If all the sides of a triangle are unequal, it is called a **scalene triangle.**
In a scalene triangle; all its angles are also unequal.

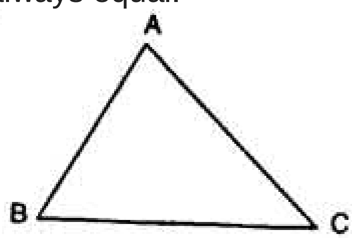


(2) Isosceles triangle : If atleast two sides of a triangle are equal, it is called an **isosceles triangle.**
In ∆ ABC, shown alongside, side AB = side AC.
∴∆ ABC is an isosceles triangle.



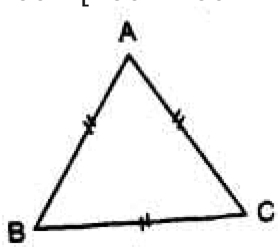
Note : (i) The angle contained by equal sides i.e. ∠BAC is called the **vertical angle** or the **angle of vertex.**
(ii) The third side (i.e. the unequal side) is called the **base** of the isosceles triangle.
(iii) The two other angles (i.e. other than the angle of vertex) are called the **base angles** of the triangle.

IMPORTANT PROPERTIES OF AN ISOSCELES TRIANGLE
The base angles i.e. the angles opposite to equal sides of an isosceles triangle are always equal.



In given triangle ABC,
(i) If side AB = side BC; then angle opposite to AB = angle opposite to BC i.e. ∠C = ∠A.
(ii) If side BC = side AC; then angle opposite to BC = angle opposite to AC i.e. ∠A = ∠B and so on.
Conversely : If any two angles of a triangle are equal; the sides opposite to these angles are also equal i.e. the triangle is isosceles.
Thus in Δ ABC,
(i) If ∠B = ∠C => side opposite to ∠B = side opposite to ∠C i.e. side AC = side AB.
(ii) If ∠A = ∠B => side BC = side AC and so on.

(3) Equilateral triangle :
If all the sides of a triangle are equal, it is called an equilateral triangle.
In the given figure, A ABC is equilateral, because AB = BC = CA.
Also, all the angles of an equilateral triangle are equal to each other and so each angle = 60°. [∵60° + 60° + 60° = 180°]



Since, all the angles of an equilateral triangle are equal, it is also known as equiangular triangle. Note : An equilateral triangle is always an isosceles triangle, but its converse is not always true.

(4) Isosceles right angled triangle : If one angle of an isosceles triangle is 90°, it is called an isosceles right angled triangle.
In the given figure, Δ ABC is an isosceles right angled triangle, because : ∠ ACB = 90° and AC = BC.
Here, the base is AB, the vertex is C and the base angles are ∠BAC and ∠ABC, which are equal.
Since, the sura of the angles of a triangle = 180″
∴∠ABC = ∠BAC = 45 [∵45° + 45° + 90° = 180°]

EXERCISE 15 (A)

Question 1.
Stale, if the triangles are possible with the following angles :
(i) 20°, 70° and 90°
(ii) 40°, 130° and 20°
(iii) 60°, 60° and 50°
(iv) 125°, 40° and 15°
Solution:
We know that, the sum of three angles of a triangle is 180°, therefore

(*i*) Sum of 20°, 70° and 90°
$$= 20° + 70° + 90° = 180°.$$
Since the sum is 180°. Hence it is possible.

(*ii*) Sum of 40°, 130° and 20°
$$= 40° + 130° + 20° = 190°.$$
Since the sum is not 180°, therefore it is not possible.

(*iii*) Sum of 60°, 60° and 50°
$$= 60° + 60° + 50° = 170°.$$
Since the sum is not 180°, therefore it is not possible.

(*iv*) Sum of 125°, 40° and 15°
$$= 125° + 40° + 15° = 180°.$$
Since the sum is 180°, therefore it is possible.

Question 2.
If the angles of a triangle are equal, find its angles.
Solution:

Since the three angles of a triangle are equal and their sum is 180°, therefore each angle will be $\frac{180^\circ}{3} = 60^\circ$.

Question 3.
In a triangle ABC, ∠A = 45° and ∠B = 75°, find ∠C.
Solution:

Since the sum of angles of a triangle is 180°

$\therefore \angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow 45^\circ + 75^\circ + \angle C = 180^\circ$

$\Rightarrow \quad 120^\circ + \angle C = 180^\circ$

$\Rightarrow \angle C = 180^\circ - 120^\circ = 60^\circ$

Question 4.
In a triangle PQR, ∠P = 60° and ∠Q = ∠R, find ∠R.
Solution:

Let $\angle Q = \angle R = x$, $\angle P = 60^\circ$

But $\angle P + \angle Q + \angle R = 180^\circ$

$\therefore 60^\circ + x + x = 180^\circ$

$\Rightarrow 60^\circ + 2x = 180^\circ$

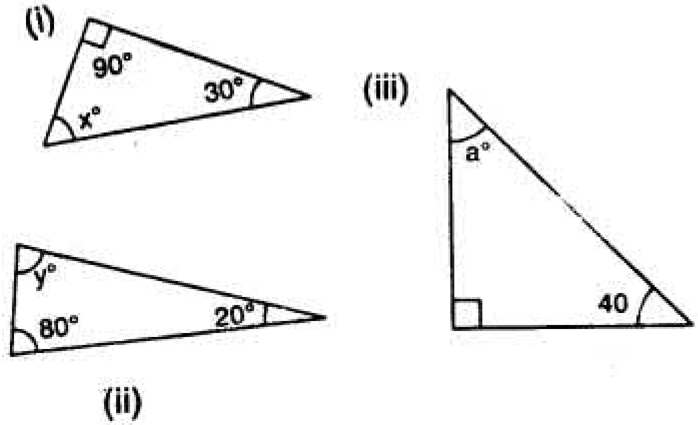
$\Rightarrow \quad 2x = 180^\circ - 60^\circ = 120^\circ$

$\Rightarrow \quad x = \frac{120^\circ}{2} = 60^\circ$

$\therefore \angle Q = \angle R = 60^\circ$

Hence, $\angle R = 60^\circ$

Question 5.
Calculate the unknown marked angles in each figure :



Solution:

We know that, the sum of three angles of a triangle is 180°, therefore

(i) In figure (i),

$90° + 30° + x = 180°$

$\Rightarrow 120° + x = 180°$

$\Rightarrow \quad x = 180° - 120° = 60°$

Hence $x = 60°$

(ii) In figure (ii),

$y + 80° + 20° = 180°$

$\Rightarrow y + 100° = 180°$

$\Rightarrow \quad y = 180° - 100° = 80°$

Hence $y = 80°$

(iii) In figure (iii),

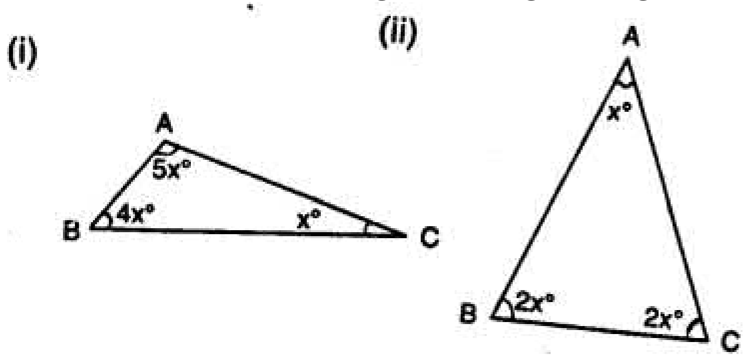
$a + 90° + 40° = 180°$

$\Rightarrow a + 130° = 180°$

$\Rightarrow \quad a = 180° - 130° = 50°$

Hence $a = 50°$

Question 6.
Find the value of each angle in the given figures:



Solution:

(*i*) In the figure (*i*),

$\angle A + \angle B + \angle C = 180°$ (Sum of angles of a triangle)

$\Rightarrow 5x° + 4x° + x° = 180°$

$\Rightarrow 10x° = 180° \quad \Rightarrow x = \frac{180°}{10} = 18°$

$\therefore \angle A = 5x° = 5 \times 18° = 90°$

$\angle B = 4x = 4 \times 18° = 72°$

and $\angle C = x = 18°$

(*ii*) In figure (*ii*),

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow x° + 2x° + 2x° = 180°$

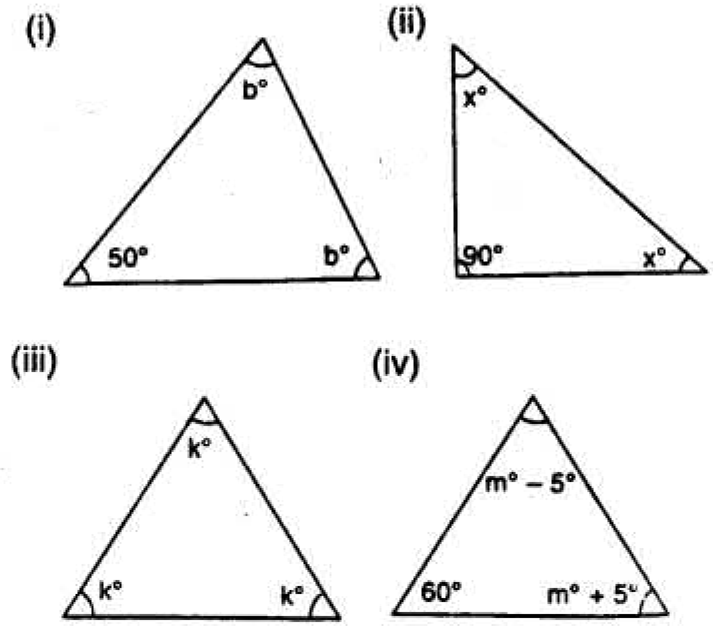
$\Rightarrow \quad 5x° = 180° \quad \Rightarrow x° = \frac{180°}{5} = 36°$

$\therefore \quad \angle A = x° = 36°$

$\angle B = 2x° = 2 \times 36° = 72°$

and $\angle C = 2x° = 2 \times 36° = 72°$

Question 7.
Find the unknown marked angles in the given figure:



Solution:

(i) In the figure (i),
$\angle A + \angle B + \angle C = 180°$
(Sum of angles of a triangle)
$b° + 50° + b° = 180°$
$\Rightarrow 2b° + 50° = 180°$
$\Rightarrow \quad 2b° = 180° - 50° = 130°$
$\Rightarrow \quad b° = \frac{130°}{2} = 65°$
Hence $\angle A = b° = 65°$
and $\angle C = b° = 65°$

(ii) In the figure (ii)
$\angle A + \angle B + \angle C = 180°$
(Sum of angles of a triangle)
$x° + 90° + x° = 180°$
$2x° + 90° = 180°$
$2x° = 180° - 90°$

$2x^\circ = 90^\circ$

$x^\circ = \frac{90^\circ}{2} = 45^\circ$

Hence $\angle A = x^\circ = 45^\circ$

and $\angle C = x^\circ = 45^\circ$

(*iii*) In the figure (*iii*)

$\angle A + \angle B + \angle C = 180^\circ$

(Sum of angles of a triangle)

$k^\circ + k^\circ + k^\circ = 180^\circ$

$3k^\circ = 180^\circ$

$k^\circ = \frac{180^\circ}{3} = 60^\circ$

Hence $\angle A = k^\circ = 60^\circ$, $\angle B = k^\circ = 60^\circ$

and $\angle C = k^\circ = 60^\circ$

(*iv*) In the figure (*iv*)

$\angle A + \angle B + \angle C = 180^\circ$

(Sum of Angles of a triangle)

$(m^\circ - 5^\circ) + 60^\circ + (m^\circ + 5^\circ) = 180^\circ$

$m^\circ - 5^\circ + 60^\circ + m^\circ + 5^\circ = 180^\circ$

$2m^\circ = 180^\circ - 65 + 5$

$2m^\circ = 120^\circ$

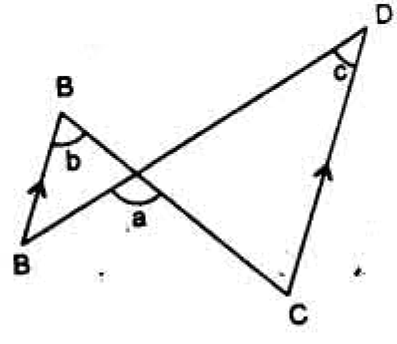
$\therefore\ m^\circ = \frac{120^\circ}{2} = 60^\circ$

Hence $\angle A = m^\circ - 5^\circ = 60^\circ - 5^\circ = 55^\circ$

and $\angle C = m^\circ + 5^\circ = 60^\circ + 5^\circ = 65^\circ$

Question 8.

In the given figure, show that: ∠a = ∠b + ∠c



(i) If ∠b = 60° and ∠c = 50° ; find ∠a.
(ii) If ∠a = 100° and ∠b = 55° : find ∠c.
(iii) If ∠a = 108° and ∠c = 48° ; find ∠b.

Solution:

$\because$ AB || CD

$\therefore b = c$ and $\angle A = \angle C$ (Alternate angles)

Now in Δ PCD,

Ext. $\angle APC = \angle C + \angle D$

$\Rightarrow a = b + c$

(i) If $b = 60°$, $c = 50°$, then

$a = b + c = 60° + 50° = 110°$

(ii) If $a = 100°$ and $b = 55°$,

then $a = b + c \quad \Rightarrow 100° = 55° + c$

$\Rightarrow c = 100° - 55° = 45°$

(iii) If $a = 108°$ and $c = 48°$,

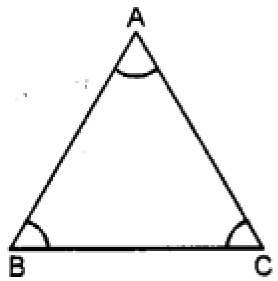
then $a = b + c \Rightarrow 108° = b + 48°$

$\Rightarrow \quad b = 108° - 48° = 60°$

Question 9.
Calculate the angles of a triangle if they are in the ratio 4 : 5 : 6.

Solution:

We know that sum of angles of a triangle is 180°



$\therefore \angle A + \angle B + \angle C = 180°$

But $\angle A : \angle B : \angle C = 4 : 5 : 6$

Let $\angle A = 4x$, $\angle B = 5x$ and $\angle C = 6x$,

then $4x + 5x + 6x = 180°$

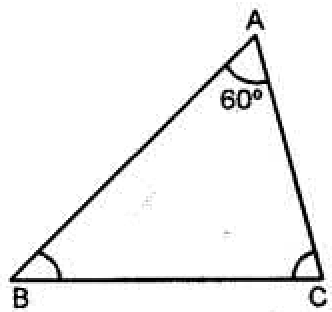
$$\Rightarrow 15x = 180° \qquad \Rightarrow x = \frac{180°}{15} = 12°$$

$\therefore \angle A = 4x = 4 \times 12° = 48°$

$\angle B = 5x = 5 \times 12° = 60°$

$\angle C = 6x = 6 \times 12° = 72°$.

Question 10.
One angle of a triangle is 60°. The, other two angles are in the ratio of 5 : 7. Find the two angles.
Solution:
In Δ ABC,
Let $\angle A = 60°$ and then $\angle B : \angle C = 5 : 7$
But $\angle A + \angle B + \angle C = 180°$
(Angles of a triangle)



$\Rightarrow 60° + \angle B + \angle C = 180°$
$\Rightarrow \angle B + \angle C = 180° - 60° = 120°$
Let $\angle B = 5x$ and $\angle C = 7x$
$\therefore 5x + 7x = 120°$
$\Rightarrow 12x = 120° \quad \Rightarrow x = \frac{120°}{12} = 10°$
$\therefore \angle B = 5x = 5 \times 10° = 50°$
$\angle C = 7x = 7 \times 10 = 70°$

Question 11.

One angle of a triangle is 61° and the other two angles are in the ratio $1\frac{1}{2} : 1\frac{1}{3}$. Find these angles.

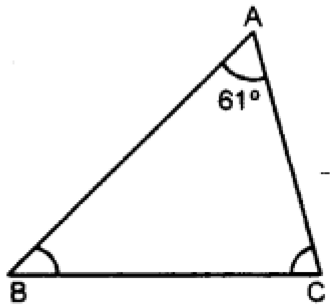
Solution:

In Δ ABC,

Let $\angle A = 61°$

But $\angle A + \angle B + \angle C = 180°$

(Angles of a triangle)



$\Rightarrow 61° + \angle B + \angle C = 180°$

$\Rightarrow \quad \angle B + \angle C = 180° - 61° = 119°$

$$\text{But } \angle B : \angle C = 1\frac{1}{2} : 1\frac{1}{3} = \frac{3}{2} : \frac{4}{3}$$

$$= \frac{9 : 8}{6} = 9 : 8$$

Let $\angle B = 9x$ and $\angle C = 8x$,

then, $9x + 8x = 119°$

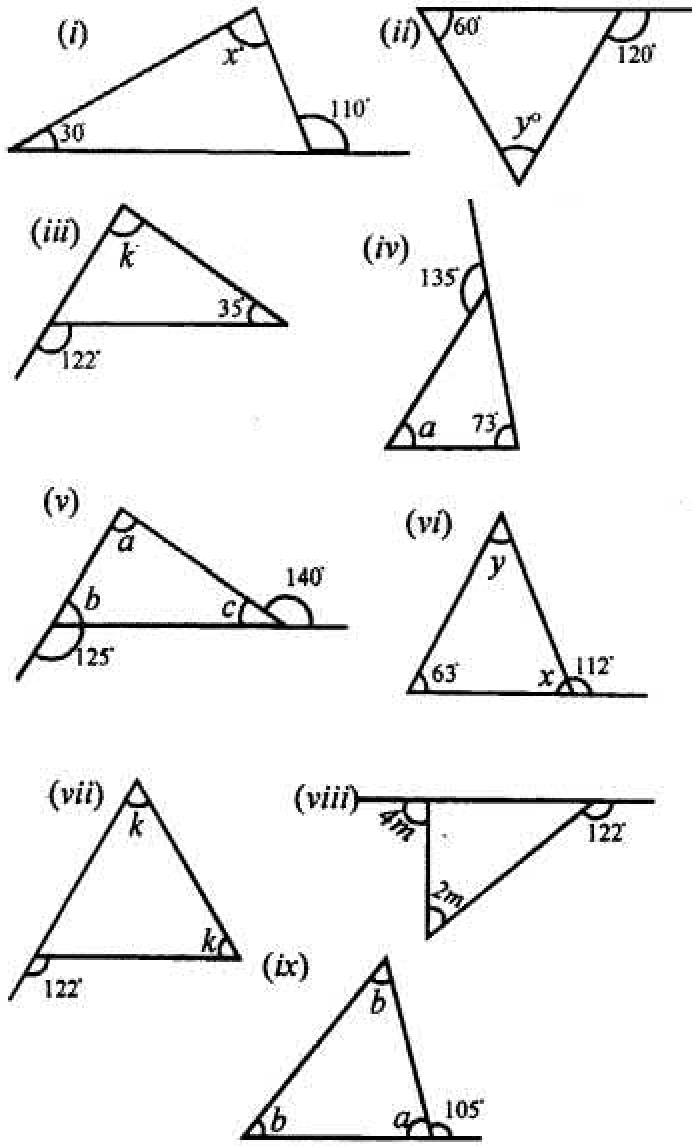
$\Rightarrow 17x = 119°$

$$\Rightarrow \quad x = \frac{119°}{17} = 7°$$

$\therefore \angle B = 9x = 9 \times 7° = 63°$

$\angle C = 8x = 8 \times 7° = 56°$

Question 12.
Find the unknown marked angles in the given figures :



Solution:

We know that in a triangle, if one side of it is produced, then

Exterior angle = sum of its interior opposite angles.

(*i*) In Fig. (*i*), $110° = x° + 30°$

$\Rightarrow x° = 110° - 30° = 80°$

(*ii*) In Fig. (*ii*), $120° = y° + 60°$

$\Rightarrow y° = 120° - 60° = 60°$

(*iii*) In Fig. (*iii*), $122° = k° + 35°$

$\Rightarrow k° = 122° - 35° = 87°$

(*iv*) In Fig. (*iv*), $135° = a° + 73°$

$\Rightarrow a° = 135° - 73° = 62°$

(*v*) In Fig. (*v*), $125° = a + c$ ...(*i*)

and $140° = a + b$ ...(*ii*)

Adding, we get

$a + c + a + b = 125° + 140°$

$\Rightarrow a + a + b + c = 265°$

But $a + b + c = 180°$

(Sum of angles of a triangle)

$\therefore \quad a + 180° = 265°$

$\Rightarrow a = 265° - 180° = 85°$

But $a + b = 140°$

$\Rightarrow 85° + b = 140°$

$\Rightarrow \quad b = 140° - 85° = 55°$

and $a + c = 125° \Rightarrow 85° + c = 125°$

$\Rightarrow \quad c = 125° - 85° = 40°$

Hence $a = 85°$, $b = 55°$ and $c = 40°$

(*vi*) In Fig. (*vi*),

$112° + x° = 180°$ (Linear pair)

$\Rightarrow \quad x = 180° - 112° = 68°$

and $112° = y + 63°$

$\Rightarrow \quad y = 112° - 63° = 49°$

Hence $x = 68°$, $y = 49°$

(*vii*) In fig. (*vii*),

$120° = a + a \quad \Rightarrow \quad 2a = 120°$

$\Rightarrow a = \frac{120°}{2} = 60°$

$\therefore a = 60°$ **Ans.**

(*viii*) In fig. (*viii*),

$140° + a = 180°$ (Linear pair)

$\Rightarrow \quad a = 180° - 140° = 40°$

Now $4m = 2m = a \Rightarrow 4m - 2m = a$

$\Rightarrow 2m = 40° \Rightarrow m = \frac{40°}{2} = 20°$

Hence $m = 20°$

(*ix*) In fig. (*ix*),

$105° = b + b \quad \Rightarrow 2b = 105°$

$\Rightarrow \; b = \frac{105°}{2} = 52{\cdot}5°$

But $a + 105° = 180°$ (Linear pair)

$\Rightarrow \quad a = 180° - 105° = 75°$

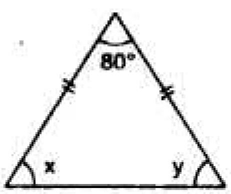
Hence $a = 75°$, $b = 52{\cdot}5°$

EXERCISE 15 (B)

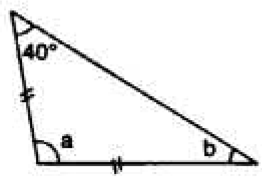
Question 1.

Find the unknown angles in the given figures:

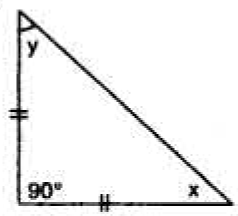
(i)



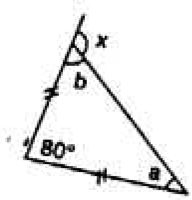
(ii)



(iii)

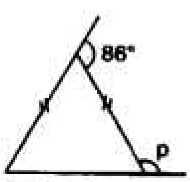


(iv)

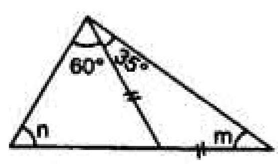


(i

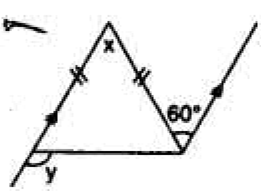
(v)



(vi)



(vii)



Solution:

(*i*) In Fig (*i*),

$x = y$ (Angles opposite to equal sides)

But $x + y + 80° = 180°$ (Angles of a triangle)

$\Rightarrow \quad x + x + 80° = 180°$

$\Rightarrow \quad 2x + 80° = 180°$

$\Rightarrow \quad 2x = 180° - 80° = 100°$

$\Rightarrow \quad x = \frac{100°}{2} = 50° \quad \therefore y = x = 50°$

Hence $x = 50°, y = 50°$

(*ii*) In Fig. (*ii*),

$b = 40°$ (Angles opposite to equal sides)

But $a + b + 40° = 180°$

(Angles of a triangle)

$\Rightarrow a + 40° + 40° = 180°$

$\Rightarrow \quad a + 80° = 180°$

$\Rightarrow \quad a = 180° - 80° = 100°$

Hence $a = 100°, b = 40°$

(*iii*) In Fig. (*iii*),

$x = y$ (Angles opposite to equal sides)

But $x + y + 90° = 180°$

(Angles of a triangle)

$\Rightarrow x + x + 90° = 180°$

$\Rightarrow \quad 2x + 90° = 180°$

$\Rightarrow \quad 2x = 180° - 90° = 90°$

$\therefore\ x = \dfrac{90°}{2} = 45° \quad \therefore\ y = x = 45°$

Hence $x = 45°, y = 45°$

(*iv*) In Fig. (*iv*),

$a = b$ (Angles opposite to equal sides)

But $a + b + 80° = 180°$

(Angles of a triangle)

$\Rightarrow a + a + 80° = 180°$

$\Rightarrow \quad 2a + 80° = 180°$

$\Rightarrow \quad 2a = 180° - 80° = 100°$

$\Rightarrow \quad a = \dfrac{100°}{2} = 50° \quad \therefore\ b = a = 50°$

$x = a + 80°$

(Exterior angle of a triangle is equal to sum of its opposite interior angles)

$= 50° + 80° = 130°$

Hence $a = 50°, b = 50°$ and $x = 130°$

(*v*) In Fig. (*v*),

Let each equal angle of an isosceles triangle be x,

then $x + x = 86° \quad \Rightarrow 2x = 86°$

$\Rightarrow x = \dfrac{86°}{2} = 43°$

But $p + x = 180°$ (Linear pair)

$p + 43° = 180°$

$\Rightarrow \quad p = 180° - 43° \quad \Rightarrow \quad p = 137°$

Hence $p = 137°$

(*vi*) In Fig. (*vi*),

$m = 35°$ (Angles opposite to equal sides)

But $m + n + (60° + 35°) = 180°$

(Angles of a triangle)

$\Rightarrow \quad m + n + 95° = 180°$

$\Rightarrow 35° + n + 95° = 180°$

$\Rightarrow \quad n + 130° = 180°$

$\Rightarrow \quad n = 180° - 130° = 50°$

Hence $m = 35°, n = 50°$

(*vii*) In Fig. (*vii*),

$x = 60°$ (Alternate angles)

Let each equal angle of an isosceles triangle be a then $a + a + x = 180°$

(Angles of a triangle)

$2a + x = 180° \Rightarrow 2a + 60° = 180°$

$\Rightarrow 2a = 180° - 60° = 120°$

$$\Rightarrow \qquad a = \frac{120°}{2} = 60°$$

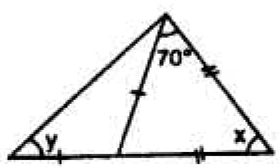
$\therefore\ y = x + a = 60° + 60° = 120°$

Hence $x = 60°$ and $y = 120°$

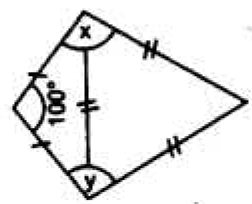
Question 2.

Apply the properties of isosceles and equilateral triangles to find the unknown angles in the given figures :

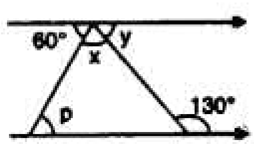
(i)



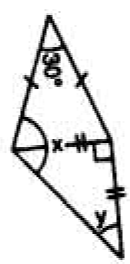
(ii)



(iii)



(iv)



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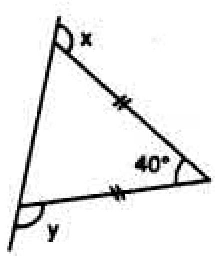

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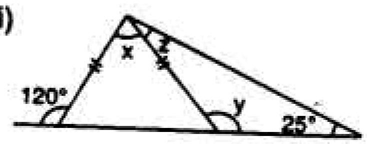


Solution:

$a = 70°$ (Angles opposite to equal sides)

But $a + 70° + x = 180°$ (Angles of a triangle)

$\Rightarrow 70° + 70° + x = 180°$

$\Rightarrow \quad 140° + x = 180°$

$\Rightarrow \quad x = 180° - 140° = 40°$

$y = b$ (Angles opposite to equal sides)

But $a = y + b$

(Exterior angle of a triangle is equal to sum of its interior opposite angles)

$\Rightarrow 70° = y + y \quad \Rightarrow 2y = 70°$

$\Rightarrow \quad y = \frac{70°}{2} = 35°$

Hence $x = 40°, y = 35°$

(*ii*) In Fig. (*ii*),

In an equilateral triangle.

each angle $= 60°$

In isosceles triangle.,

Let each base angle $= a$

$\therefore a + a + 100° = 180°$

$\Rightarrow \quad 2a + 100° = 180°$

$\Rightarrow \quad 2a = 180° - 100° = 80°$

$\therefore \ a = \frac{80°}{2} = 40° \ \therefore \ x = 60° + 40° = 100°$

and $y = 60° + 40° = 100°$

(*iii*) In Fig. (*iii*), $130° = x + p$

(Exterior angle of a triangle is equal to the sum of its interior opposite angles)

$\because$ Lines are parallel (Given)

$\therefore \quad p = 60°$ (Alternate angle)

and $y = a$

But $a + 130° = 180°$ (Linear pair

$\Rightarrow a = 180° - 130° = 50° \ \therefore \ y = 50°$

and $x + p = 130°$

$\Rightarrow x + 60° = 130° \Rightarrow x = 130° - 60° = 70$

Hence $x = 70°, y = 50°$ and $p = 60°$

(iv) In Fig. (iv),

$x = a + b$

But $b = y$ (Angles opposite to equal sides)

Similarly $a = c$

But $a + c + 30° = 180°$

$\Rightarrow a + a + 30° = 180° \Rightarrow 2a + 30° = 180°$

$\Rightarrow 2a = 180° - 30° = 150°$

$\Rightarrow a = \frac{150°}{2} = 75°$ and $b + y = 90°$

$\Rightarrow y + y = 90° \qquad \Rightarrow 2y = 90°$

$\Rightarrow \quad y = \frac{90°}{2} = 45° \qquad \Rightarrow b = 45°$

Hence $x = a + b = 75° + 45° = 120°$

and $y = 45°$

(v) In Fig. (v),

$a + b + 40° = 180°$ (Angles of a triangle)

$\Rightarrow a + b = 180° - 40° = 140°$

But $a = b$ (Angles opposite to equal sides)

$\therefore a = b = \frac{140°}{2} = 70°$

$\therefore x = b + 40° = 70° + 40° = 110°$

(Exterior angle of a triangle is equal to the sum of its interior opposite angles)

Similarly $y = a + 40°$

$= 70° + 40° = 110°$

Hence $x = 110°, y = 110°$

(vi) In the Fig. (vi),

$a = b$ (Angles opposite to equal sides)

$\therefore y = 120°$

But $a + 120° = 180°$ (Linear pair)

$\Rightarrow a = 180° - 120° = 60°$

$\therefore b = 60°$

But $x + a + b = 180°$ (Angles of a triangle)

$\Rightarrow x + 60° + 60° = 180°$

$\Rightarrow \quad x + 120° = 180°$

$\therefore x = 180° - 120° = 60°$

$b = z + 25$

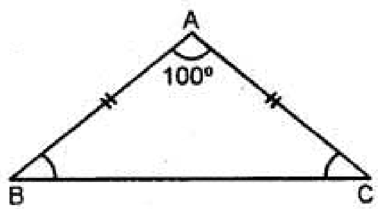
(Exterior angle of a triangle is equal to the sum of its interior opposite angles)

$\Rightarrow 60° = z + 25°$

$\Rightarrow \quad z = 60° - 25° = 35°$

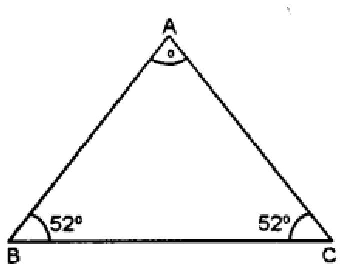
Hence $x = 60°, y = 120°$ and $z = 35°$

Question 3.
The angle of vertex of an isosceles triangle is 100°. Find its base angles.
Solution:
In Δ ABC.



$\because$ AB = AC.
$\therefore$ $\angle B = \angle C$
But $\angle A = 100°$
and $\angle A + \angle B + \angle C = 180°$
(Angles of a triangle)
$\Rightarrow 100° + \angle B + \angle B = 180°$
$\Rightarrow 2\angle B = 180° - 100°$ $\quad \Rightarrow 2\angle B = 80°$
$\therefore \quad \angle B = \frac{80°}{2} = 40°$
Hence $\angle B = \angle C = 40°$

Question 4.
One of the base angles of an isosceles triangle is 52°. Find its angle of vertex.
Solution:
Each of the base angles of an isosceles
Δ ABC = 52°
$\therefore \angle B = \angle C = 52°$



But $\angle A + \angle B + \angle C = 180°$
(Angles of a triangle)
$\Rightarrow \angle A + 52° + 52° = 180°$
$\Rightarrow \quad \angle A + 104° = 180°$
$\Rightarrow \quad \angle A = 180° - 104° = 76°$
Hence $\angle A = 76°$

Question 5.
In an isosceles triangle, each base angle is four times of its vertical angle. Find all the angles of the triangle.
Solution:

Let vertical angle of an isosceles triangle = x

$\therefore$ Each base angle = $4x$

$\therefore x + 4x + 4x = 180°$

(Sum of angles of a triangle)

$\Rightarrow 9x = 180° \quad \Rightarrow \quad x = \frac{180°}{9} = 20°$

$\therefore$ Vertical angle = $20°$

and each of the base angle = $4x$

$= 4 \times 20° = 80°$

Question 6.
The vertical angle of an isosceles triangle is 15° more than each of its base angles. Find each angle of the triangle.
Solution:

Let each angle of the base of the isosceles triangle = $x°$

Then vertical angle = $x + 15°$

Now $x + x + x + 15° = 180°$

(Sum of angles of a triangle)

$\Rightarrow 3x + 15° = 180°$

$\Rightarrow \quad 3x = 180° - 15° = 165°$

$\therefore x = \frac{165°}{3} = 55°$

Hence each base angle = $55°$

and vertical angle = $55° + 15° = 70°$

Question 7.
The base angle of an isosceles triangle is 15° more than its vertical angle. Find its each angle.
Solution:
Let vertical angle of the isosceles triangle $= x°$

$\therefore$ Each base angle $= x + 15°$

$\therefore x + 15° + x + 15° + x° = 180°$ (Sum of angles of a triangle)

$\Rightarrow 3x + 30° = 180°$

$\Rightarrow 3x = 180° - 30° = 150°$

$\therefore x = \frac{150°}{3} = 50°$

Hence vertical angle $= 50°$

and each base angle $= 50° + 15° = 65°$

Question 8.
The vertical angle of an isosceles triangle is three times the sum of its base angles. Find each angle.
Solution:
Let each base angle of an isosceles, triangle $= x$

then its vertical $= 3(x + x) = 3 \times 2x = 6x$.

$\therefore 6x + x + x = 180°$ (Sum of angles of a triangle)

$\Rightarrow 8x = 180° \quad \Rightarrow x = \frac{180°}{8} = 22 \cdot 5°$

$\therefore$ Each base angle $= 22 \cdot 5°$

and vertical angle $= 3 \times (22 \cdot 5 + 22 \cdot 5)$

$= 3 \times 45 = 135°$

Question 9.
The ratio between a base angle and the vertical angle of an isosceles triangle is 1 : 4. Find each angle of the triangle.
Solution:
Ratio between base angle and vertical angle of an isosceles triangle = 1 : 4

Let each base angle = x

then vertical angle = $4x$

$\therefore x + x + 4x = 180°$

(Sum of angles of a triangle)

$\Rightarrow 6x = 180°$ $\Rightarrow$

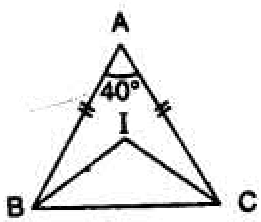
$x = \frac{180°}{6} = 30°$

$\therefore$ Each base angle = $x = 30°$

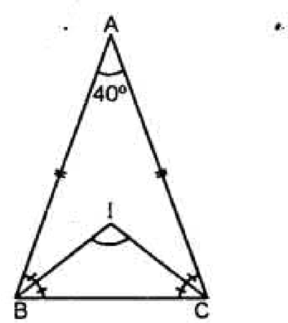
and vertical angle = $4x$

$= 4 \times 30° = 120°$

Question 10.
In the given figure, BI is the bisector of∠ABC and CI is the bisector of ∠ACB. Find ∠BIC.



Solution:
In Δ ABC,
BI is the bisector of ∠ABC and CI is the bisector of ∠ACB.



$\because AB = AC$

$\therefore \angle B = \angle C$

(Angles opposite to equal sides)

But $\angle A = 40°$

and $\angle A + \angle B + \angle C = 180°$

(Angles of a triangle)

$\Rightarrow 40° + \angle B + \angle B = 180°$

$\Rightarrow \quad 40° + 2\angle B = 180°$

$\Rightarrow \quad 2\angle B = 180° - 40° = 140°$

$\Rightarrow \quad \angle B = \frac{140°}{2} = 70°$

$\therefore \angle ABC = \angle ACB = 70°$

But BI and CI are the bisectors of $\angle ABC$ and $\angle ACB$ respectively.

$\therefore \angle IBC = \frac{1}{2}\angle ABC = \frac{1}{2}(70°) = 35°$

and $\angle ICB = \frac{1}{2}\angle ACB = \frac{1}{2} \times 70° = 35°$

Now in Δ IBC,

$\angle BIC + \angle IBC + \angle ICB = 180°$

(Angles of a triangle)

$\Rightarrow \angle BIC + 35° + 35° = 180°$

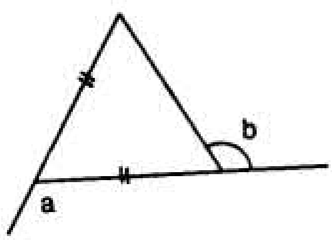
$\Rightarrow \angle BIC + 70° = 180°$

$\Rightarrow \quad \angle BIC = 180° - 70° = 110°$

Hence $\quad \angle BIC = 110°$

Question 11.

In the given figure, express a in terms of b.



Solution:

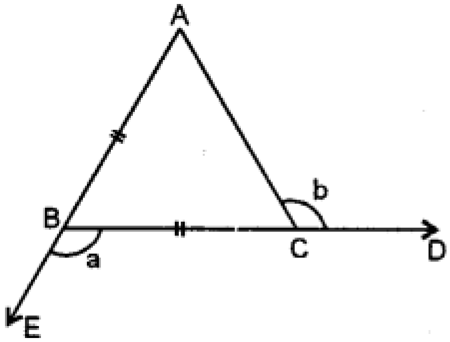
In Δ ABC,

$BC = BA$

$\therefore \angle BCA = \angle BAC$

and Ext. $\angle CBE = \angle BCA + \angle BAC$

$\Rightarrow \quad a = \angle BCA + \angle BCA$



$\Rightarrow a = 2\,\angle BCA$ ...(i)

But $\angle ACB = 180° - b$

($\because$ $\angle ACD$ and $\angle ACB$ are linear pair)

$\Rightarrow \angle BCA = 180° - b$ ...(ii)

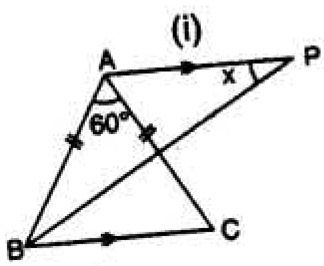
$\therefore\ a = 2\,\angle BCA = 2\,(180° - b)$

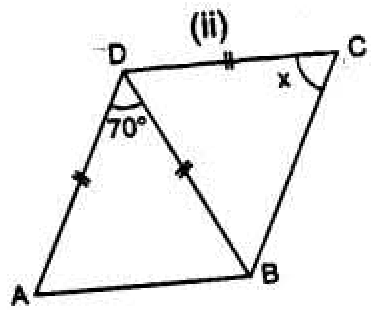
$= 360° - 2b$

Question 12.

(a) In Figure (i) BP bisects ∠ABC and AB = AC. Find x.

(b) Find x in Figure (ii) Given: DA = DB = DC, BD bisects ∠ABC and∠ADB = 70°.





Solution:

(a) In figure (i),

AB = AC, and BP bisects $\angle ABC$

AP||BC is drawn.

Now $\angle PBC = \angle PBA$

($\because$ PB is the bisector of $\angle ABC$)

$\because$ AP||BC

$\therefore\ \angle APB = \angle PBC$ (Alternate angles)

$\Rightarrow x = \angle PBC$ ...(i)

In Δ ABC, $\angle A = 60°$

and $\angle B = \angle C$ ($\because$ AB = AC)

But $\angle A + \angle B + \angle C = 180°$

(Angles of a triangle)

$\Rightarrow\ 60° + \angle B + \angle C = 180°$

$\Rightarrow \quad 60° + \angle B + \angle B = 180°$

$\Rightarrow 2\angle B = 180° - 60° = 120°$

$\therefore \quad \angle B = \frac{120°}{2} = 60°$

$\Rightarrow \frac{1}{2}\angle B = \frac{60°}{2} = 30° \quad \Rightarrow \quad \angle PBC = 30°$

$\therefore$ From (*i*),

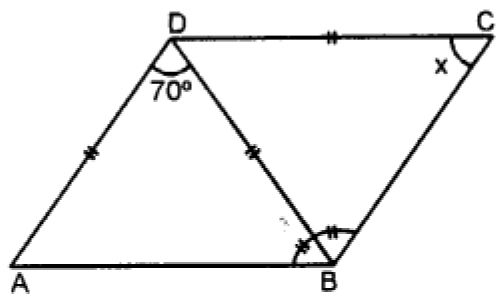
$x = 30°$

(*b*) In the figure (*ii*),

DA = DB = DC

BD bisects $\angle ABC$

and $\angle ADB = 70°$



But $\angle ADB + \angle DAB + \angle DBA = 180°$

(Angles of a triangle)

$\Rightarrow 70° + \angle DBA + \angle DBA = 180°$

($\because$ DA = DB)

$\Rightarrow 70° + 2\angle DBA = 180°$

$\Rightarrow \quad 2\angle DBA = 180° - 70°$

$= 110°$

$\therefore \angle DBA = \frac{110°}{2} = 55°$

$\because$ BD is the bisector of $\angle ABC$,

$\therefore \angle DBA = \angle DBC, = 55°$

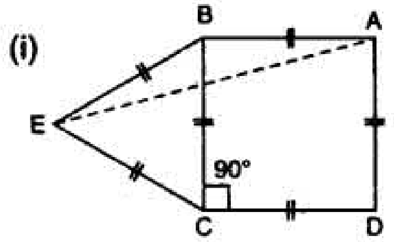
But in Δ DBC,

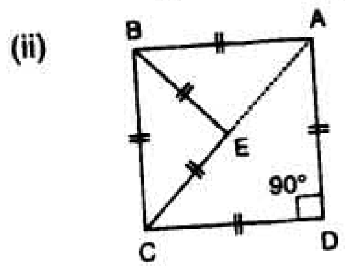
DB = DC

$\therefore \angle DCB = \angle DBC$

$\Rightarrow \quad x = 55°$.

Question 13.
In each figure, given below, ABCD is a square and △ BEC is an equilateral triangle.
Find, in each case : (i) ∠ABE(ii) ∠BAE





Solution:

We know that the sides of a square are equal and each angle is of 90°

Three sides of an equilateral triangle are equal and each angle is of 60°. Therefore,

In fig. (*i*), ABCD is a square and Δ BEC is an equilateral triangle.

(*i*) $\angle ABE = \angle ABC + \angle CBE$
$= 90° + 60° = 150°$

(*ii*) But in Δ ABE
$\angle ABE + \angle BEA + \angle BAE = 180°$
(Angles of a triangle)
$\Rightarrow 150° + \angle BAE + \angle BAE = 180°$
$(\because AB = BE)$
$\Rightarrow 150° + 2\angle BAE = 180°$
$\Rightarrow 2\angle BAE = 180° - 150° = 30°$
$\therefore \angle BAE = \frac{30°}{2} = 15°$

In figure (*ii*),
∵ ABCD is a square and Δ BEC is an equilateral triangle,

(*i*) $\therefore \angle ABE = \angle ABC - \angle CBE$
$= 90° - 60° = 30°$

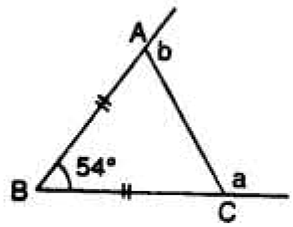
(*ii*) In Δ ABE, $\angle ABE + \angle AEB + \angle BAE = 180°$
(Angles of a triangle)
$\Rightarrow 30° + \angle BAE + \angle BAE = 180°$
$(\because AB = BE)$

$\Rightarrow 30° + 2\angle BAE = 180°$

$\Rightarrow 2\angle BAE = 180° - 30° = 150°$

$\Rightarrow \quad \angle BAE = \frac{150°}{2} = 75°$

Question 14.
In $\triangle$ ABC, BA and BC are produced. Find the angles a and h. if AB = BC.



Solution:
In Δ ABC, sides BA and BC are produced

$\angle ABC = 54°$; $AB = BC$

Now in Δ ABC,

$\angle BAC + \angle BCA + \angle ABC = 180°$

(Angles of a triangle)

$\Rightarrow \angle BAC + \angle BAC + 54° = 180°$ ($\because$ AB = BC)

$\Rightarrow 2\angle BAC = 180° - 54°$

$\Rightarrow 2\angle BAC = 126°$

$\therefore \quad \angle BAC = \frac{126°}{2} = 63°$ and $\angle BCA = 63°$

$\angle BAC + b = 180°$ (Linear pair)

$\Rightarrow 63° + b = 180°$

$\Rightarrow \quad b = 180° - 63° = 117°$

and $\angle BCA + a = 180°$ (Linear pair)

$\therefore 63° + a = 180°$

$\Rightarrow a = 180° - 63° = 117°$

Hence $a = 117°$, $b = 117°$

EXERCISE 15 (C)

Question 1.

Construct a ∆ABC such that:

(i) AB = 6 cm, BC = 4 cm and CA = 5.5 cm

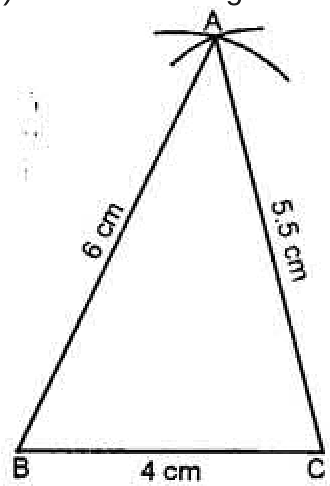
(ii) CB = 6.5 cm, CA = 4.2 cm and BA = 51 cm

(iii) BC = 4 cm, AC = 5 cm and AB = 3.5 cm

Solution:

(i) Steps of Construction :

(i) Draw a line segment BC = 4 cm.



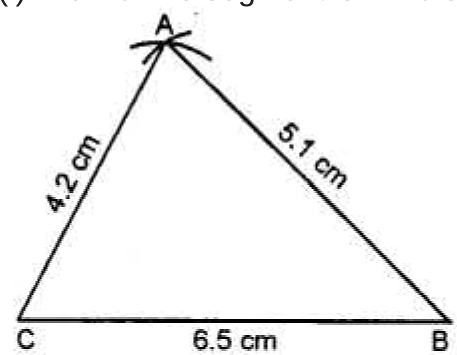
(ii) With centre B and radius 6 cm draw an arc.

(iii) With centre C and radius 5.5 cm, draw another arc intersecting the First are at A.

(iv) Join AB and AC. ∆ABC is the required triangle.

(ii) Steps of Construction :

(i) Draw a line segment CB = 6 5 cm



(ii) With centre C and radius 4.2 cm draw an arc.

(iii) With centre B and radius 5.1 cm draw another arc intersecting the first arc at A.

(iv) Join AC and AB.

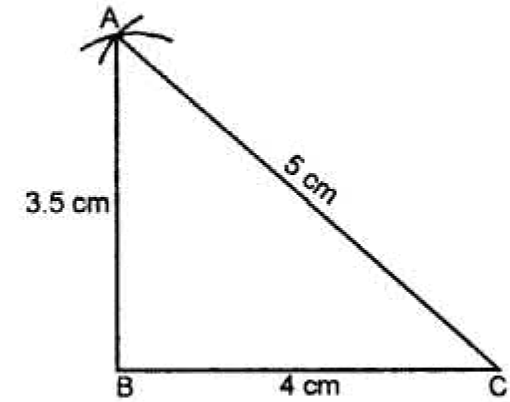
∆ ABC is the required triangle.

(iii) Steps of Construction :

(i) Draw a line segment BC = 4 cm.

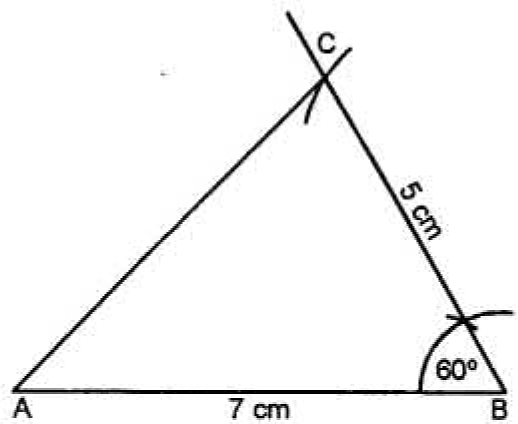
(ii) With centre B and radius 3.5 cm, draw an arc

(iii) With centre C and radius 5 cm, draw another arc which intersects the first arc at A.

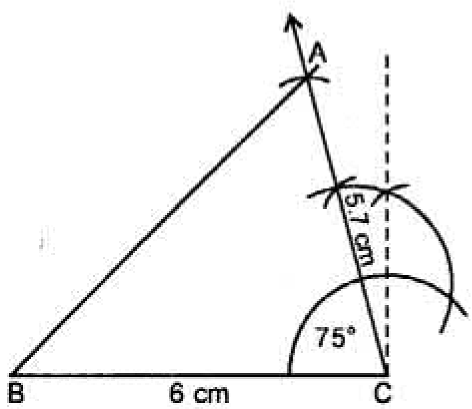


(iv) Join AB and AC.
Δ ABC is the required triangle.

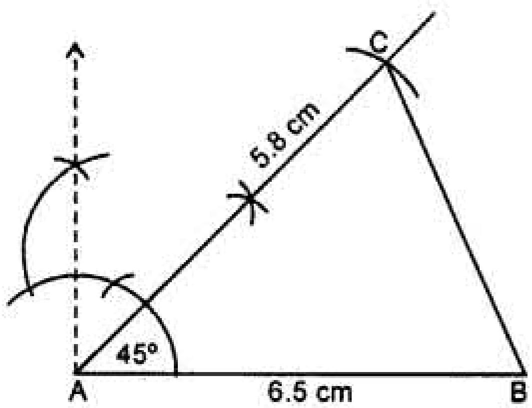
Question 2.
Construct a A ABC such that:
(i) AB = 7 cm, BC = 5 cm and ∠ABC = 60°
(ii) BC = 6 cm, AC = 5.7 cm and ∠ACB = 75°
(iii) AB = 6.5 cm, AC = 5.8 cm and ∠A = 45°
Solution:
(i) Steps of Construction :
(i) Draw a line segment AB = 7 cm.



(ii) At B, draw a ray making an angle of 60° and cut off BC = 5 cm
(iii) Join AC,
ΔABC is the required triangle.
(ii) Steps of Construction :
(i) Draw a line segment BC = 6 cm.
(ii) At C, draw a ray making an angle of 75° and cut off CA = 5.7 cm.
(iii) JoinAB
Δ ABC is the required triangle.

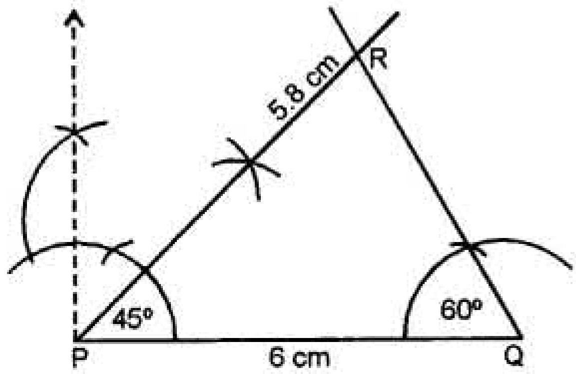


(iii) Steps of Construction :
(i) Draw a line segment AB = 6.5 cm



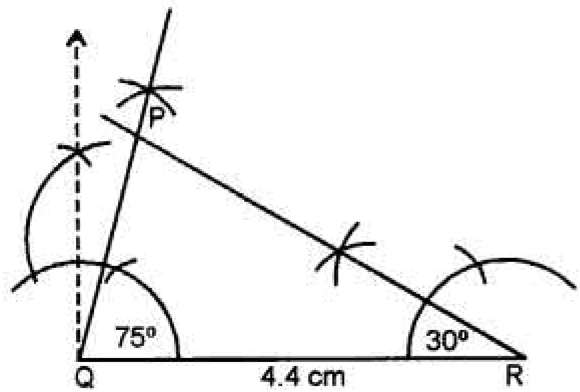
(ii) At A, draw a ray making an angle of 45° and cut off AC = 5.8 cm
(iii) JoinCB.
Δ ABC is the required triangle.

Question 3.
Construct a Δ PQR such that :
(i) PQ = 6 cm, ∠Q = 60° and ∠P = 45°. Measure ∠R.
(ii) QR = 4.4 cm, ∠R = 30° and ∠Q = 75°. Measure PQ and PR.
(iii) PR = 5.8 cm, ∠P = 60° and ∠R = 45°.
Measure ∠Q and verify it by calculations
Solution:
(i) Steps of Construction:
(i) Draw a line segment PQ = 6 cm.
(ii) At P, draw a ray making an angle of 45°
(iii) At Q, draw another ray making an angle of 60° which intersects the first ray at R.
Δ PQR is the required triangle.
On measuring ∠R, it is 75°.



(ii) Steps of Construction :

(i) Draw a line segment QR = 44 cm.



(ii) At Q, draw a ray making an angle of 75°

(iii) At R, draw another arc making an angle of 30° ; which intersects the first ray at R

∆ PQR is the required triangle.

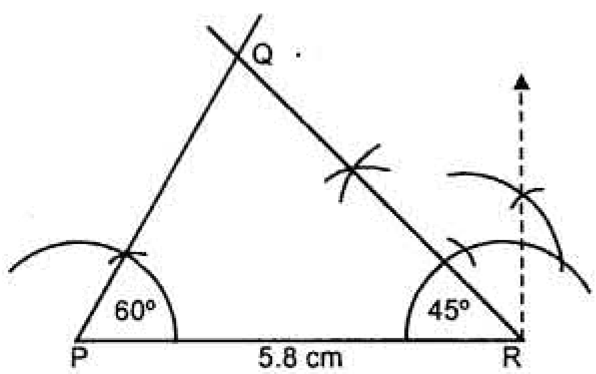
On measuring the lengths of PQ and PR, PQ = 2.1 cm and PR = 4. 4 cm.

(iii) Steps of Construction :

(i) Draw a line segment PR = 5.8 cm

(ii) At P, construct an angle of 60°

(iii) At R, draw another angle of 45° meeting each other at Q.



∆ PQR is the required triangle. On measuring ∠Q, it is 75°

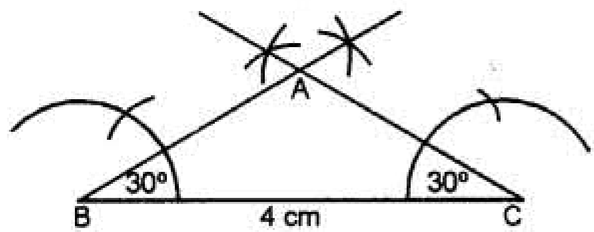
Verification : We know that sum of angles of a triangle is 180°

$\therefore \angle P + \angle Q + \angle R = 180°$

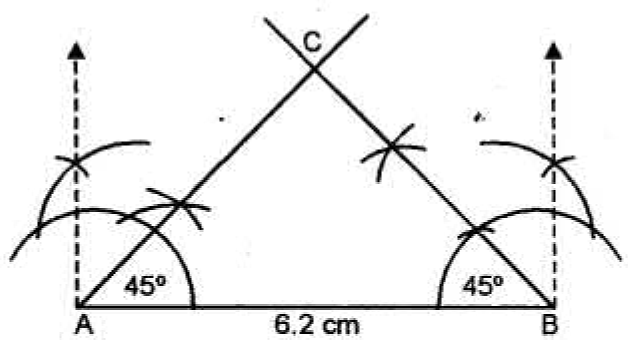
$\Rightarrow 60° + \angle Q + 45° = 180°$

⇒ ∠Q + 105° = 180°
⇒ ∠Q = 180° – 105° = 75°.

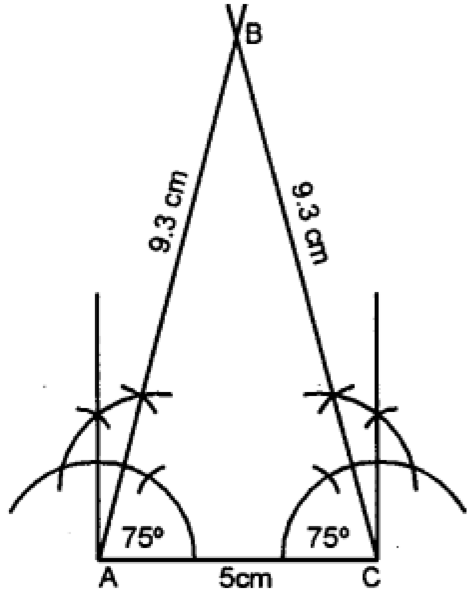
Question 4.
Construct an isosceles A ABC such that:
(i) base BC = 4 cm and base angle = 30°
(ii) base AB = 6-2 cm and base angle = 45°
(iii) base AC = 5 cm and base angle = 75°.
Measure the other two sides of the triangle.
Solution:
(i) Steps of Construction :
We know that in an isosceles triangle base angles are equal.



(i) Draw a line segment BC = 4 cm.
(ii) At B and C, draw rays making an angle of 30° each intersecting each other at A.
∆ ABC is the required triangle.
On measuring the equal sides each is 2.5 cm (approx.) in length.
(ii) Steps of Construction :
We know that in an isosceles triangle, base angles are equal.

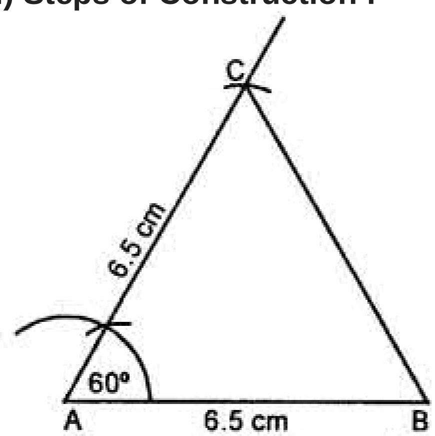


(i) Draw a line segment AB = 6.2 cm
(ii) At A and B, draw rays making an angle of 45° each which intersect each other at C.
∆ABC is the required triangle.
On measuring the equal sides, each is 4.3 cm (approx.) in length.
(iii) Steps of Construction :
We know that base angles of an isosceles triangles are equal.

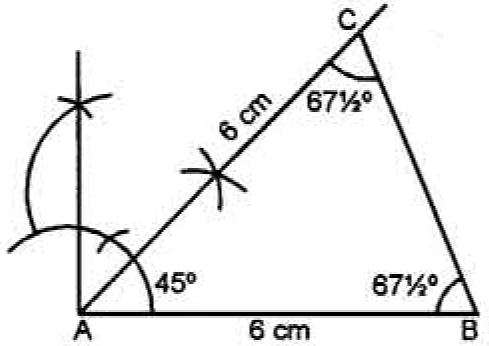


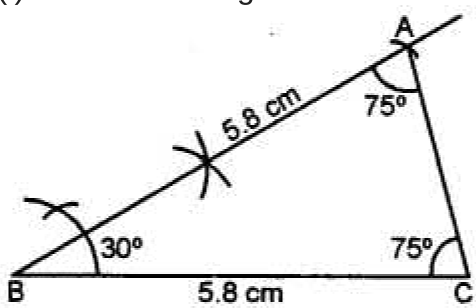
(i) Draw a line segment AC = 5cm.
(ii) At A and C, draw rays making an angle of 75° each which intersect each other at B.
∆ ABC is the required triangle.
On measuring the equal sides, each is 9.3 cm in length.

Question 5.
Construct an isosceles ∆ABC such that:
(i) AB = AC = 6.5 cm and ∠A = 60°
(ii) One of the equal sides = 6 cm and vertex angle = 45°. Measure the base angles.
(iii) BC = AB = 5-8 cm and ZB = 30°. Measure ∠A and ∠C.
Solution:
(i) Steps of Construction :



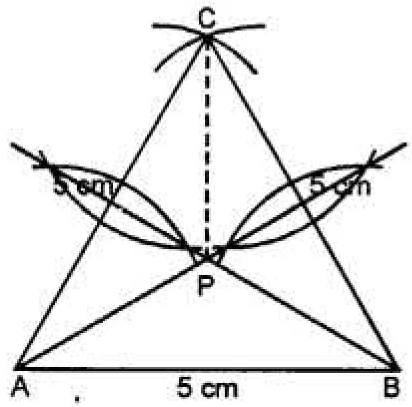
(i) Draw a line segment AB = 6.5 cm.
(ii) At A, draw a ray making an angle of 60°.
(iii) Cut off AC = 6.5 cm
(iv) JoinBC.

∆ABC is the required triangle.
(ii) Steps of Construction :
(i) Draw a line segment AB = 6 cm


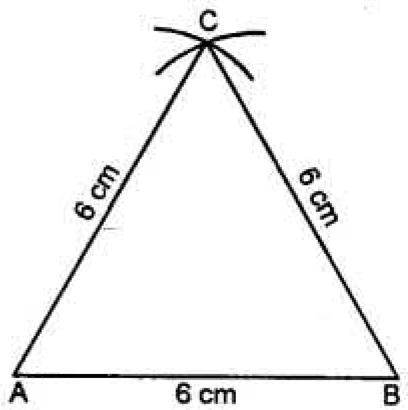
(ii) At A, construct an angle equal to 45°
(iii) Cut off AC = 6 cm
(iv) JoinBC.
∆ ABC is the required triangle.
On measuring, ∠B and ∠C, each is equal 1° to, $67\frac{1}{2}$°
(iii) Steps of Construction :
(i) Draw a line segment BC = 5.8 cm


(ii) At B, draw a ray making an angle of 30°.
(iii) Cut off BA = 5.8 cm
(iv) Join AC.
∆ ABC is the required triangle On measuring ∠C and ∠A, each is equal to 75°.

Question 6.
Construct an equilateral A ABC such that:
(i) AB = 5 cm. Draw the perpendicular bisectors of BC and AC. Let P be the point of intersection of these two bisectors. Measure PA, PB and PC.
(ii) Each side is 6 cm.
Solution:
(i) Steps of Construction :
(i) Draw a line segment AB = 5 cm.



(ii) With centres A and B and radius 5 cm each, draw two arcs intersecting each other at C.
(iii) Join AC and BC $\triangle ABC$ is the required triangle.
(iv) Draw the perpendicular bisectors of sides AC and BC which intersect each other at P-
(v) Join PA, PB and PC.
On measuring, each is 2.8 cm.
(ii) Steps of Construction :
(i) Draw a line segment AB = 6 cm.

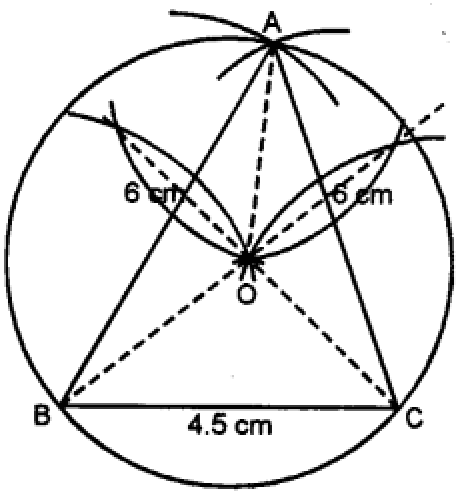


(ii) At A and B as centre and 6 cm as radius draw two arcs intersecting each other at C.
(iii) Join AC and BC.
$\triangle ABC$ is the required triangle.

Question 7.
(i) Construct a $\triangle$ ABC such that AB = 6 cm, BC = 4.5 cm and AC = 5.5 cm. Construct a circumcircle of this triangle.
(ii) Construct an isosceles $\triangle PQR$ such that PQ = PR = 6.5 cm and $\angle PQR = 75°$. Using ruler and compasses only construct a circumcircle to this triangle.
(iii) Construct an equilateral triangle ABC such that its one side = 5.5 cm. Construct a circumcircle to this triangle.
Solution:

(*i*) Steps of Construction :

(*i*) Draw a line segment BC = 4·5 cm



(*ii*) With centre B and radius 6 cm, draw are arc

(*iii*) With centre C and radius 5·5 cm, draw another arc intersecting the first arc at A.

(*iv*) Join AB and AC.

Δ ABC is the required triangle.

(*v*) Draw the perpendicular bisectors of AB and AC. Which intersect each other at O.

(*vi*) Join OB, OC and OA.

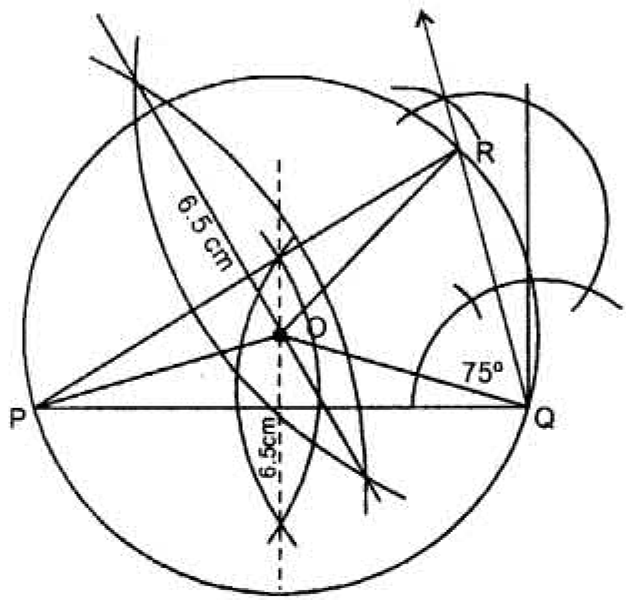
(*vii*) With centre O, and radius OA, draw a circle which passes through A, B and C.

This is the required circum circle of Δ ABC.

(*ii*) Steps of Construction :

(*i*) Draw a line segment PQ = 6.5 cm

(*ii*) At Q, draw a ray making an angle of 75°.



(*iii*) Through P, with a radius of 6.5 cm, draw an arc which intersects the angle ray at R.

(*iv*) Join PR,

Δ PQR is the required triangle.

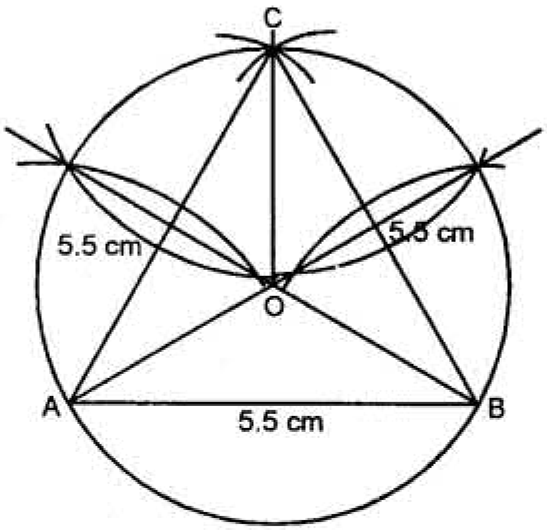
(*v*) Draw the perpendicular bisectors of sides PQ and PR intersecting each other at O.

(*vi*) Join OP, OQ and OR.

(*vii*) With centre O and radius equal to OP or OQ or OR draw a circle which passes through P,Q and R. This is the required circum circle of Δ PQR

(*iii*) Steps of Construction :

(*i*) Draw a line segment AB = 5·5 cm



(*ii*) With centres A and B and radius 5·5 cm, draw two arcs intersecting each other at C.

(*iii*) Join AC and BC.

ΔABC is the required triangle.

(*iv*) Draw perpendicular bisectors of sides AC and BC which intersect each other at O.

(*v*) Join OA, OB and OC.

(*vi*) With centre O and radius OA or OB or OC, draw a circle which passes through A, B and C. This is the required circumcircle.

Question 8.

(i) Construct a $\triangle$ABC such that AB = 6 cm, BC = 5.6 cm and CA = 6.5 cm. Inscribe a circle to this triangle and measure its radius.

(ii) Construct an isosceles $\triangle$ MNP such that base MN = 5.8 cm, base angle MNP = 30°. Construct an incircle to this triangle and measure its radius.

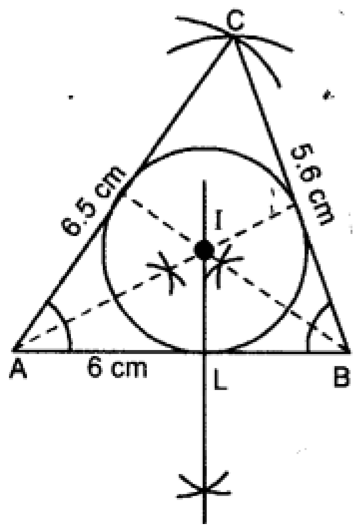
(iii) Construct an equilateral $\triangle$DEF whose one side is 5.5 cm. Construct an incircle to this triangle.

(iv) Construct a $\triangle$ PQR such that PQ = 6 cm, $\angle$QPR = 45° and angle PQR = 60°. Locate its incentre and then draw its incircle.

Solution:

(i) Steps of Construction :

(i) Draw a line segment AB = 6 cm.



(ii) With centre A and radius 6·5 cm and with centre B and radius 5·6 cm, draw arcs intersecting each other at C.

(iii) Join AC and BC.

(iv) Draw the angle bisector of ∠A and ∠B intersecting each other at I.

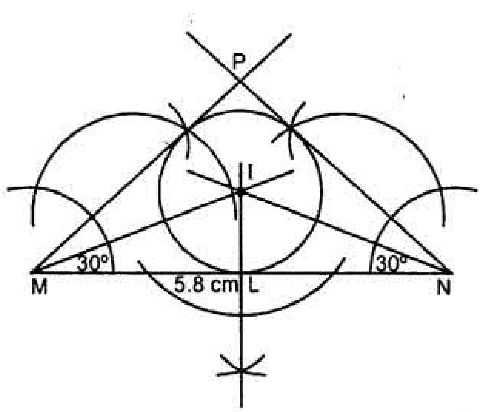
(v) From I, draw I L ⊥ AB

(vi) With centre I and radius IL, draw a circle which touches the sides of Δ ABC internally.

On measuring the required incircle whose radius is 1.6 cm.

(ii) Steps of Construction :

(i) Draw a line segment MN = 5.8 cm.



(*ii*) At M and N, draw two rays making an angle of 30° each which intersect each other at P.

(*iii*) Now draw the angle bisectors of $\angle M$ and $\angle N$ which intersect each other at I.

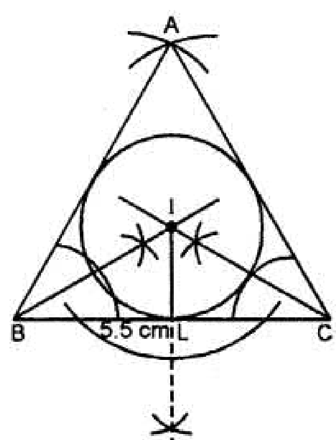
(*iv*) From I, draw perpendicula - IL on MN.

(*v*) With centre I and radius IL, draw a circle which touches the sides of the Δ PMN internally.

On measuring the required incircle and its radius is 0·6 cm.

(*iii*) Steps of Construction :

(*i*) Draw a lines segment BC = 5·5 cm.



(*ii*) With centres B and C and radius 5·5 cm each draw two arcs intersecting each other at A.

(*iii*) Join AB and AC.

(*iv*) Draw the perpendicular bisectors of $\angle B$ and $\angle C$ intersecting each other at I.

(*v*) From I, draw IL $\perp$ BC

(*vi*) With centre I and radius IL, draw a circle which touches the sides of the Δ ABC internally.

This is the required incircle.

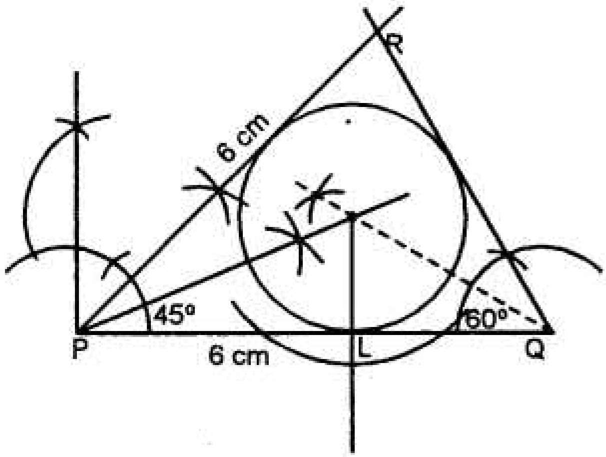
(iv) Steps of Construction :

(*i*) Draw a line segment PQ = 6 cm.

(*ii*) At P draw a rays making an angle of 45° and at Q, making an angle of 60°, intersecting each other at R.

(*iii*) Draw the bisectors of ∠P and ∠Q intersecting each other at I.

(*iv*) From I, draw IL ⊥ PQ.



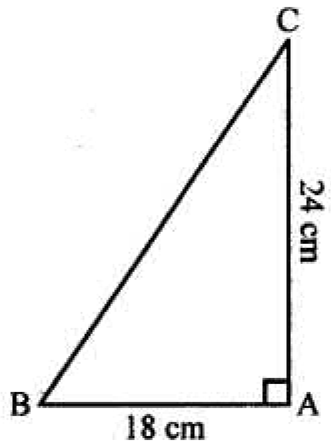
(*v*) With centre I and radius IL, draw a circle which touches the sides of Δ PQR internally. This is the required incircle whose I is incentre.

CHAPTER - 16
PYTHAGORAS THEOREM

EXERCISE 16

Question 1.
Triangle ABC is right-angled at vertex A. Calculate the length of BC, if AB = 18 cm and AC = 24 cm.

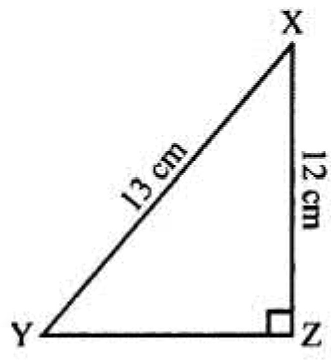
Solution:
Given : ΔABC right angled at A and AB = 18 cm, AC = 24 cm.



To find : Length of BC.
According to Pythagoras Theorem,
$BC^2 = AB^2 + AC^2$
$= 18^2 + 24^2 = 324 + 576 = 900$
$\therefore BC = \sqrt{900} = \sqrt{30x30} = 30$ cm

Question 2.
Triangle XYZ is right-angled at vertex Z. Calculate the length of YZ, if XY = 13 cm and XZ = 12 cm.

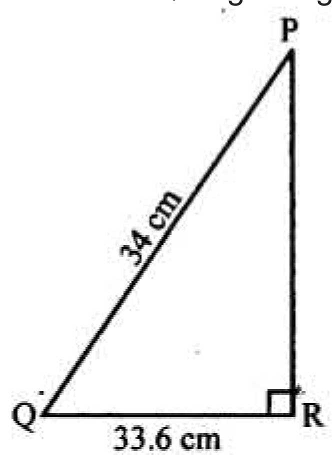
Solution:
Given : ΔXYZ right angled at Z and XY = 13 cm, XZ = 12 cm.
To find : Length of YZ.
According to Pythagoras Theorem,
$XY^2 = XZ^2 + YZ^2$
$13^2 = 12^2 + YZ^2$



$169 = 144 + YZ^2$
$169 - 144 = YZ^2$
$25 = YZ^2$
$\therefore YZ = \sqrt{25}$cm $\sqrt{5x5}$ = 5 cm

Question 3.
Triangle PQR is right-angled at vertex R. Calculate the length of PR, if:
PQ = 34 cm and QR = 33.6 cm.

Solution:
Given : ΔPQR right angled at R and PQ = 34 cm, QR = 33.6 cm.

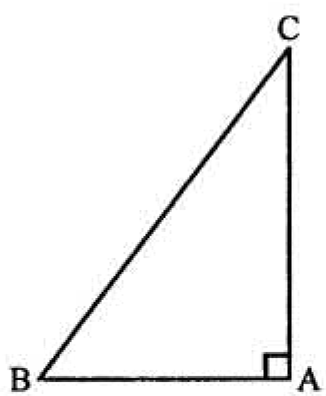


To find : Length of PR.
According to Pythagoras Theorem,
$PR^2 + QR^2 = PQ^2$
$PR^2 + 33.6^2 = 34^2$
$PR^2 + 1128.96 = 1156$
$PR^2 = 1156 - 1128.96$
$\therefore PR = \sqrt{27.04}$ = 5.2 cm

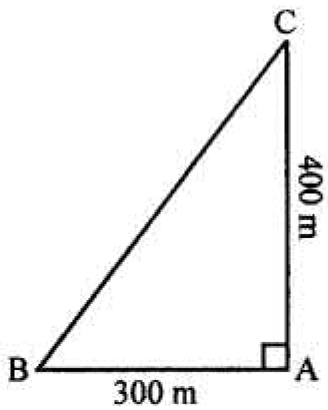
Question 4.
The sides of a certain triangle are given below. Find, which of them is right-triangle
(i) 16 cm, 20 cm and 12 cm
(ii) 6 m, 9 m and 13 m

Solution:
(i) 16 cm, 20 cm and 12 cm
The given triangle will be a right-angled triangle if square of its largest side is equal to the sum of the squares on the other two sides.
i.e., If $(20)^2 = (16)^2 = (12)^2$
$(20)^2 = (16)^2 + (12)^2$
$400 = 256 + 144$
$400 = 400$
So, the given triangle is right angled.
(ii) 6 m, 9 m and 13 m
The given triangle will be a right-angled triangle if square of its largest side is equal to the sum of the squares on the other two sides.
i.e., If $(13)^2 = (9)^2 + (6)^2$
$169 = 81+36$ $169 \neq 117$
So, the given triangle is not right angled.

Question 5.
In the given figure, angle BAC = 90°, AC = 400 m and AB = 300 m. Find the length of BC.



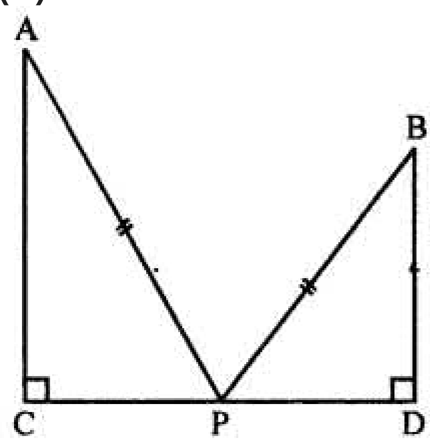
Solution:
AC = 400 m
AB = 300 m
BC = ?



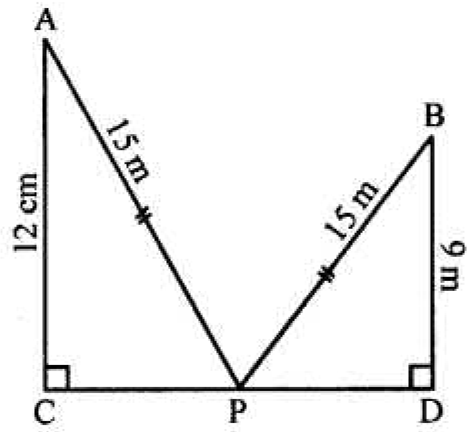
According to Pythagoras Theorem,
$BC^2 = AB^2 + AC^2$
$BC^2 = (300)^2 + (400)^2$
$BC^2 = 90000 + 160000$
$BC^2 = 250000$
$BC = \sqrt{250000} = 500$ m

Question 6.
In the given figure, angle ACP = ∠BDP = 90°, AC = 12 m, BD = 9 m and PA= PB = 15 m. Find:

(i) CP
(ii) PD
(iii) *CD*



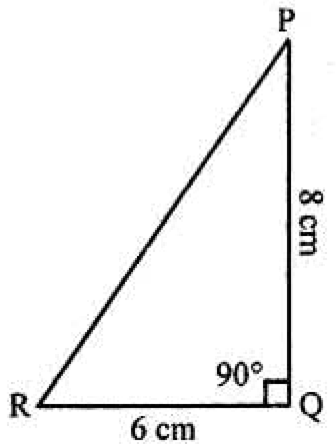
Solution:
Given : AC = 12 m
BD = 9 m
PA = PB= 15 m



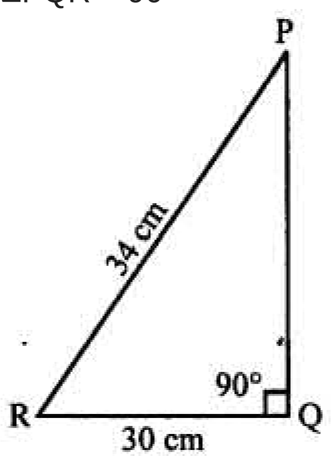
(i) In right angle triangle ACP
$(AP)^2 = (AC)^2 + (CP)^2$
$15^2 = 12^2 + CP^2$
$225 = 144 + CP^2$
$225 – 144 = CP^2$
$81 = CP$
$\sqrt{81} = CP$
∴ CP = 9 m
(ii) In right angle triangle BPD
$(PB)^2 = (BD)^2 + (PD)^2$
$(15)^2 = (9)^2 + PD^2$
$225 = 81 + PD^2$
$225-81 = PD^2$
$144 = PD^2$
$\sqrt{144} = PD$
∴ PD = 12 m
(iii) CP = 9 m
PD = 12 m
∴ CD = CP + PD
= 9+ 12 = 21 m

Question 7.
In triangle PQR, angle Q = 90°, find :
(i) PR, if PQ = 8 cm and QR = 6 cm
(ii) PQ, if PR = 34 cm and QR = 30 cm

Solution:
(i) Given:
PQ = 8 cm
QR = 6 cm
PR = ?
∠PQR = 90°



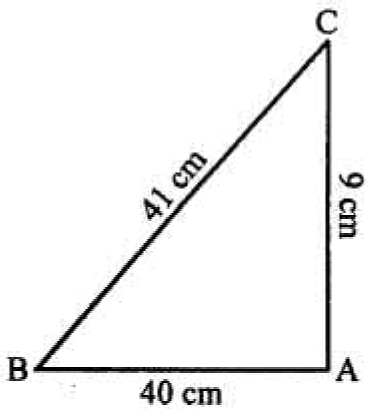
According to Pythagoras Theorem,
$(PR)^2 = (PQ)^2 + (QR)^2$
$PR^2 = 8^2 + 6^2$
$PR^2 = 64 + 36$
$PR^2 = 100$
$\therefore PR = \sqrt{100} = 10$ cm
(ii) Given :
PR = 34 cm
QR = 30 cm
PQ = ?
$\angle PQR = 90°$



According to Pythagoras Theorem,
$(PR)^2 = (PQ)^2 + (QR)^2$
$(34)^2 = PQ^2 + (30)^2$
$1156 = PQ^2 + 900$
$1156-900 = PQ^2$
$256 = PQ^2$
$\therefore PQ = 16$ cm

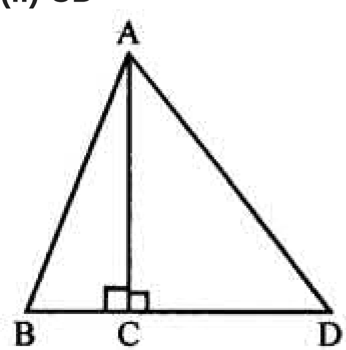
Question 8.
Show that the triangle ABC is a right-angled triangle; if:
AB = 9 cm, BC = 40 cm and AC = 41 cm

Solution:
AB = 9 cm
CB = 40 cm
AC = 41 cm



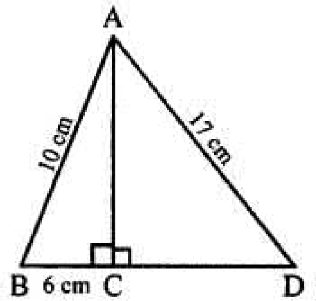
The given triangle will be a right angled triangle if square of its largest side is equal to the sum of the squares on the other two sides.
According to Pythagoras Theorem,
$(AC)^2 = (BC)^2 + (AB)^2$
$(41)^2 = (40)^2 + (9)^2$
1681 = 1600 + 81
1681 = 1681
Hence, it is a right-angled triangle ABC.

Question 9.
In the given figure, angle ACB = 90° = angle ACD. If AB = 10 m, BC = 6 cm and AD = 17 cm, find :
(i) AC
(ii) CD



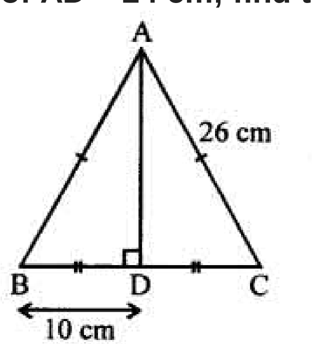
Solution:
Given:

△ABD
∠ACB = ∠ACD = 90°
and AB = 10 cm, BC = 6 cm and AD = 17 cm



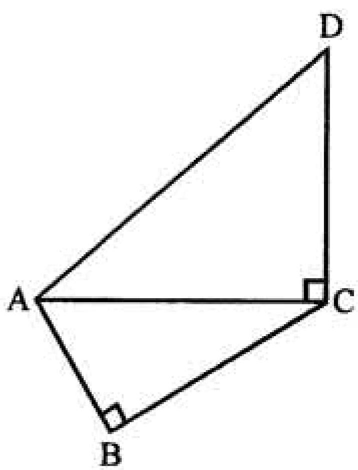
To find:
(i) Length of AC
(ii) Length of CD
Proof:
(i) In right-angled triangle ABC
BC = 6 cm, AB = 110 cm
According to Pythagoras Theorem,
$AB^2 = AC^2 + BC^2$
$(10)^2 = (AC)^2 + (6)^2$
$100 = (AC)^2 + 36$
$AC^2 = 100-36 = 64$ cm
$AC^2 = 64$ cm
∴ AC = $\sqrt{8x8}$ = 8 cm
(ii) In right-angle triangle ACD
AD = 17 cm, AC = 8 cm
According to Pythagoras Theorem,
$(AD)^2 = (AC)^2 + (CD)^2$
$(17)^2 = (8)^2 + (CD)^2$
$289 – 64 = CD^2$
$225 = CD^2$
CD =$\sqrt{15x15}$ = 15 cm

Question 10.
In the given figure, angle ADB = 90°, AC = AB = 26 cm and BD = DC. If the length of AD = 24 cm; find the length of BC.



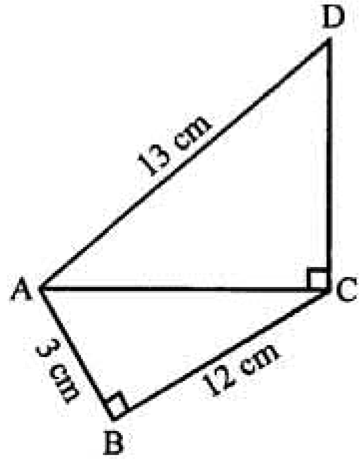
Solution:
Given:
ΔABC
$\angle ADB = 90°$ and $AC = AB = 26$ cm
$AD = 24$ cm
To find : Length of BC In right angled ΔADC
$AB = 26$ cm, $AD = 24$ cm
According to Pythagoras Theorem,
$(AC)^2 = (AD)^2 + (DC)^2$
$(26)^2 = (24)^2 + (DC)^2$
$676 = 576 + (DC)^2$
$\Rightarrow (DC)^2 = 100$
$\Rightarrow DC = \sqrt{100} = 10$ cm
$\therefore$ Length of $BC = BD + DC$
$= 10 + 10 = 20$ cm

Question 11.
In the given figure, AD = 13 cm, BC = 12 cm, AB = 3 cm and angle ACD = angle ABC = 90°. Find the length of DC.



Solution:
Given :
$\Delta ACD = \angle ABC = 90°$
and $AD = 13$ cm, $BC = 12$ cm, $AB = 3$ cm

To find : Length of DC.

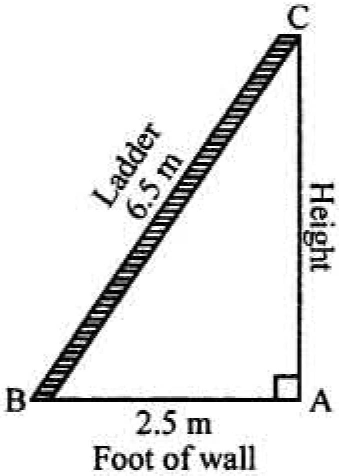


(i) In right angled $\triangle ABC$
AB = 3 cm, BC = 12 cm
According to Pythagoras Theorem,
$(AC)^2 = (AB)^2 + (BC)^2$
$(AC)^2 = (3)^2 + (12)^2$
$(AC) = \sqrt{9+144} = \sqrt{153}$ cm
(ii) In right angled triangle ACD
AD = 13 cm, AC = $\sqrt{153}$
According to Pythagoras Theorem,
$DC^2 = AB^2 - AC^2$
$DC^2 = 169 - 153$
DC = $\sqrt{16}$ = 4 cm
∴ Length of DC is 4 cm

Question 12.

A ladder, 6.5 m long, rests against a vertical wall. Ifthe foot of the ladcler is 2.5 m from the foot of the wall, find upto how much height does the ladder reach?

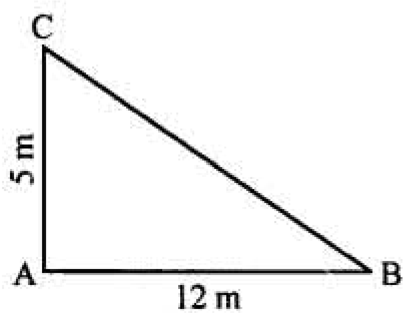
Solution:
Given :
Length of ladder = 6.5 m
Length of foot of the wall = 2.5 m
To find : Height AC According to Pythagoras Theorem,
$(BC)^2 = (AB)^2 + (AC)^2$



$(6.5)^2 = (2.5)^2 + (AC)^2$
$42.25 = 6.25 + AC^2$
$AC^2 = 42.25 – 6.25 = 36$ m
AC = $\sqrt{6x6}$ = 6 m
∴ Height of wall = 6 m

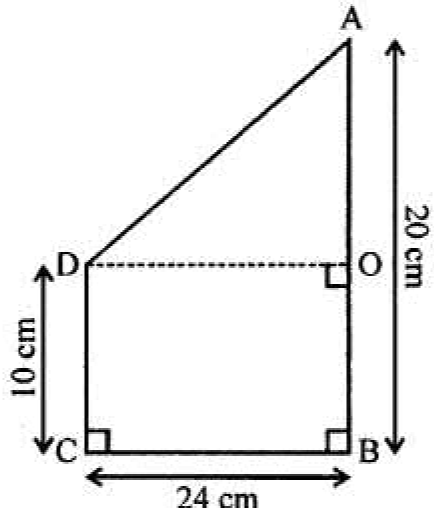
Question 13.
A boy first goes 5 m due north and then 12 m due east. Find the distance between the initial and the final position of the boy.

Solution:
Given : Direction of north = 5 m i.e. AC Direction of east = 12 m i.e. AB



To find: BC
According to Pythagoras Theorem,
In right angled AABC
$(BC)^2 = (AC)^2 + (AB)^2$
$(BC)^2 = (5)^2 + (12)^2$
$(BC)^2 = 25 + 144$
$(BC)^2 = 25 + 144$
$(BC)^2= 169$
∴ BC = $\sqrt{169}$ = $\sqrt{13x13}$ = 13 m

Question 14.
Use the information given in the figure to find the length AD.

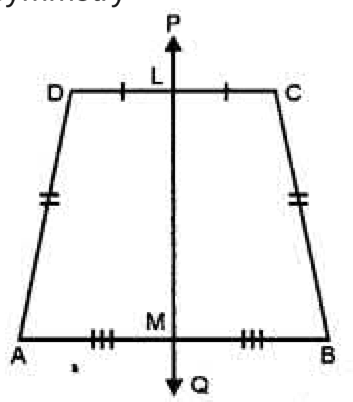


Solution:
Given :
AB = 20 cm
$\therefore$AO = $\frac{AB}{2}$ = $\frac{20}{2}$ =10cm
BC = OD = 24 cm
To find : Length of AD
In right angled triangle
AOD $(AD)^2 = (AO)^2 + (OD)^2$
$(AD)^2 = (10)^2 + (24)^2$
$(AD)^2 = 100 + 576$
$(AD)^2 = 676$
$\therefore$ AD = $\sqrt{26x26}$
AD = 26 cm

CHAPTER - 17
SYMMETRY

POINTS TO REMEMBER

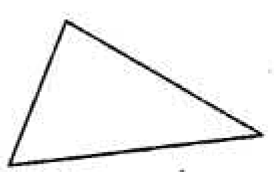
1. Symmetry : A geometrical figure is said to be symmetric about a line if on folding about that line, the two parts of the figure exactly concide each other. The given figure is symmetric about the line PQ. The line is said to be a line of symmetry or an axis of symmetry



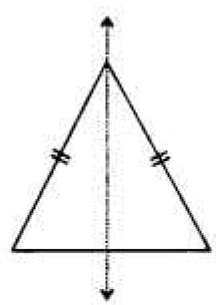
2. Lines of symmetry of given geometrical figures :

It is not necessary that every figure under consideration will definitely have a line symmetry. If we consider different types of triangle ; we find :

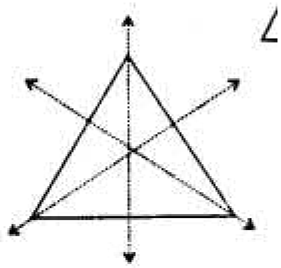
1. A scalene triangle has no line of Symmetry : i.e. we can not have a line in a scalene triangle about which if the figure (triangle) is folded, the two parts of the figure will coincide.



2. An isosceles triangle has only one line of symmetry. The bisector of angle of vertex which is also the perpendicular bisector of its base.

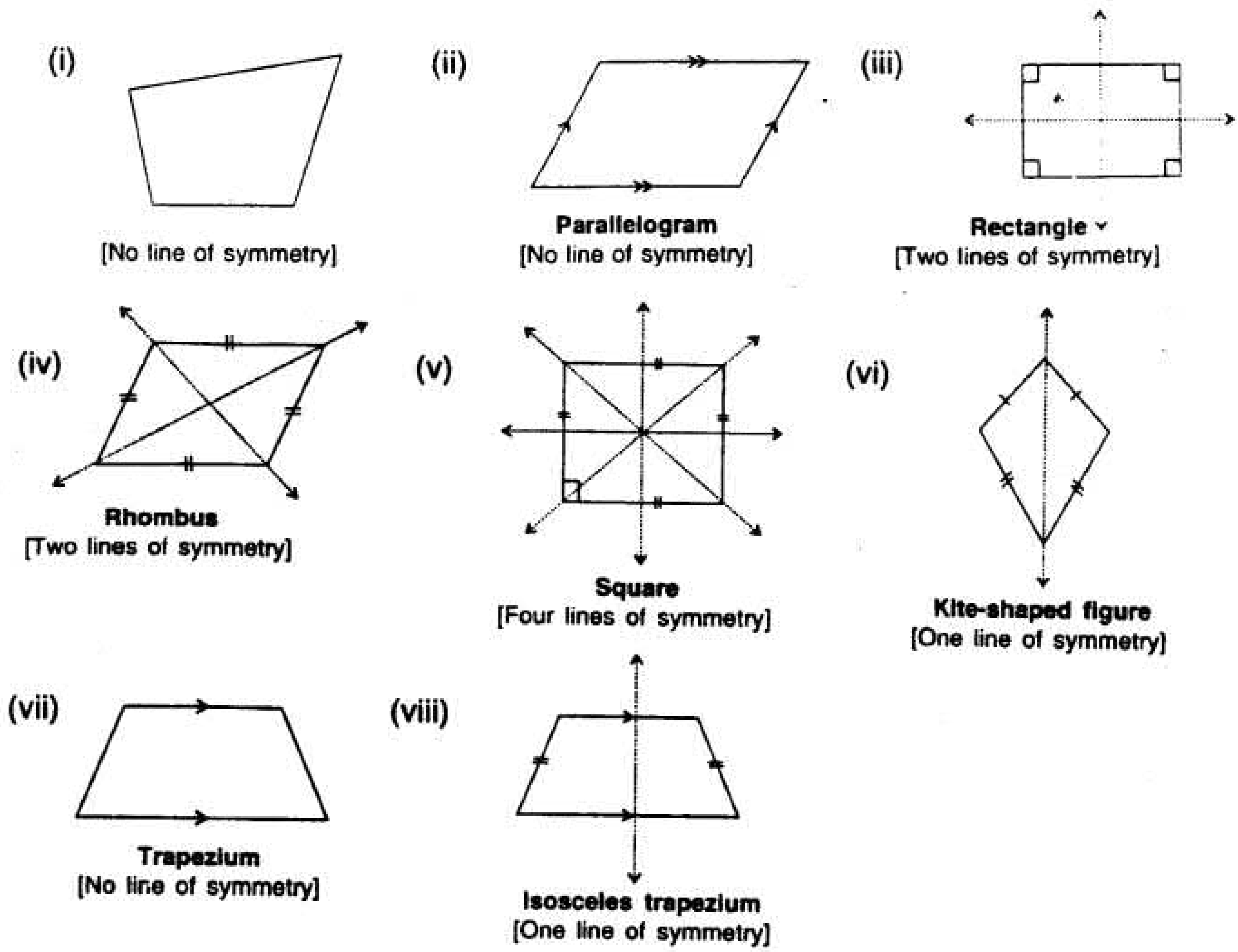


3. An equilateral triangle has three lines of symmetry.



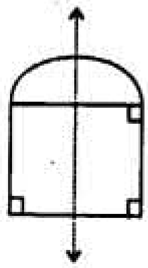
The bisectors of the angle of vertices which are also the perpendicular bisectors of its sides.

4. Line/lincs of symmetry of differed types of quadrilaterals are shown below by dotted lines :

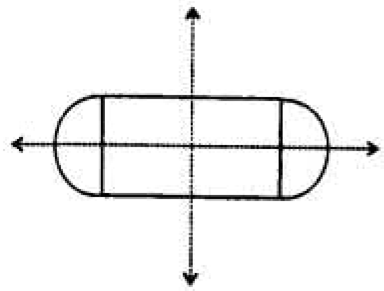


5. In each of the following, the dotted line/lines are the line/lines of symmetry of the given figure:

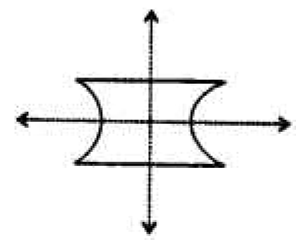
(i)



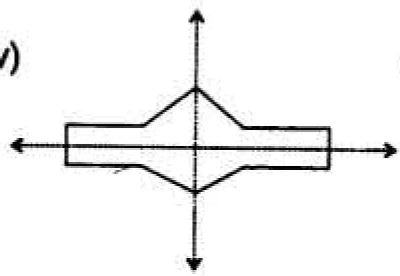
(ii)



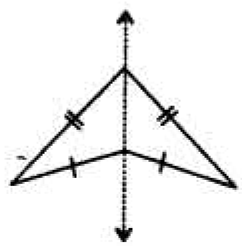
(iii)



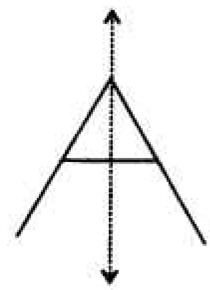
(iv)



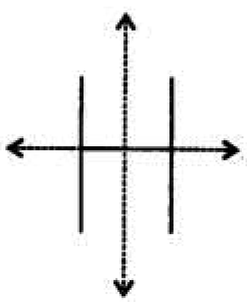
(v)



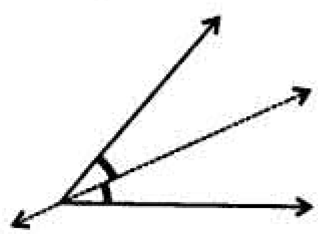
(vi)



(vii)



(vii)



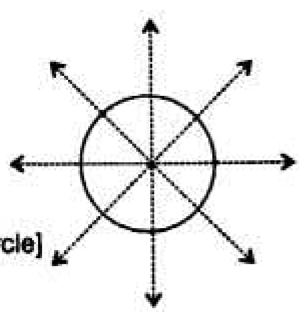
6. As shown below:

(i) a circle has infinite lines of symmetry ; every line through its centre is line of symmetry'.

(ii) a semi-circle has one line of symmetry.

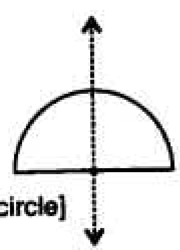
(iii) a quadrant (one-fourth) of a circle has one line of symmetry' and so on.

(i)



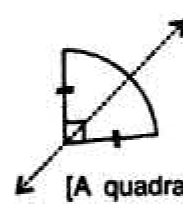
[A circle]

(ii)



[A semi-circle]

(ii)



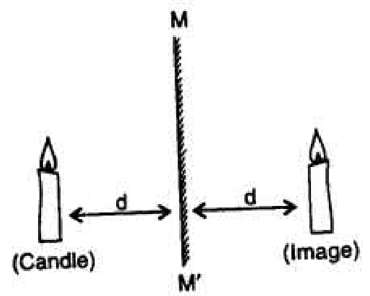
[A quadrant]

Note : It is clear from the question numbers 4 and 5, given above that:

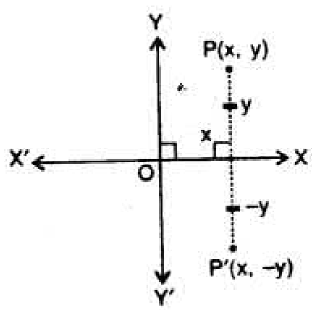
1. The largest number of lines of symmetry' of a triangle is three (3).

2. The largest number of lines of symmetry of a quadrilateral is four (4).

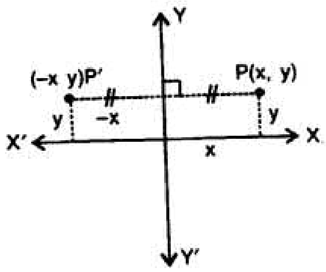
i. e. as the number of sides in a triangle is 3 ; the largest number of lines of symmetry in it is 3 and as the number of sides in a quadrilateral is 4 ; the largest number of lines of symmetry is 4.
In the same way :
1. The largest number of lines of symmetry of a pentagon is 5, as a pentagon has 5 sides.
2. The hexagon has 6 sides and so the largest number of lines of symmetry' of a hexagon is 6.
In general, we can say, that if a polygon has n sides ; M
the largest number of lines of symmetry, it can have, is n.
3. Reflection (Image): Tire given figure shows a candle place 'd' distance before a plane mirror MM', the image of the candle is obtained in the mirror at the same distance 'd' behind the mirror. Geometrically, the line joining the candle (c) and its reflection c' is (Candle) perpendicular bisector of the mirror line MM'.



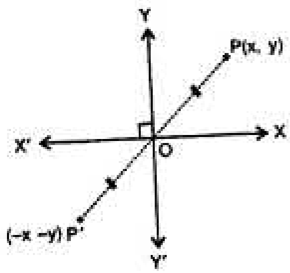
4. Reflection in x-axis : Reflection if x-axis means the x-axis is considered as the plane mirror, the given point as the object and then to find its image.



Let P (x, y) be a point and. as shown in the figure, when it is reflected in x-axis to point P'; the co-ordinates of image point P' are (x, – y).
i. e. reflection of P (x, y) in x-axis = P' (x, – y)
In other words :
Image of P (x, y) in x-axis = P' (x, – y)
We can say, when a point (x, y) is reflected in x-axis, the sign of its second component (ordinate) changes i.e. tire sign of y changes and so the image of (x, y) in x-axis is (x, – y).
5. Reflection in y-axis : As is clear from the figure, given alongside, the reflection P (x, y) in y-axis is point P' (- x, y).
We can say, when a point (x,y) is reflected in y-axis, the sign of its first component (abscissa) change i.e. the sign of x changes and so the image of (x, y) in y- axis is (- x,y).



6. Reflection in Origin : When a point P (x, y) is reflected in origin, the sign of both of its components change i.e. the image of P (x, y) is P' (- x, -y) as shown along side in the figure.

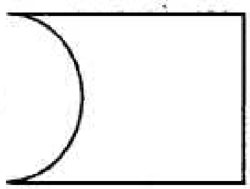


EXERCISE 17 (A)

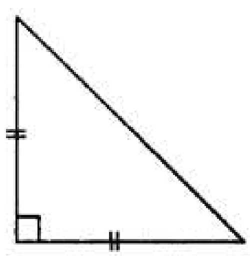
Question 1.

For each figure, given below, draw the line (s) of symmetry, if possible :

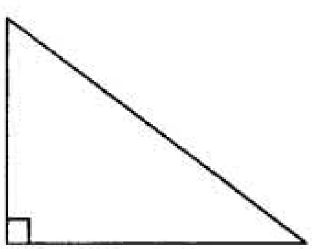
(i)

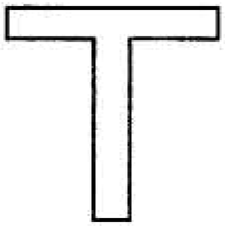


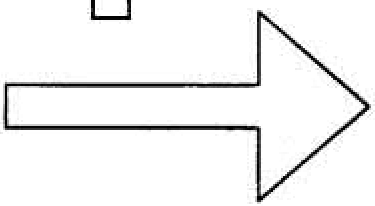
(ii)

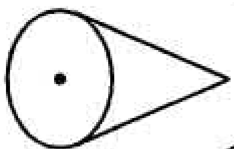


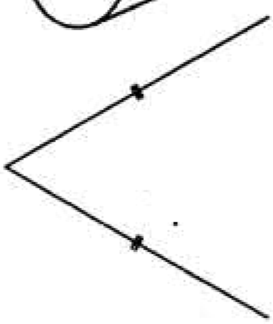
(iii)

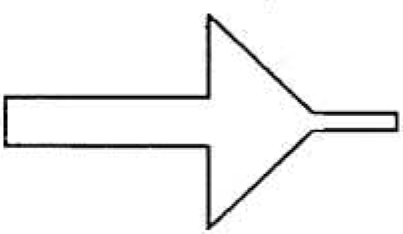


(iv) 

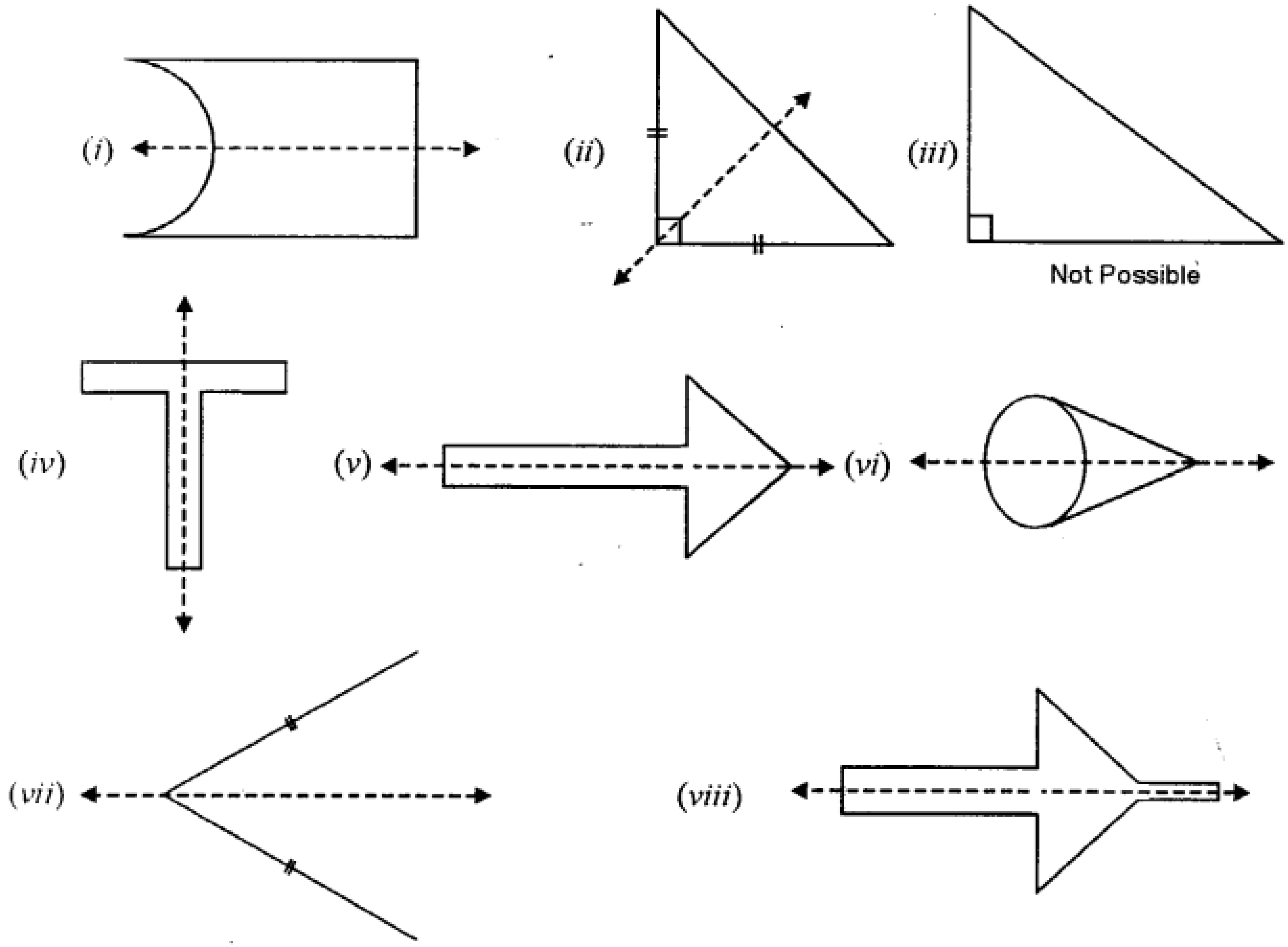
(v) 

(vi) 

(vii) 

(viii) 

Solution:

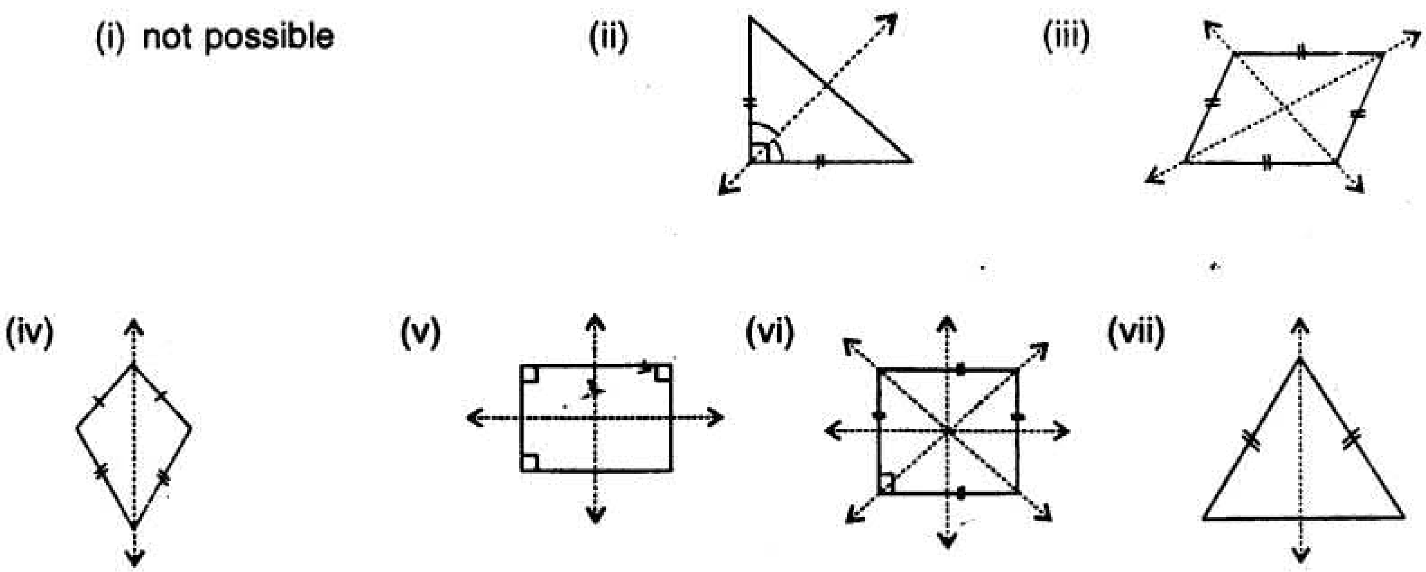


Question 2.
Write capital letters A to Z of English alphabet ; and in each case, if possible, draw the largest number of lines of symmetry.
Solution:
Line or lines of symmetry’ is possible in the following alphabets. For others alphabets it is not possible


A B C D E
F G H I J K
Not Possible
Not Possible
Not Possible
L M N O P
Not Possible
Not Possible
Not Possible
Q R S T U
Not Possible
Not Possible
Not Possible
V W X Y Z
Not Possible

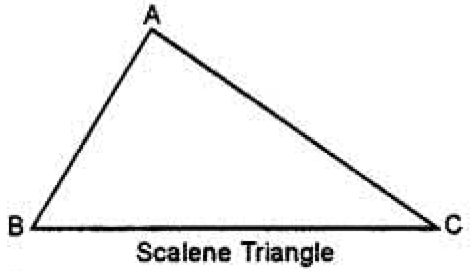
Question 3.
By drawing a free hand sketch of each of the following, draw in each case, the line (s) of symmetry,
if any:
(i) a scalene triangle
(ii) an isosceles right angled triangle
(iii) a rhombus
(iv) a kite shaped figure triangle.
(v) a rectangle
(vi) a square
(vii) an isosceles

Solution:

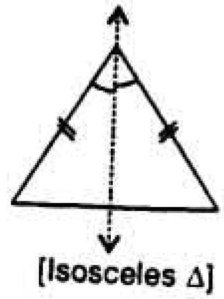


Question 4.
Draw a triangle with :
(i) no line of symmetry,
(ii) only one line of symmetry,
(iii) exactly two lines of symmetry,
(iv) exactly three lines of symmetry,
(v) more than three lines of symmetry.
In each case, if possible, represent the line/ lines of symmetry by dotted lines.
Also, write the special name of the triangle drawn.

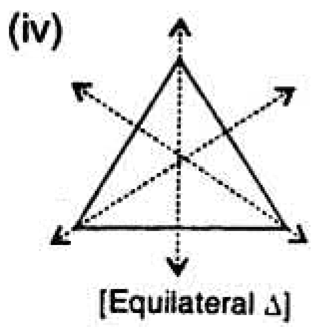
Solution:
(i) Scalene triangle : It has no line of symmetry.



(ii) Isosceles Triangle : It has one line of symmetry as shown.



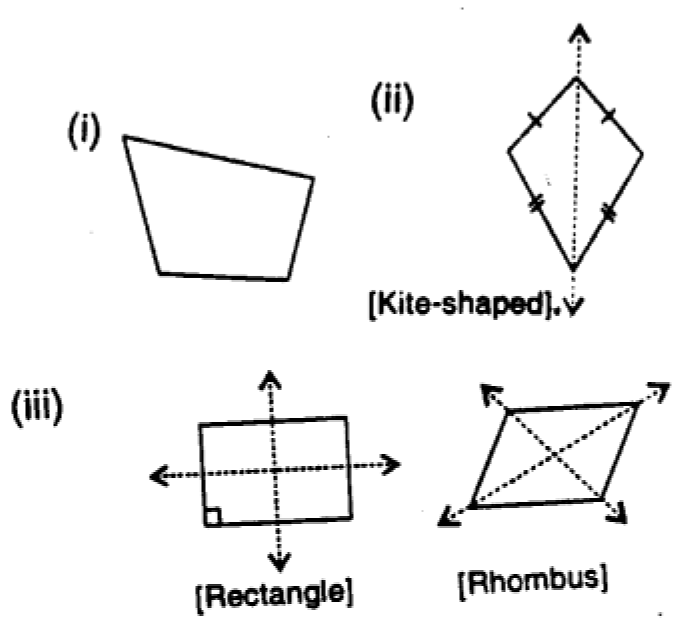
(iii) It is not possible.
(iv) Equilateral Triangle : It has three lines of symmetry as shown



(v) It is not possible.

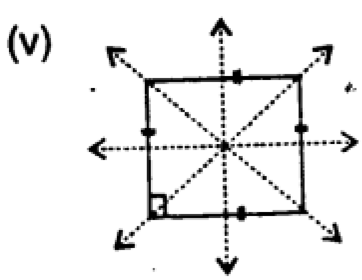
Question 5.
Draw a quadrilateral with :
(i) no line of symmetry.
(ii) only one line of symmetry.
(iii) exactly two lines of symmetry.
(iv) exactly three lines of symmetry.
(v) exactly four lines of symmetry.
(vi) more than four lines of symmetry.
In each case, if possible, represent the line/ lines of symmetry by dotted lines.
Also, write the special name of the quadrilateral drawn.

Solution:



(*iv*) Not possible

(*v*)

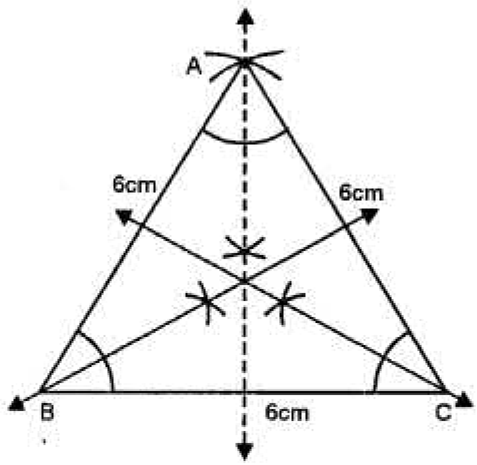


(*vi*) Not possible

Question 6.
Construct an equilateral triangle with each side 6 cm. In the triangle drawn, draw all the possible lines of symmetry.

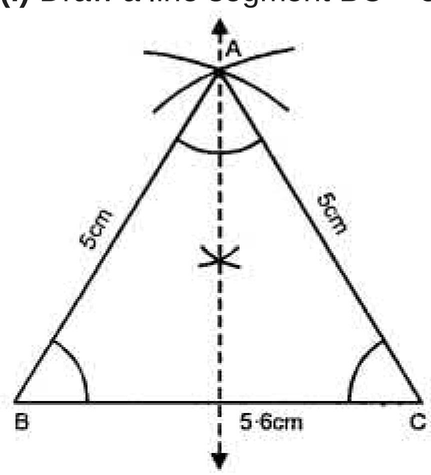
Solution:
Steps of Construction :

(i) Draw a line segment BC = 6 cm



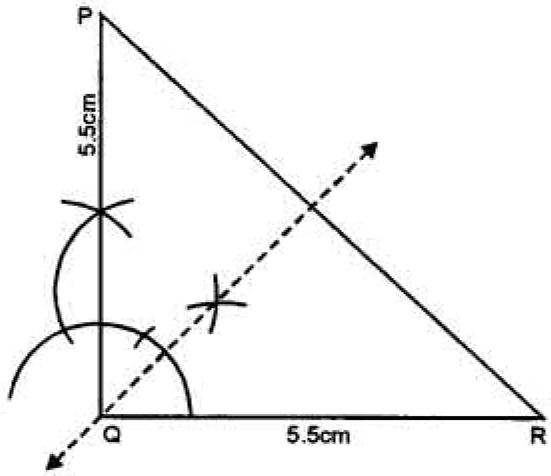
(ii) With centres B and C and radius 6 cm, draw two arcs intersecting each other at A.
(iii) Join AB and AC
∆ ABC is the required equilateral triangle,
(iv) Draw the angle bisectors of ∠A, ∠B and ∠C.
These are the lines of symmetry which are three in numbers as the triangle is equilateral.

Question 7.
Construct a triangle ABC in which AB = AC = 5cin and BC = 5.6 cm. If possible, draw its lines of symmetry.
Solution:
Steps of Construction :
(i) Draw a line segment BC = 5.6 cm



(ii) With centres B and C and radius 5 cm, draw two arcs intersecting each other at A.
(iii) Join AB and AC.
∆ ABC is an isosceles triangle.
(iv) Draw the bisector of ∠A. This is the only one line of symmetry as the triangle is an isosceles.

Question 8.
Construct a triangle PQR such that PQ = QR = 5 .5 cm and angle PQR = 90°. If possible, draw its lines of symmetry.
Solution:
∴ ∠PQR = 90°, and ∠P = ∠R
(opposite sides are equal)
∴∠P + ∠R = 90°
Hence ZP = ZR = $\frac{90^\circ}{2}$ = 45°
Steps of Construction:
(i) Draw a line segment QR = 5.5 cm

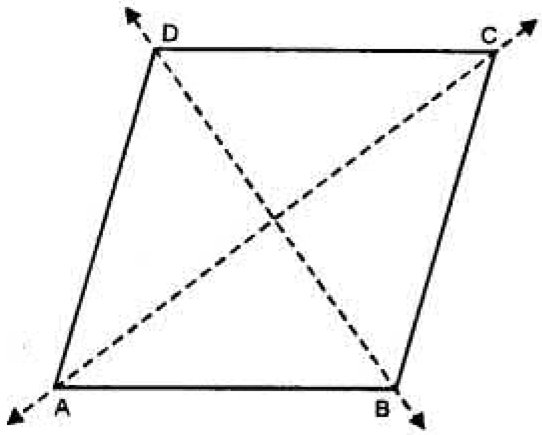


(ii) At Q, draw a ray making an angle of 90° and cut off QP = 5.5 cm.
(iii) Join PR.
∆PQR is an isosceles triangle.
(iv) Draw the angle bisector of ∠PQR. It is the line of symmetry. Since the triangle is an isosceles.
∴It has only one line of symmetry.

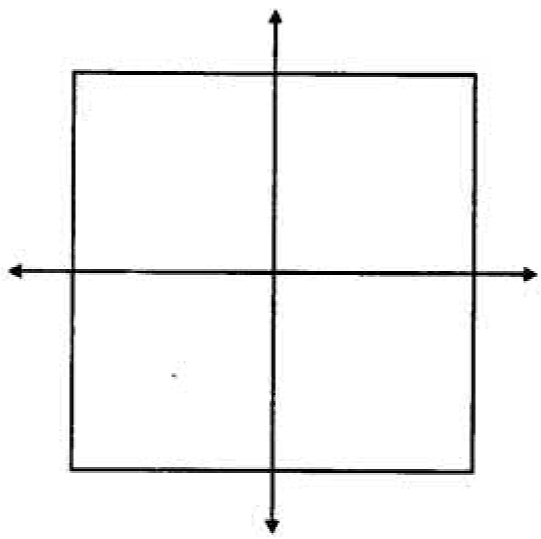
Question 9.
If possible, draw a rough sketch of a quadrilateral which has exactly two lines of symmetry.

Solution:
Since the quadrilateral has exactly two lilies of symmetry

∴It must be a rectangle or a rhombus



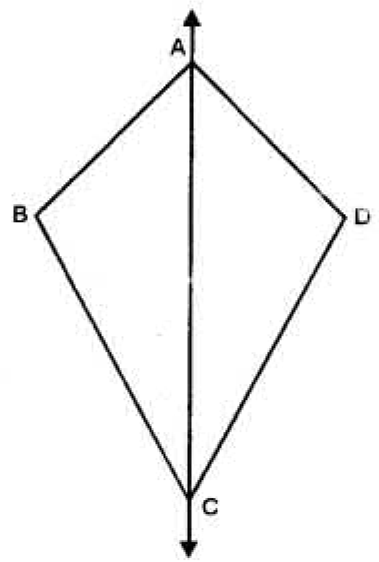
Rhombus



Rectangle

Question 10.
A quadrilateral ABCD is symmetric about its diagonal AC. Name tire sides of this quadrilateral which are equal.

Solution:
The quadrilateral ABCD is symmetric about its diagonal AC.

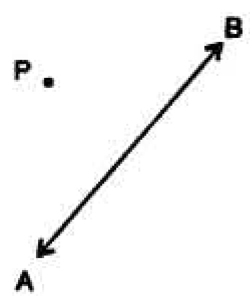


∴It must be a kite shaped.
Hence side AB = AD and BC = DC.

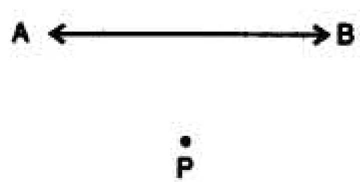
EXERCISE 17 (B)

Question 1.
In each figure, given below, find the image of the point P in the line AB :

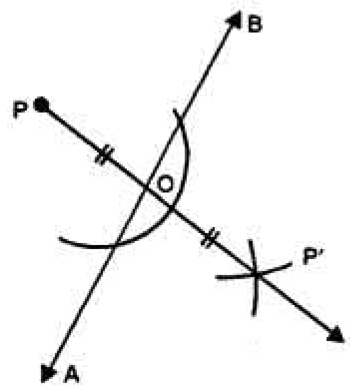
(i)



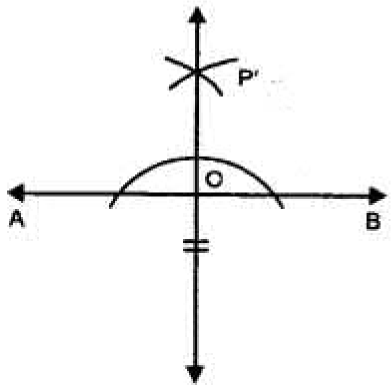
(ii)



Solution:
Steps of Construction : Fig. (i) and (ii)
(i) From P, draw a perpendicular to the given line AB meeting it at O.



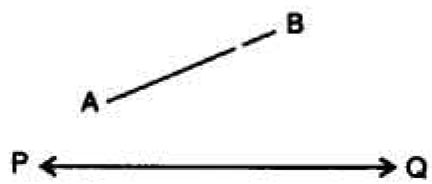
(ii) Produce PO to P' such that OP' = PO.
P' is the required image of P in AB.



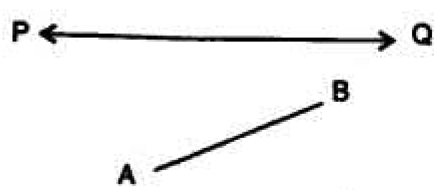
Question 2.

In each figure, given below, find the image of the line segment AB in the line PQ :

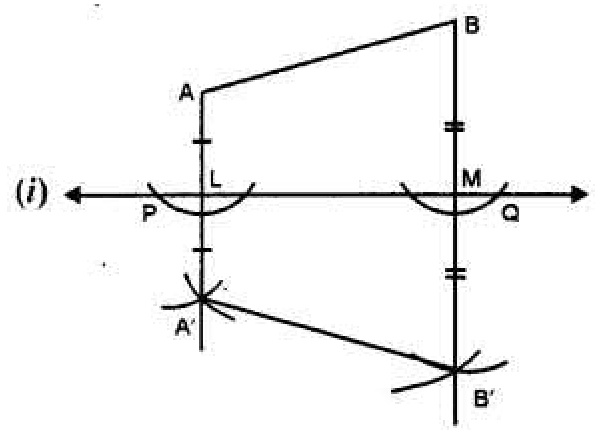
(i)

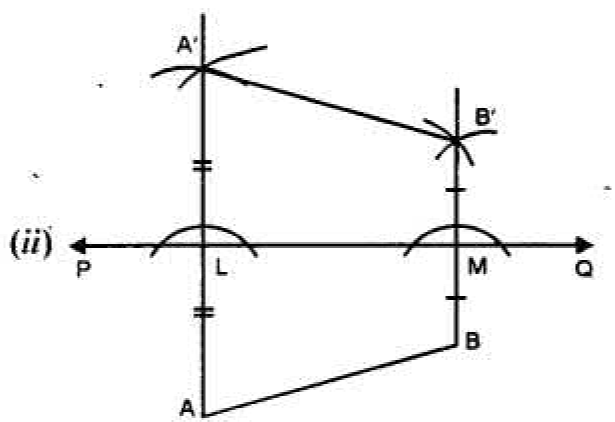


(ii)



Solution:





Steps of Construction :
(i) From A and B, draw perpendiculars on PQ intersecting PQ at L and M.
(ii) Produce AL to A' such that AL = LA' and produce BM to B' such that BM = MB' A'B' is the image of the line segment AB in PQ.

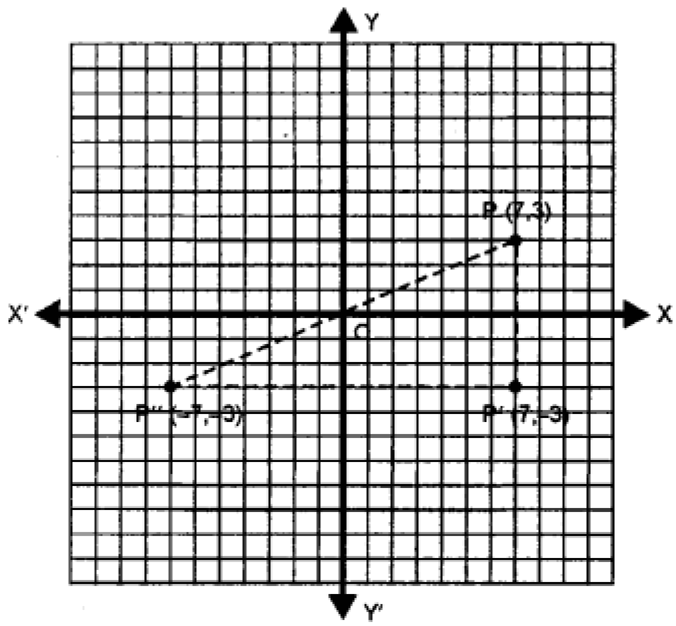
Question 3.
Complete the following table :

Point	Reflection in		
	x-axis	y-axis	origin
(*i*) (8, 2)	…………	…………	…………
(*ii*) (5, 6)	…………	…………	…………
(*iii*) (4, – 5)	…………	…………	…………
(*iv*) (6, – 2)	…………	…………	…………
(*v*) (– 3, 7)	…………	…………	…………
(*vi*) (– 4, 5)	…………	…………	…………
(*vii*) (– 2, – 7)	…………	…………	…………
(*viii*) (– 6, – 3)	…………	…………	…………
(*ix*) (4, 0)	…………	…………	…………
(*x*) (– 7, 0)	…………	…………	…………
(*xi*) (0, – 6)	…………	…………	…………
(*xii*) (0, 8)	…………	…………	…………
(*xiii*) (0, 0)	…………	…………	…………

Solution:

Point	Reflection in		
	x-axis	y-axis	origin
(*i*) (8, 2)	(8, – 2)	(– 8, 2)	(– 8, – 2)
(*ii*) (5, 6)	(5, – 6)	(– 5, 6)	(– 5, – 6)
(*iii*) (4, – 5)	(4, 5)	(– 4, – 5)	(– 4, 5)
(*iv*) (6, – 2)	(6, 2)	(– 6, – 2)	(– 6, 2)
(*v*) (– 3, 7)	(– 3, – 7)	(3, 7)	(3, – 7)
(*vi*) (– 4, 5)	(– 4, – 5)	(4, 5)	(4, – 5)
(*vii*) (– 2, – 7)	(– 2, 7)	(2, – 7)	(2, 7)
(*viii*) (– 6, –3)	(– 6, 3)	(6, – 3)	(6, 3)
(*ix*) (4, 0)	(4, 0)	(– 4, 0)	(– 4, 0)
(*x*) (– 7, 0)	(–7, 0)	(7, 0)	(7, 0)
(*xi*) (0, – 6)	(0, 6)	(0, – 6)	(0, 6)
(*xii*) (0, 8)	(0, – 8)	(0, – 8)	(0, – 8)
(*xiii*) (0, 0)	(0, 0)	(0, 0)	(0, 0)

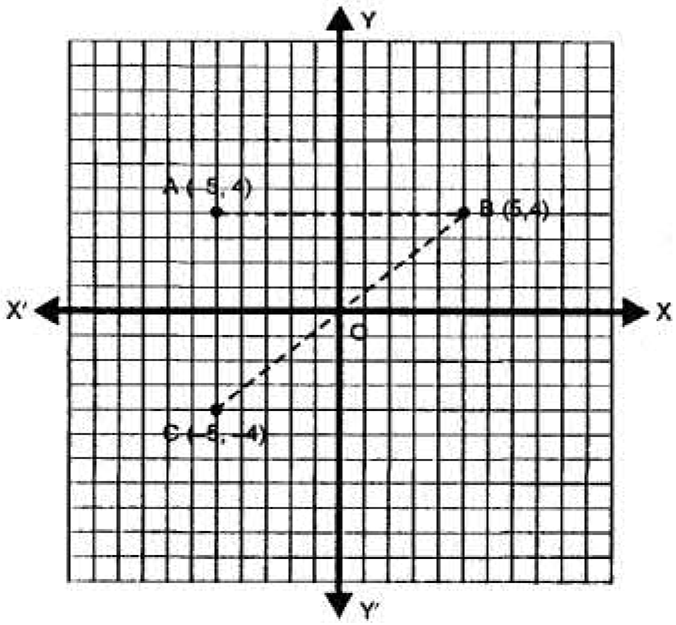
Question 4.
A point P (7,3)|is reflected in x-axis to point P'. The point P' is further reflected in v-axis to point P" Find :
(i) the co-ordinates of P'
(ii) the co-ordinates of P"
(iii) the image of P (7, 3) in origin.
Solution:
(i) Image of point P (7,3) when reflected in x-axis is P' whose co-ordinates will be (7,-3)



(ii) Image of point P' (7,-3) when reflected in y-axis, is P" whose co-ordinates will be (-7,-3)
(iii) The image of P (7, 3) in origin is P" whose co-ordinates are (- 7, – 3).

Question 5.
A point A (- 5, 4) is reflected in y-axis to point B. The point B is further reflected in origin to point C. find :
(i) the co-ordinates of B
(ii) the co-ordinates of C
(iii) the image of A (- 5, 4) in x-axis.
Solution:
(i) Image of point A (- 5,4) when reflected in y-axis is B whose co-ordinates will be (5,4)
(ii) Image of B (5, 4) when reflected in origin is C whose co-ordinates will be (- 5, – 4)

(iii) Image of A (- 5,4) in x-axis is C whose co-ordinates are (- 5, – 4)



Question 6.

The point P (3, – 8) is reflected in origin to point Q. The Point Q is further reflected in x-axis to point R. Find :

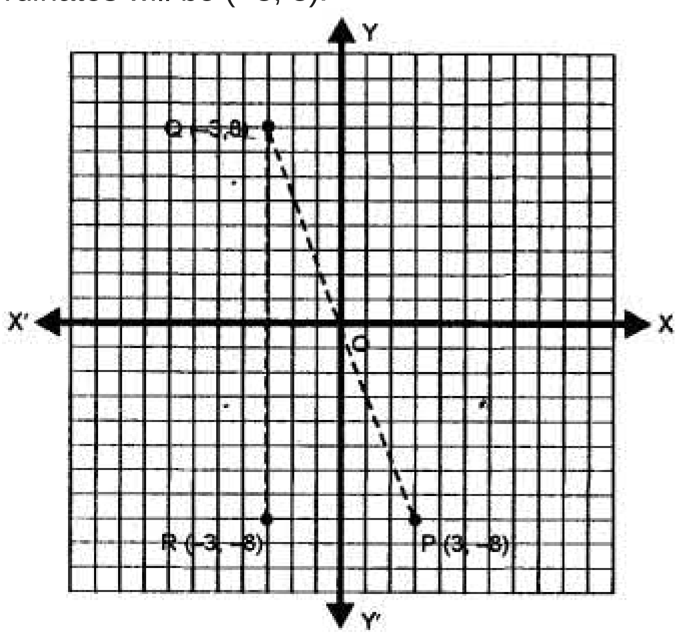
(i) the co-ordinates of Q

(ii) the co-ordinates of R

(iii) the image of P (3, – 8) in y-axis.

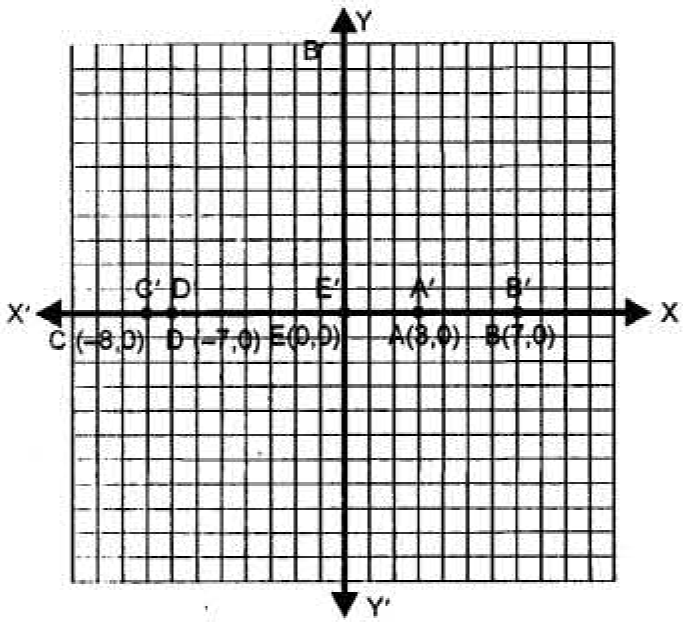
Solution:

(i) The image of the given point P (3, – 8) when reflected in origin is Q whose co-ordinates will be (- 3, 8).



(ii) The image of Q (- 3, 8) when reflected in x-axis is R whose co-ordinates will be (-3,- 8)
(iii) Tlie image of P (3, 8) in y-axis is R whose co-ordinates are (- 3, – 8).

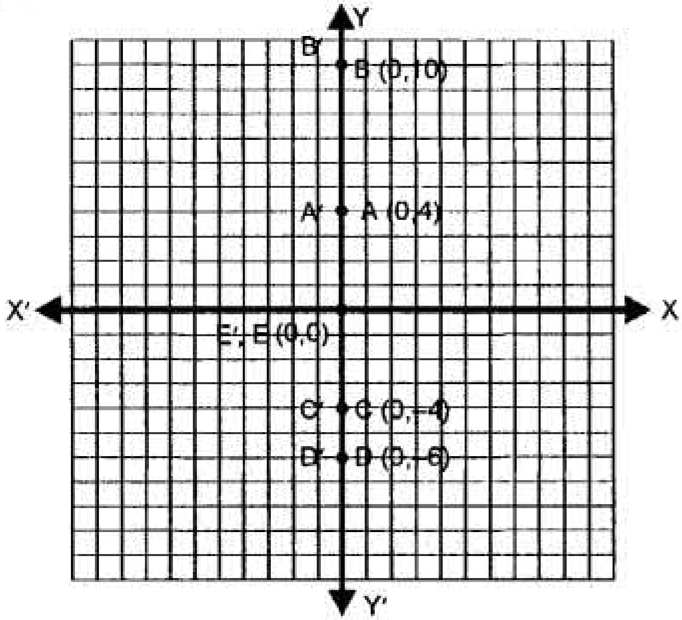
Question 7.
Each of the points A (3, 0), B (7, 0), C (- 8, 0), D (- 7, 0) and E (0, 0) is reflected in x-axis to points A', B', C', D' and E' respectively. Write the co-ordinates of each of the image points A', B', C', D' and E'.
Solution:
The points are given :
A (3, 0), B (1, 0), C (-8, 0), D (- 7, 0) and E (0, 0)



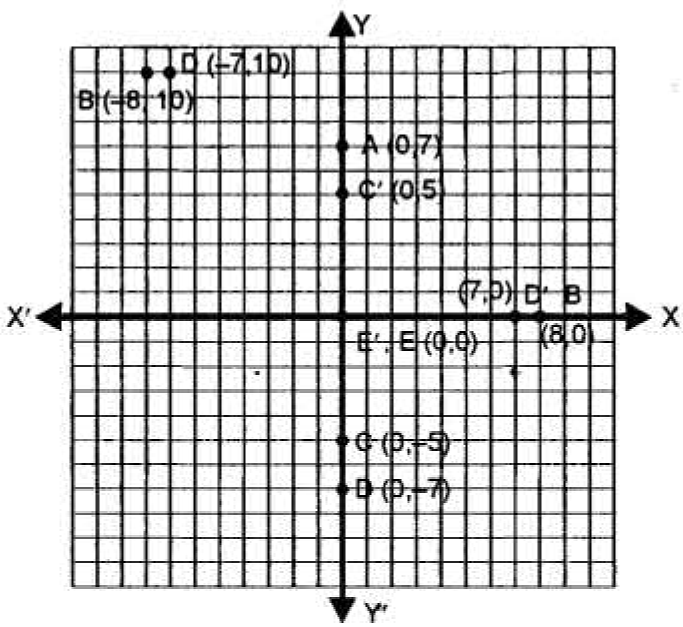
This images will be when reflected in x-axis. A' (3, 0), B' (7, 0), C' (- 8, 0) D' (- 7, 0) and E' (0, 0) as the given points lie on x-axis.

Question 8.
Each of the points A (0, 4), B (0, 10), C (0, – 4), D (0, – 6) and E (0, 0) is reflected in y-axis to points A', B', C', D' and E' respectively. Write the co-ordinates of each of the image points A', B', C', D' and E'.
Solution:
The given points
A (0, 4), B (0, 10), C (0, – 4), D (0, – 6) and E (0, 0) are reflected in y-axis. The co – ordinates of their images will be A' (0, 4), B' (0, 10), C' (0, – 4) D' (0, – 6) and E' (0, 0)

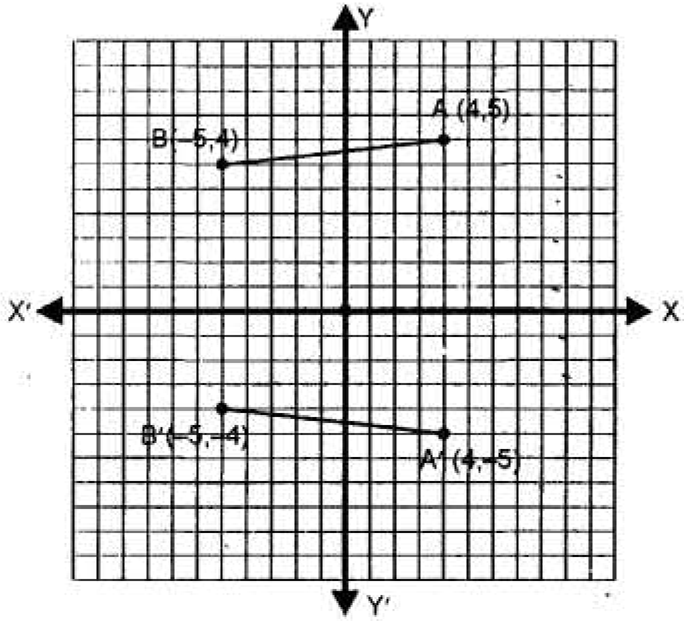
as they all lie on y-axis.



Question 9.
Each of the points A (0, 7), B (8. 0), C (0, -5), D (- 7, 0) and E (0, 0) are reflected in origin to points A', B', C', D' and E' respectively. Write the co-ordinates of each of the image points A', B', C', D' and E'.
Solution:
The points A (0, 7), B (8, 0), C (0, – 5). D (- 7.0) and E (0,0) are reflected in origin.
So, the co-ordinates of their images will be A' (0,-7), B' (- 8, 0), C' (0,5), D' (7, 0) and E' (0, 0)



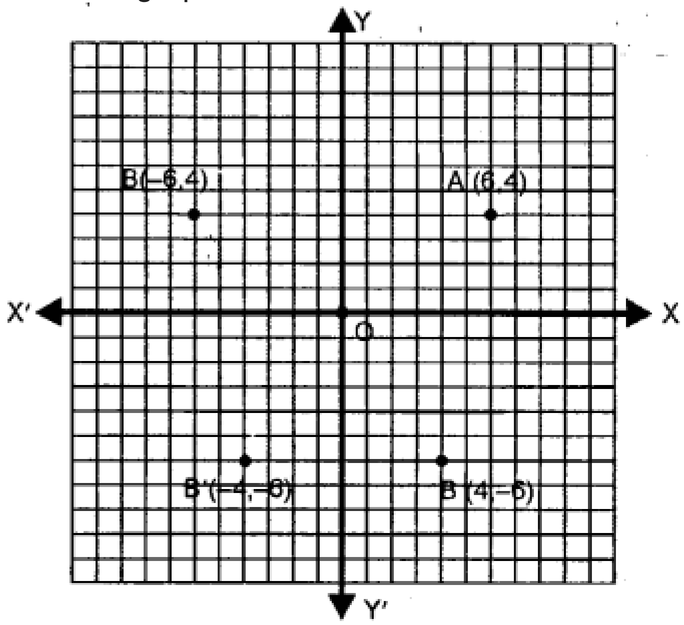
Question 10.
Mark points A (4, 5) and B (- 5, 4) on a graph paper. Find A’, the image of A in x-axis and B’, the image of B in x-axis.
Mark A’ and B’ also on the same graph paper.
(ii) Join AB and A’ B’ and
find if AB = A’ B’ ?
Solution:
The given points :
A (4, 5) and B (- 5, 4) have been marked on the graph.



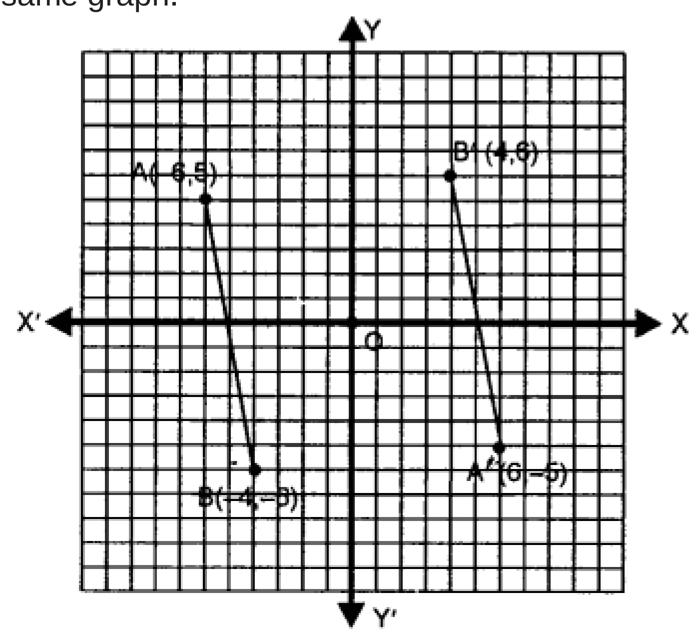
The image of A in x-axis A (4, – 5) and image of B in x-axis is B’ (-5, -4) which have been also plotted on the same graph.
AB and A’ B’ are joined. We see that AB = A’ B’.

Question 11.
Mark points A (6, 4) and B (4, – 6) on a graph paper.
Find A’, the image of A in y-axis and B’, the image of B in y-axis. Mark A’ and B’ also on the same graph paper.
Solution:
The given points are
A (6, 4) and B (4, – 6)
The images of A and B is y-axis are A’ (- 6, 4) and B’ (- 4, – 6) respectively as shown in

the same graph.



Question 12.
Mark points A (- 6, 5) and B (- 4, – 6) on a graph paper. Find A', the image of A in origin and B', the image of B in origin. Mark A' and B' also on the same graph paper. Join AB and A' B'. Is AB = A' B' ?
Solution:
The given points are A (- 6, 5) and B (- 4, – 6). The images of A and B in the origin are A' and B' where co-ordinates are A' (6, – 5) and B' (4, 6) which have been plotted on the same graph.



AB and A' B' are joined we see that AB = A' B'.

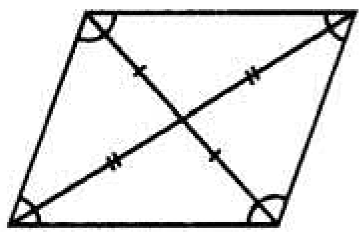
EXERCISE 17 (C)

Question 1.

How many lines of symmetry does a rhombus have?

Solution:

It has two lines of symmetry.



Question 2.

What is the order of rotational symmetry of a rhombus?

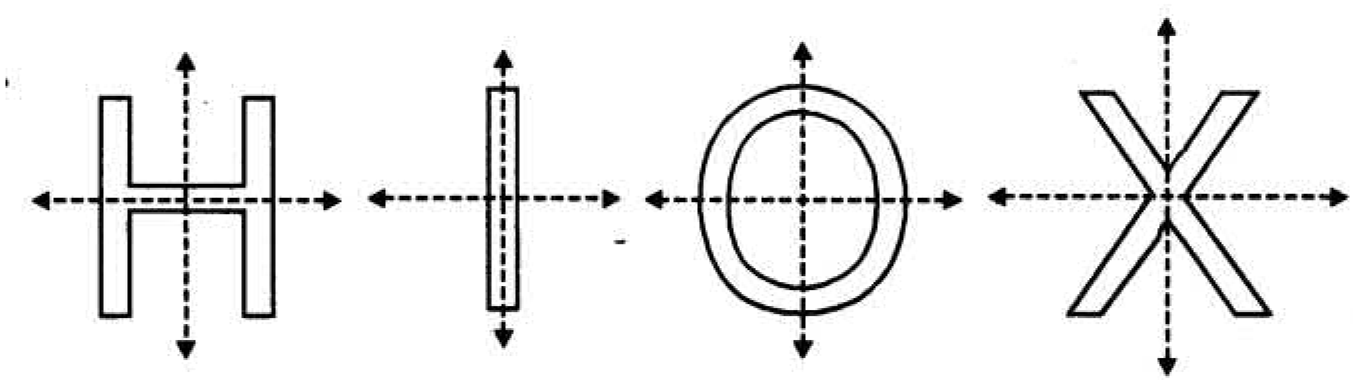
Solution:

The order of rotational symmetry can be defined as the number of times that a shape appears exactly the same during a full 360° rotation. Order of rotational symmetry of a rhombus is 2.

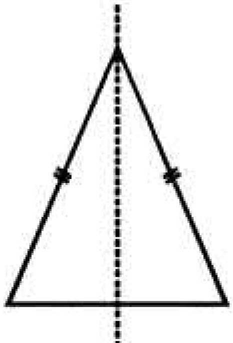
Question 3.

Show that each of the following figures has two lines of symmetry and a rotational.

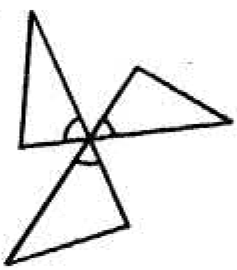
Solution:

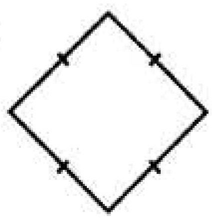


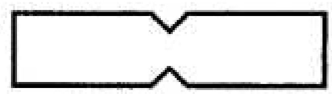
Question 4.
Name a figure that has a line of symmetry but does not have any roational symmetry.
Solution:
Isosceles triangle has only line symmetry and no rotational symmetry.

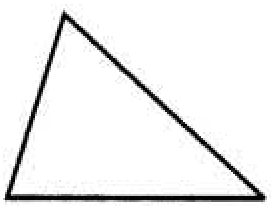


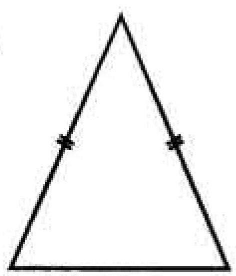
Question 5.
In each of the following figures, draw all possible lines of symmetry and also write the order of rotational symmetry:

(i)


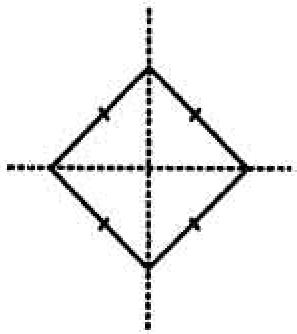
(ii)


(iii)


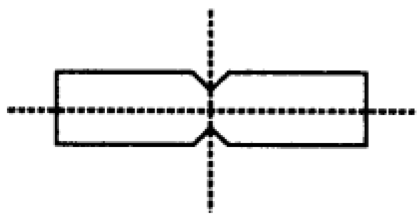
(iv)


(v)


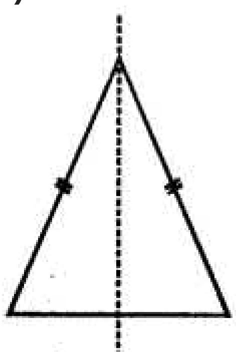
Solution:
(i) It has no line of symmetry and order of rotational symmetry is 3
(ii) It has 2 line of symmetry and order of rotational symmetry is 4.



(iii) It has 2 line of symmetry and order of rotational symmetry is 2.



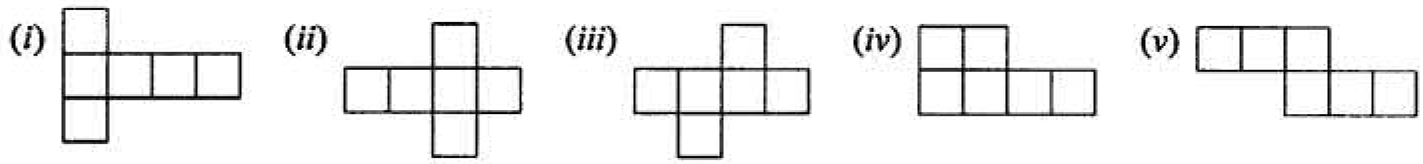
(iv) It has no line of symmetry and order of rotational symmetry is 0.
(v) It has 1 line of symmetry and order of rotational symmetry is 0.



CHAPTER - 18
RECOGNITION OF SOLIDS

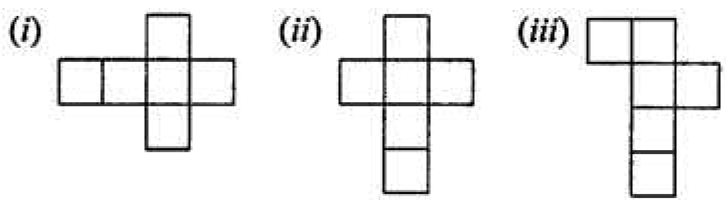
EXERCISE 18

Question 1.
Identify the nets which can be used to form cubes

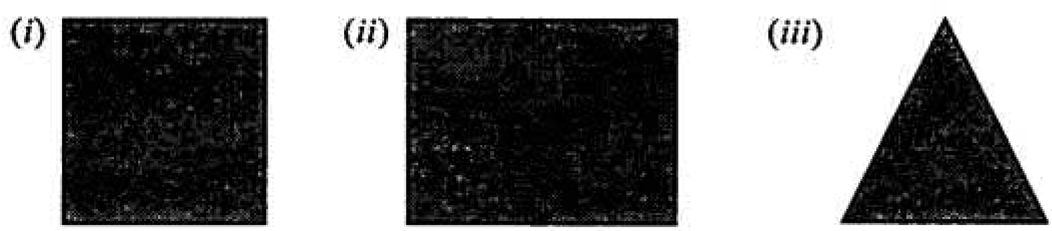


Solution:
Nets for a cube are (ii) , (iii) and (v).

Question 2.
Draw at least three different nets for making cube.
Solution:



Question 3.
The figure, given below, shows shadows of some 3D objects, when seen under the lamp of an overhead projector :

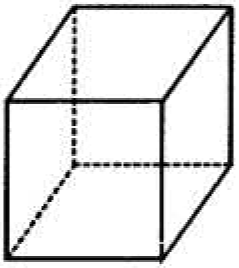


A square A rectangle A triangle

In each case, name the object.

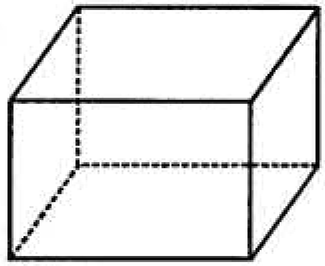
Solution:

(i)



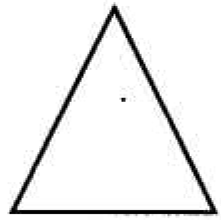
Cube

(ii)



Cuboid

(iii)



Cone

Question 4.
Using Euler’s formula, find the values of a, b, c and d.

Faces	*a*	5	20	6
Vertices	6	*b*	12	*d*
Edges	12	9	*c*	12

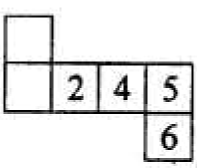
Solution:

Faces	*a*	5	20	6
Vertices	6	*b*	12	*d*
Edges	12	9	*c*	12

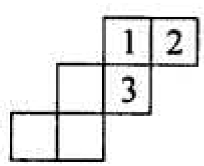
(i) a + 6 – 12 = 2 ⇒ a = 2 – 6 + 12 = 14 – 6 = 8
(ii) b + 5- 9 = 2 ⇒6 = 2 + 9- 5 = 6
(iii) 20+ 12 — c = 2 ⇒32 – c = 2 ⇒ c = 32-2 ⇒ c = 30
(iv) 6 + d-12=2 ⇒ d – 6 = 2 ⇒ d = 2 + 6 = 8

Question 5.
Dice are cubes with dot or dots on each face. Opposite faces of a die always have a total of seven on them.
Below are given two nets to make dice (cube), the numbers inserted in each square indicate the number of dots in it.

(i)



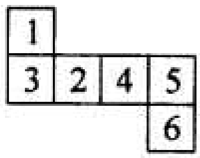
(ii)



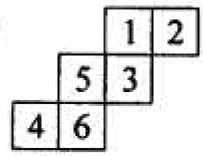
Insert suitable numbers in each blank so that numbers in opposite faces of the die have a total of seven dots.

Solution:

(i)

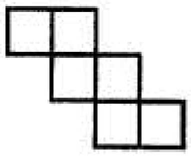


(ii)

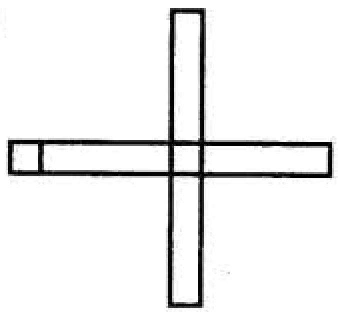


Question 6.
The following figures represent nets of some solids. Name the solids

(i)

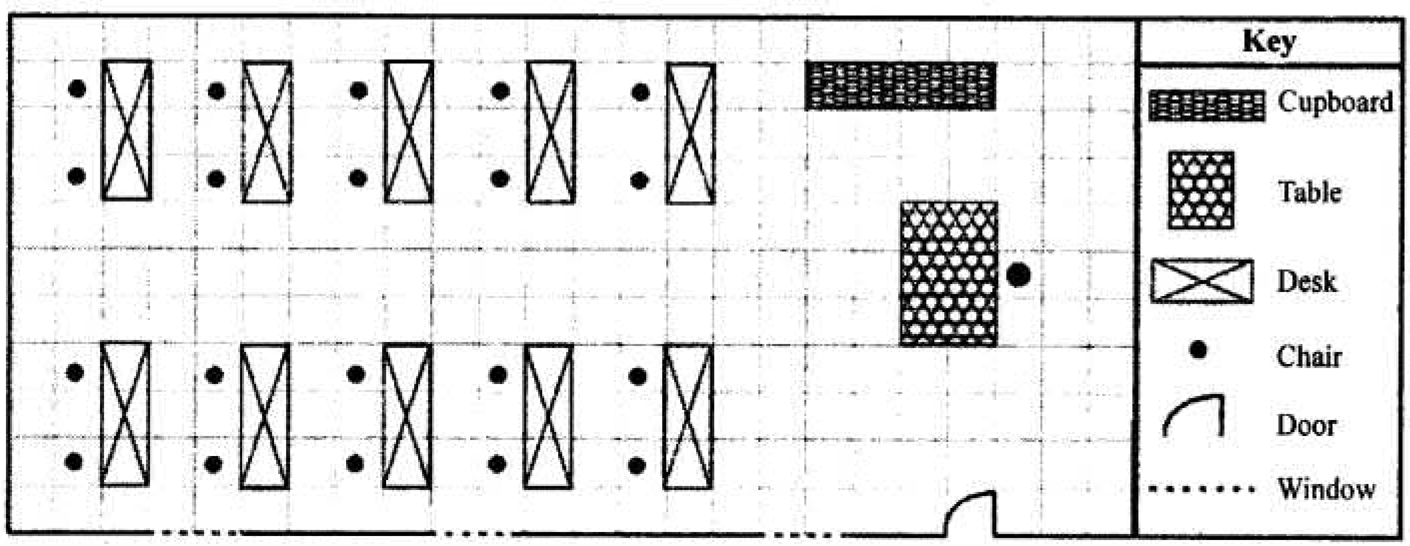


(ii)



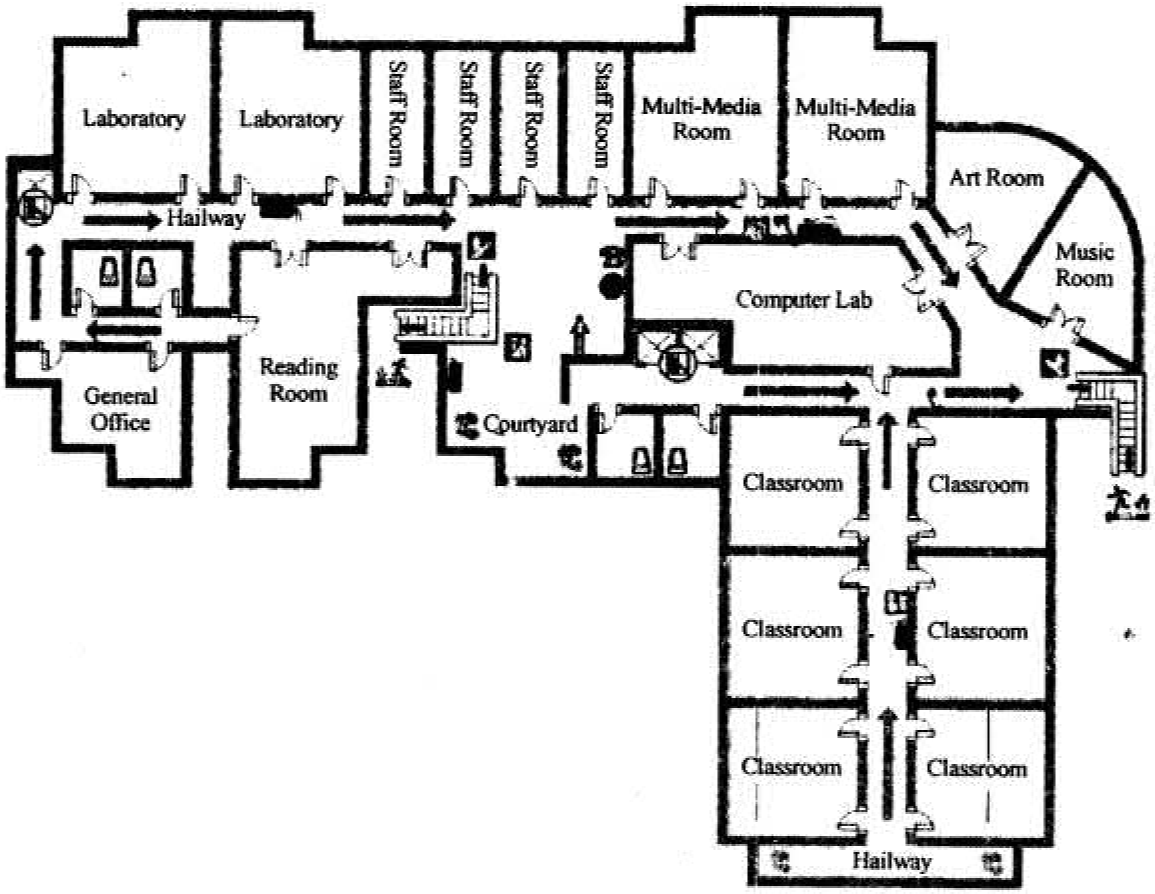
Solution:
The given nets are of the solid as given below :
(i) Cube
(ii) Cuboid

Question 7.
Draw a map of your class room using proper scale and symbols for different objects.
Solution:



Question 8.
Draw a map of your school compound using proper scale and symbols for

various features like play ground, main building, garden, etc.
Solution:

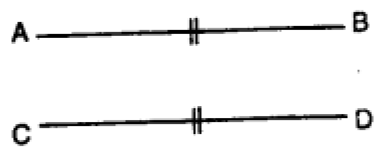


Question 9.
In the map of India, the distance between two cities is 13.8 cm.
Taking scale : 1 cm = 12 km, find the actual distance between these two cities.
Solution:
The scale for a map is given to be 1 cm = 12 km
The distance between these two cities = 13.8 cm on the map
∴ Actual distance between these two cities
= 12 x 13.8 km = 165.6 km

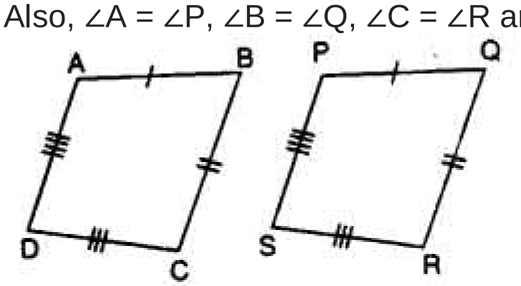
CHAPTER - 19
CONGRUENT TRIANGLES

POINTS TO REMEMBER

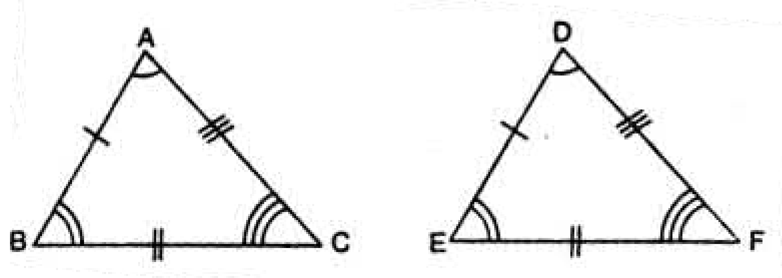
1. Meaning of Congruency : If two geometrical figures coincide exactly, by placing one over the other, the figures are said to be congruent to each other.
1. Two lines AB and CD are said to be congruent if, on placing AB on CD, or CD on AB ; the two AB and CD exactly coincide.
It is possible only when AB and CD are equal in length.Two figures ABCD and PQRS are said to be



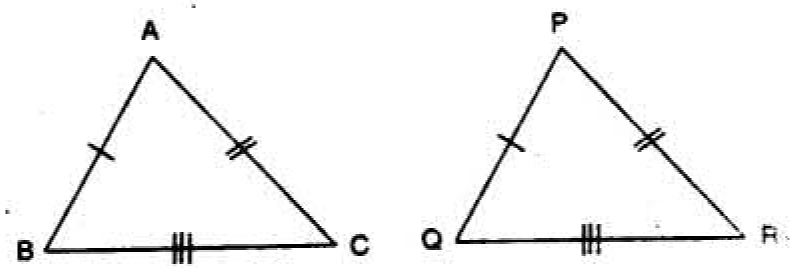
congruent if, on placing ABCD on PQRS or PQRS on ABCD the two figures exactly coincide i.e. A and P coincide. B and Q coincide, C and R coincide and D and S coincide.
It is possible only when :
AB = PQ, BC = QR, CD = RS and AD = PS
Also, $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$ and $\angle D = \angle S$.



2. Congruency in Triangles : Let triangle ABC is placed over triangle DEF ; such that, vertex A falls on vertex D and side AB falls on side DE ; then if the two triangles coincide with each other in such a way that B falls on E ; C falls on F ; side BC coincides with side EF and side AC coincides with side DF, then the two triangles are congruent to each other.
The symbol used for congruency is " ≡ " or " ≅ "
$\therefore \Delta$ ABC is congruent to Δ DEF is written as :
Δ ABC = Δ DEF or ΔABC = ΔDEF.

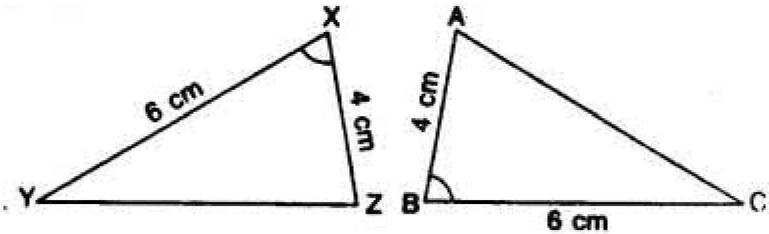


3. Corresponding Sides and Corresponding Angles : In case of congruent triangles ABC and DEF, drawn above ; when A ABC is placed over A DEF to cover it exactly; then the sides of the two triangles, which coincide with each other, are called corresponding sides. ‘ Thus, the side AB and DE are corresponding sides; sides BC and EF are corresponding sides and sides AC and DF are also corresponding sides.
In the same way. the angles of the two triangles which coincide with each other, are called corresponding angles. Thus, three pairs of corresponding angles are ∠A and ∠D ; ∠B and ∠E and also ∠C and ∠F.
Note : The corresponding parts of congruent triangles are always equal (congruent).
∴(i) AB = DE, BC = EF and AC = DF. i.e. corresponding sides are equal.
Also (ii) ∠A = ∠D, ZB = ∠E and ∠C = ∠F
i.e. corresponding angles are equal.
4. Conditions of Congruency:
1. If three sides of one triangle are equal to three sides of the other triangle, each to each, then the two triangles are congruent.
The test is known as : side, side, side and is abbreviated as S.S.S.
In triangle ABC and PQR, given alongside:
AB = PQ ; BC = QR and AC = PR
And, so A ABC is congruent to Δ PQR e. Δ ABC = Δ PQR by S.S.S.
Because in congruent triangles, corresponding sides and corresponding angles are equal.
∴ ∠A = ∠P : ∠B = ∠Q and ∠C =∠R

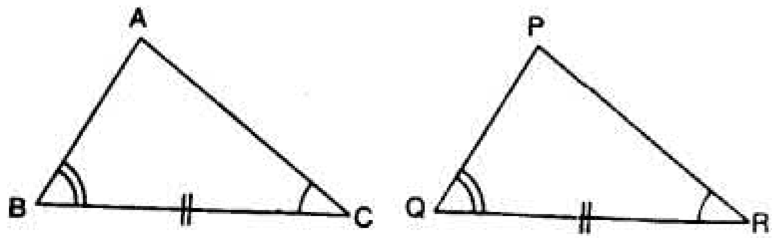


2. If two sides and the included angle of one triangle are equal to two sides and the included angle of the other triangle, each to each, then the triangles are congruent.
This test is known as : side, angle, side and is abbreviated as S.A.S. In the given triangles, AB = XZ ; BC = XY and ∠ABC = ∠ZXY
∴ Δ ABC ≅ Δ ZXY
Note : Triangles will be congruent by S.A.S., only when the angles included by the corresponding equal sides are equal.
The pairs of corresponding sides of these two congruent triangles are : AB and ZX ; BC and XY ; AC and ZY
The pairs of corresponding angles are :

∠B and ∠X ; ∠A and ∠Z : ∠C and ∠Y.

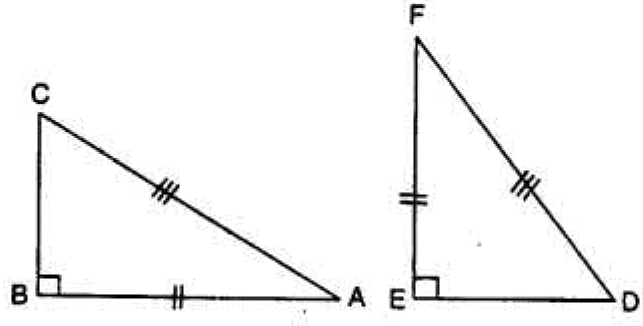


3. If two angles and the included side of one triangle are equal to the two angles and the included side of the other triangle ; then the triangles are congruent.
This test is known as : angle, side, angle and is abbreviated as A.S.A.
In the given figure :
BC = QR;
∠B = ∠Q and ∠C = ∠R
∴ ∆ABC = ∆ PQR.. (by A.S.A.)



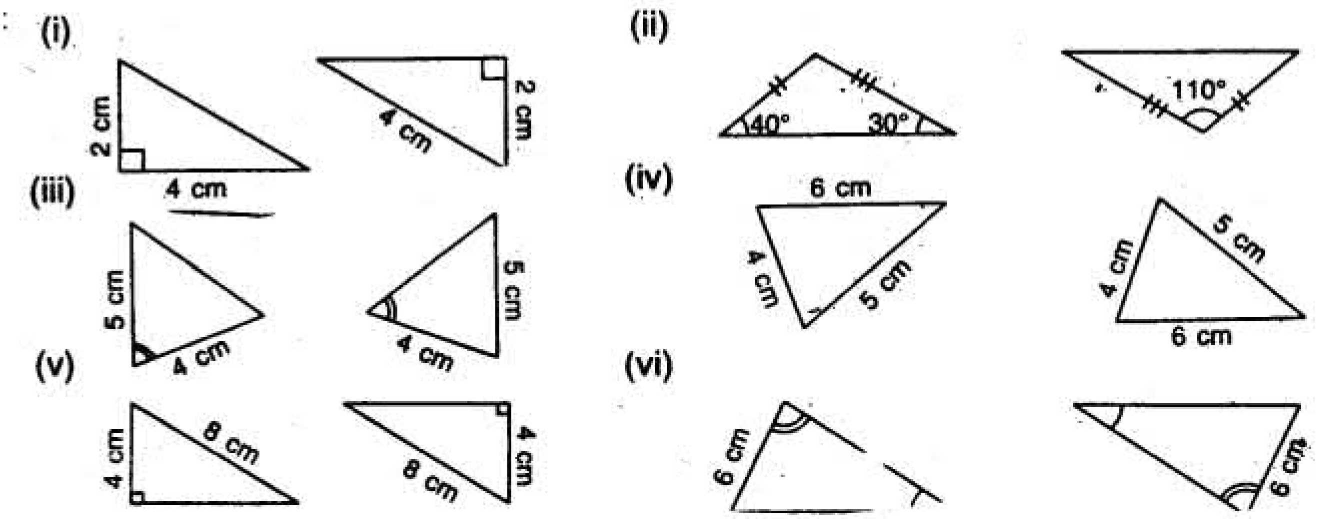
4. If the hypotenuse and one side of a right angled triangle are equal to the hypotenuse and one side of another right angled triangle, then the two triangles are congruent.
This test is known as : right angle, hypotenuse, side and is abbreviated as R.H.S.
In the given figure :
∠B = ∠E = 90° ; AB = FE
and hypotenuse AC = hypotenuse FD
∴ ∆ ABC = ∆ FED (byRH.S.)
The corresponding angles in this case are :
∠A and ∠F ; ∠B and ∠E ; ∠C and ∠D and the corresponding sides are :
AB and EF ; AC and FD ; BC and ED.
Since the triangles are congruent, therefore all its corresponding sides are equal and corresponding angles are also equal.
∴ BC = ED ; ∠A = ∠F and ∠C = ∠D
Note : If three angles of a triangle are equal to the three angles of the other triangle, then the triangle are not necessarily congruent.
For congruency at least one pair of corresponding sides must be equal.

∴A.A.A. is not a test of congruency.



EXERCISE 19

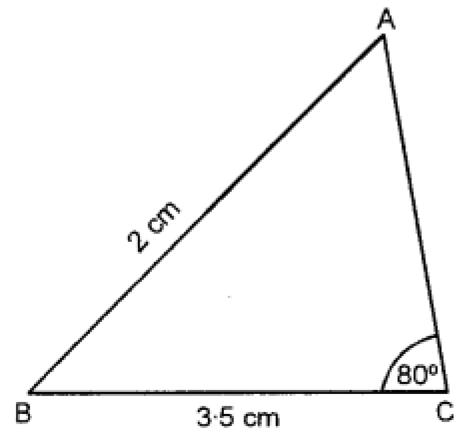
Question 1.
State, whether the pairs of triangles given in the following figures are congruent or not:

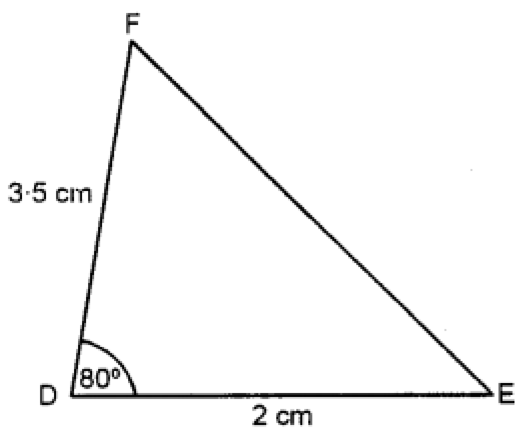


(vii) Δ ABC in which AB = 2 cm, BC = 3.5 cm and ∠C = 80°.
and, Δ DEF in which DE = 2 cm, DF = 3.5 cm and ∠D = 80°.

Solution:
(i) In these triangles, corresponding sides are not equal. Hence these are not congruent triangles.
(ii) In the first A, third angle
= 180°-(40°+ 30°)
= 180° – 70°
= 110°
Now in these two triangles the sides and included angle of the one are equal to the corresponding sides and included angle.
Hence these are congruent triangles

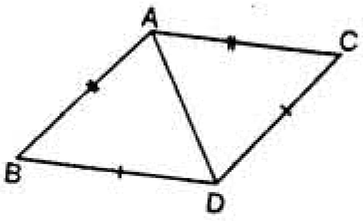
(S.A.S. axiom)
(iii) In these triangles, corresponding two sides are equal but included angles are not-equal. Hence these are not congruent triangles.
(iv) In these triangles, corresponding three sides are equal.
Hence these are congruent triangles.
(S.S.S. Axiom)
(v) In these right triangles, one side and diagonal of the one, are equal to the corresponding side and diagonal are equal. Hence these are congruent triangles. –
(R.H.S. Axiom)
(vi) In these triangles two sides and one angle of the one are equal to the corresponding sides and one angle of the other are equal.
Hence these are congruent triangles.
(S.S. A. Axiom).
(vii) In A ABC. AB = 2 cm, BC = 3.5 cm and ∠C = 80° and in ∆ DEF,
DE = 2 cm, DF = 3.5 cm and ∠D = 80°





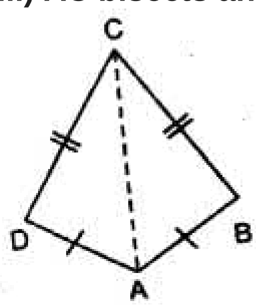
From the figure we see that two corresponding sides are equal but their included angles are not equal.
Hence, these are not congruent triangles

Question 2.
In the given figure, prove that:
$\triangle ABD \cong \triangle ACD$



Solution:
Proof :
In Δ ABD and Δ ACD,
AD = AD (common)
AB = AC (given)
BD = DC (given)
$\therefore \Delta ABD \cong \Delta ACD$ (S.S.S. Axiom)
Hence proved.

Question 3.
Prove that:
(i) $\triangle ABC \equiv \triangle ADC$
(ii) $\angle B = \angle D$
(iii) AC bisects angle DCB



Solution:

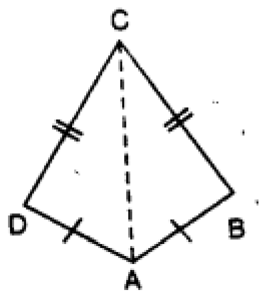
Given : In the figure,

AB = AD, CB = CD

To prove : $\Delta ABC \cong \Delta ADC$

$\angle B = \angle D$

AC bisects angle DCB



Proof : In ΔABC and ΔADC,

AC = AC (common)

AB = AD (given)

CB = CD (given)

(i) $\therefore \Delta ABC \cong \Delta ADC$ (SSS axiom)

$\therefore \angle B = D$ (c.p.c.t.)

$\angle BCA = \angle DCA$

$\therefore$ AC bisects $\angle DCB$

Question 4.

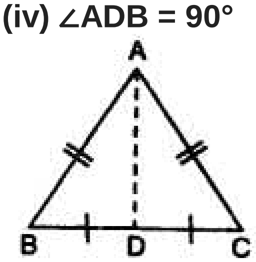
Prove that:

(i) $\triangle ABD \equiv \triangle ACD$

(ii) $\angle B = \angle C$

(iii) $\angle ADB = \angle ADC$

(iv) $\angle ADB = 90°$

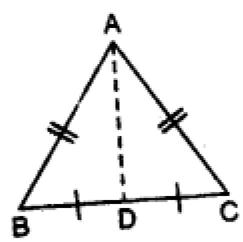


Solution:

Given : In the figure,

AD = AC

BD = CD



To prove :

(*i*) $\Delta ABD \cong \Delta ACD$ (*ii*) $\angle B = \angle C$

(*iii*) $\angle ADB = \angle ADC$ (*iv*) $\angle ADB = 90^\circ$

Proof : In ΔABD and ΔACD

AD = AD (common)

AD = AC (given)

BD = CD (given)

(*i*) $\therefore \Delta ABD \cong \Delta ACD$

(SSS axiom)

(*ii*) $\therefore \angle B = \angle C$

(c.p.c.t.)

(*iii*) $\angle ADB = \angle ADC$

(c.p.c.t.)

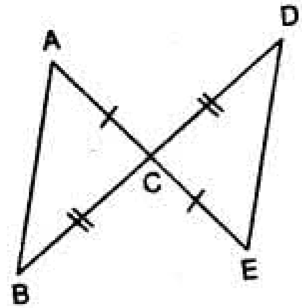
But $\angle ADB + \angle ADC = 180^\circ$

(Linear pair)

$\therefore \angle ADB = \angle ADC$

$$= \frac{180^\circ}{2} = 90^\circ$$

Question 5.
In the given figure, prove that:
(i) △ACB ≅ △ECD
(ii) AB = ED



Solution:

(*i*) In Δ ACB and Δ ECD,

AC = CE (given)

∠ACB = ∠DCE (vertically opposite angles)

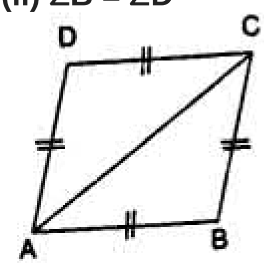
BC = CD (given)

∴ Δ ACB ≅ Δ ECD (S.A.S. Axiom)

(*ii*) Hence AB = ED (c.p.c.t.)

Hence proved.

Question 6.
Prove that:
(i) △ ABC ≅ △ ADC
(ii) ∠B = ∠D



Solution:

Proof : (*i*) In Δ ABC and Δ ADC

AC = AC (common)

AB = DC (given)

BC = AD (given)

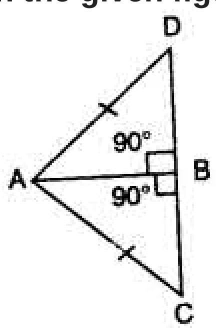
$\therefore$ Δ ABC $\cong$ Δ ADC (S.S.S. Axiom)

(*ii*) Hence $\angle B = \angle D$ (c.p.c.t.)

Hence proved

Question 7.

In the given figure, prove that: BD = BC.



Solution:

Proof :

In right Δ ABD and Δ ABC

Side AB = side AB (common)

Hypotenuse AD = Hypotenuse AC (given)

$\therefore$ Δ ABD $\cong$ Δ ABC (R.H.S. Axiom)

Hence BD = BC (c.p.c.t.)

Hence proved.

Question 8.

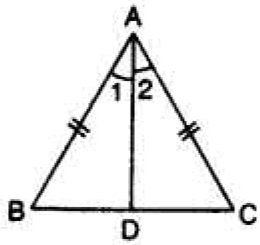
In the given figure ;

$\angle 1 = \angle 2$ and AB = AC. Prove that:

(i) $\angle B = \angle C$

(ii) BD = DC

(iii) AD is perpendicular to BC.



Solution:

Proof :

In Δ ADB and Δ ADC,

AB = AC (given)

$\angle 1 = \angle 2$ (given)

AD = AD (common)

$\therefore \Delta ADB \cong \Delta ADC$ (S.A.S. Axiom)

(*i*) Hence $\angle B = \angle C$ (c.p.c.t.)

(*ii*) BD = DC (c.p.c.t.)

(*iii*) and $\angle ADB = \angle ADC$ (c.p.c.t.)

But $\angle ADB + \angle ADC = 180°$ (Linear pair)

$\therefore \angle ADB = \angle ADC = 90°$

Hence AD is perpendicular to BC.

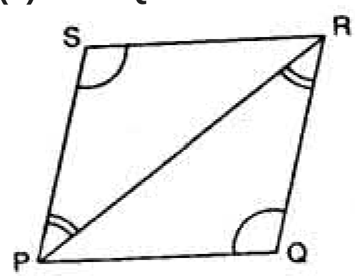
Hence proved.

Question 9.

In the given figure prove tlyat:

(i) PQ = RS

(ii) PS = QR



Solution:

Proof : In Δ PQR and Δ PSR,

PR = PR (common)

$\angle PRQ = \angle RPS$ (given)

$\angle PQR = \angle PSR$ (given)

$\therefore \Delta PQR \cong \Delta PSR$ (A.A.S. Axiom)

Hence (*i*) PQ = RS (c.p.c.t.)

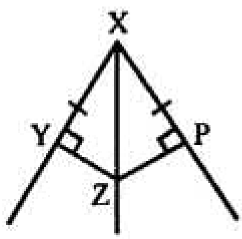
(*ii*) QR = PS (c.p.c.t.)

or PS = QR

Hence proved.

Question 10.

(i) Δ XYZ $\cong$ Δ XPZ
(ii) YZ = PZ
(iii) $\angle YXZ = \angle PXZ$



Solution:

In right Δ XYZ and Δ XPZ,

Side XY = Side XP (given)

Hypotenuse XZ = Hypotenuse XZ (common)

(*i*) $\therefore \Delta XYZ \cong \Delta XPZ$ (R.H.S. Axiom)

Hence (*ii*) YZ = PZ (c.p.c.t.)

and (*iii*) $\angle YXZ = \angle PXZ$ (c.p.c.t.)

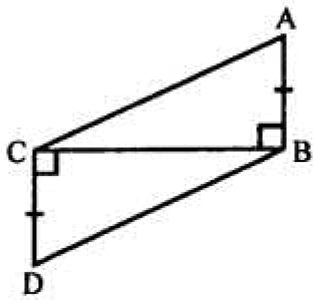
Hence proved.

Question 11.

In the given figure, prove that:

(i) $\triangle ABC \cong \triangle DCB$

(ii) AC=DB



Solution:

Proof :

In Δ ABC and Δ DCB,

CB = CB	(common)
$\angle ABC = \angle BCD$	(each 90°)
and AB = CD	(given)
(*i*) $\therefore \Delta ABC \cong \Delta DCB$	(S.A.S. Axiom)
(*ii*) Hence AC = DB	(c.p.c.t.)

Hence proved.

Question 12.

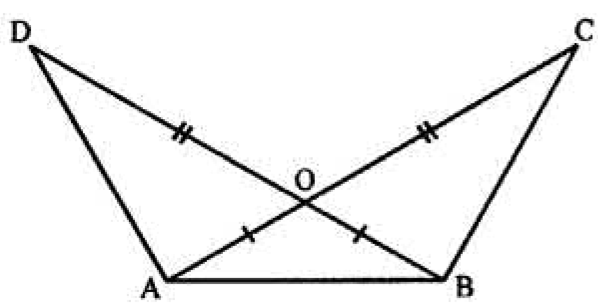
In the given figure, prove that:

(i) $\triangle AOD \cong \triangle BOC$

(ii) AD = BC

(iii) $\angle ADB = \angle ACB$

(iv) $\triangle ADB \cong \triangle BCA$



Solution:

Proof :

In Δ AOD and Δ BOC

OA = OB (given)

$\angle AOD = \angle BOC$

(vertically opposite angles)

OD = OC (given)

(i) $\therefore \Delta AOD \cong \Delta BOC$ (S.A.S. Axiom)

Hence (ii) AD = BC (c.p.c.t.)

and (iii) $\angle ADB = \angle ACB$ (c.p.c.t.)

(iv) $\Delta ADB \cong \Delta BCA$

$\Delta ADB = \Delta BCA$ (Given)

AB = AB (Common)

$\therefore \Delta AOB \cong \Delta BCA$

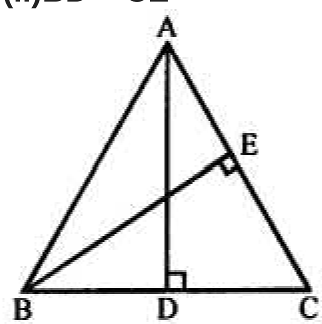
Hence proved.

Question 13.

ABC is an equilateral triangle, AD and BE are perpendiculars to BC and AC respectively. Prove that:

(i) AD = BE

(ii)BD = CE



Solution:

In Δ ABC,

AB = BC = CA,

AD $\perp$ BC, BE $\perp$ AC.

Proof : In Δ ADC and Δ BEC

$\angle ADC = \angle BEC$ (each 90°)

$\angle ACD = \angle BCE$ (common)

and AC = BC

(sides of an equilateral triangle)

$\therefore \Delta ADC \cong \Delta BEC$ (A.A.S. Axiom)

Hence (*i*) AD = BE (c.p.c.t.)

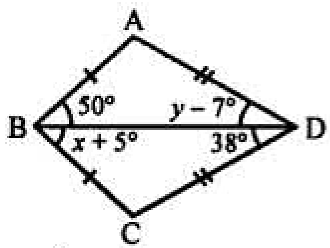
and (*ii*) BD = CE (c.p.c.t.)

Hence proved.

Question 14.

Use the informations given in the following figure to prove triangles ABD and CBD are congruent.

Also, find the values of x and y.



Solution:

Given : In the figure AB = BC, AD = DC

$\angle ABD = 50$, $\angle ADB = y - 7°$

$\angle CBD = x + 5°$, $\angle CDB = 38°$

To find : The value of x and y

In ΔABD and ΔCBD

BD = BD (common)

AB = BC (given)

AD = CD (given)

$\therefore \Delta ABD \cong \Delta CBD$ (SSS axiom)

$\therefore \angle ABD = \angle CBD$

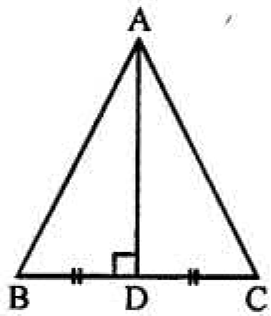
$\Rightarrow 50 = x + 5° \Rightarrow x = 50° - 5° = 45°$

and $\angle ADB = \angle CDB$

$\Rightarrow y - 7° = 38° \Rightarrow y = 38° + 7° = 45°$

Hence $x = 45°$, $y = 45°$

Question 15.
The given figure shows a triangle ABC in which AD is perpendicular to side BC and BD = CD. Prove that:
(i) △ABD ≅ △ACD
(ii) AB=AC
(iii) ∠B = ∠C



Solution:

(i) In the given figure ΔABC

$AD \perp BC$, $BD = CD$

In ΔABD and ΔACD

$AD = AD$ (Common)

$\angle ADB = \angle ADC$ (each 90%)

$BD = CD$ (Given)

$\therefore$ $\Delta ABD \cong \Delta CAD$ (By SAS Rule)

(ii) Side $AB = AC$ (c.p.c.t.)

(iii) $\angle B = \angle C$

Reason, since $\Delta ADB \cong \Delta ADC$

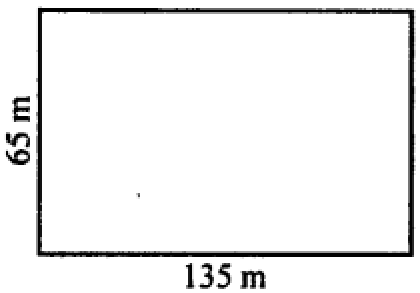
$\therefore$ $\angle B = \angle C$

Hence proved.

CHAPTER - 20
MENSURATION

EXERCISE 20 (A)

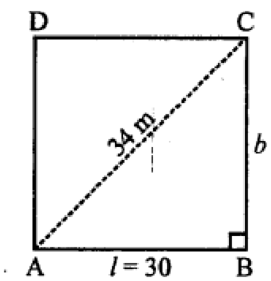
Question 1.
The length and the breadth of a rectangular plot are 135 m and 65 m. Find, its perimeter and the cost of fencing it at the rate of ₹60 per m.
Solution:
Given :
Length (l) = 135 m
Breadth (b) = 65 m



Perimeter = 2 (l + b)
= 2(135 + 65)
= 2(200) = 400 m
∴Perimeter of rectangular plot is = 400 m
Cost of fencing per m = ₹60
∴Cost of fencing 400 m = ₹60 x 400 m = ₹24000

Question 2.
The length and breadth of a rectangular field are in the ratio 7 : 4. If its perimeter is 440 m, find its length and breadth. Also, find the cost of fencing it @ ₹150 per m.
Solution:
Given : Perimeter = 440 m
Let the length of rectangular field = lx and breadth = 4x
2(l + b) = Perimeter
2(7x + 4x) = 440 m
2(11x) = 440 m
22x = 440 m
x = $\frac{440}{22}$
x = 11 m
∴Length = 7x = 7 x 11 = 77 m
Breadth = Ax = 4 x 11 = 44 m
Cost of fencing per m = ₹150
Cost of fencing 440 m = ₹150 x 440 = ₹66,000

Question 3.
The length of a rectangular field is 30 m and its diagonal is 34 m. Find the breadth of the field and its perimeter.
Solution:
Length = 30 m
Diagonals = 34 m



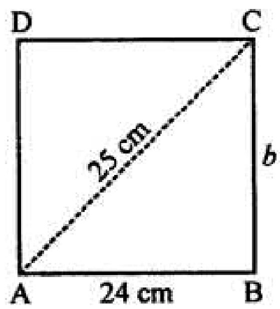
Let the breadth of the rectangle = b m
Applying Pythagoras Theorem in triangle ABC,
We get,
$AC^2 = AB^2 + BC^2$
$(34)^2 = (30)^2 + b^2$
$1156 = 900 + b^2$
$1156 - 900 = b^2$
$256 = b^2$
$\Rightarrow b = \sqrt{256} = 16$ m
Perimeter $= 2(l + b)$
$= 2(30 + 16) = 2 \times 46 = 92$ m

Question 4.
The diagonal of a square is $12\sqrt{2}$ cm. Find its perimeter.
Solution:
Diagonal of square = Its side x $\sqrt{2}$
Side $\sqrt{2} = \sqrt{2}\,\sqrt{2}$
i.e. side = 12 cm
Perimeter of a square = 4 x Side
= 4 x 12 = 48 cm

Question 5.
Find the perimeter of a rectangle whose length = 22.5 m and breadth = 16 dm.
Solution:
Length = 22.5 m
Breadth = 16 dm = 1.6 m
Perimeter of rectangle = 2(l + b)

– 2(22.5 + 1.6)
– 2(24.1) = 48.2 m

Question 6.
Find the perimeter of a rectangle with length = 24 cm and diagonal = 25 cm
Solution:
Length of a rectangle (l) = 24 cm Diagonal = 25 cm



Let breadth of the rectangle = b m
Applying Pythagoras Theorem in triangle ABC,
We get, $(AC)^2 = (AB)^2 + (BC)^2$
$(25)^2 = (24)^2 + (b)^2$
$625 = 576 + (b)^2$
$625 – 576 = b^2$
49 = A2
$\sqrt{7x7}$ =b
∴b = 7 cm
Now, perimeter of the rectangle
= 2(1 + b)
= 2(24 + 7)
= 2(31)
= 62 cm

Question 7.
The length and breadth of rectangular piece of land are in the ratio of 5 : 3. If the total cost of fencing it at the rate of ₹48 per metre is ₹19,200, find its length and breadth.
Solution:
Ratio in length and breadth of a rectangular piece of land = 5:3
Cost of fencing =₹ 19,200
and rate = ₹48 per m
∴Perimeter = $\frac{19200}{48}$= 400 m 48
Let length = 5x.
Then breadth = 3x
∴Perimeter = 2(l + b)
400 = 2(5x + 3x)
400 = 2 x 8x= 16x

∴16x = 400

⇒ x = $\frac{400}{16}$ = 25

∴Length of the land = 5x= 5 x 25 = 125 m and breadth = 3x = 3 x 25 = 75 m

Question 8.
A wire is in the shape of square of side 20 cm. If the wire is bent into a rectangle of length 24 cm, find its breadth.

Solution:
Side of square = 20 cm
Perimeter of square = 4 x 20 = 80 cm
Or perimeter of rectangle = 80 cm
Length of a rectangle = 24 cm
∴ Perimeter of a rectangle = 2(l + b)
b = $\frac{80}{2}$ – 24
b = 40 – 24 = 16 m

Question 9.
If P = perimeter of a rectangle, l= its length and b = its breadth find :
(i) P, if l = 38 cm and b = 27 cm
(ii) b, if P = 88 cm and l = 24 cm
(iii) l, if P = 96 m and b = 28 m

Solution:
(i) Length (l) = 38 cm
Breadth (b) = 27 cm
Perimeter of a rectangle = $2(l + b)$
= 2(38 + 27)
= 2(65) = 130 cm

(ii) Perimeter of a rectangle = 88 cm
Length (l) = 24 cm
Let breadth = b
$P = 2(l + b)$

$$b = \frac{P}{2} - l$$

$$b = \frac{88}{2} - 24 \text{ cm} = 44 \text{ cm} - 24 \text{ cm}$$

∴ Breadth of a rectangle = 20 cm

(iii) Perimeter of a rectangle = 96 m
Breadth (b) = 28 m
Let length = l
$P = 2(l + b)$

$$l = \frac{P}{2} - b$$

$$= \frac{96}{2} - 28 = 48 - 42 = 20 \text{ m}$$

∴ Length of a rectangle = 20 m

Question 10.
The cost of fencing a square field at the rate of
Cost of fencing 440 m = ₹150 x 440 = ₹75 per meter is
Cost of fencing 440 m = ₹150 x 440 = ₹67,500. Find the perimeter and the side of the square field.
Solution:

Length of the fence × its rate = ₹67,500

$$\Rightarrow \text{Length of the fence} = ₹\frac{67500}{75} = 900 \text{ m}$$

∴ Perimeter of a square field = length of its fence = 900 m

Since, perimeter of a square = 4 × Length of its side

⇒ Length of the side of the square

$$= \frac{\text{Perimeter}}{4} = \frac{900}{4} = 225 \text{ m}$$

Question 11.
The length and the breadth of a rectangle are 36 cm and 28 cm. If its perimeter is equal to the perimeter of a square, find the side of the square.
Solution:

Length of rectangle = 36 cm

Breadth of rectangle = 28 cm

Perimeter of the rectangle = $2(l + b)$

$= 2(36 + 28)$

$= 2(64) = 128$ cm

Given, perimeter of the square = perimeter of rectangle = 128 cm

$$\therefore \text{Side of the square} = \frac{\text{Perimeter}}{4}$$

$$= \frac{128}{4} = 32 \text{ cm}$$

Question 12.
The radius of a circle is 21 cm. Find the circumference (Take π = $3\frac{1}{7}$).
Solution:

Given, radius (r) = 21 cm and $\pi = \frac{22}{7}$

Circumference of the circle = $2\pi r$

$= 2 \times \frac{22}{7} \times 21$ cm

$= 2 \times 22 \times 3$ cm $= 132$ cm

Question 13.
The circumference of a circle is 440 cm. Find its radius and diameter. (Take π = $\frac{22}{7}$
Solution:

(*i*) Circumference of circle = 440 cm

Radius $= \frac{C}{2\pi} = \frac{440 \times 7}{2 \times 22}$ cm

$= \frac{3088}{44} = 70$ cm

(*ii*) Diameter = 2 × radius
= 2 × 70 = 140 cm

Question 14.
The diameter of a circular field is 56 m. Find its circumference and cost of fencing it at the rate of ₹80 per m. (Take n = $\frac{22}{7}$)
Solution:

Given, Diameter of a circular field = 56 m

∴ Radius $= \frac{56}{2} = 28$ m

Circumference of the circle = $2\pi r$

$= 2 \times \frac{22}{7} \times 28$ m

$= 2 \times 22 \times 4$ m $= 176$ m
Cost of fencing of 176 m is
= 176 m × ₹80 per m = ₹1,40,780

Question 15.
The radii of two circles are 20 cm and 13 cm. Find the difference between their circumferences. (Take π = $\frac{22}{7}$)
Solution:

Radius of 1st circle = 20 cm
Circumference of the circle = $2\pi r$

$= 2 \times \frac{22}{7} \times 20$

$= \frac{880}{7} = 122.8$ cm

Radius of 2nd circle = 13 cm
Circumference of the circle = $2\pi r$

$= 2 \times \frac{22}{7} \times 13 = \frac{572}{7} = 81.7$

$\therefore$ Difference of circumference of two circles
= 122.8 – 81.7 cm = 41.1 cm

Question 16.
The diameter of a circle is 42 cm, find its perimeter. If the perimeter of the circle is doubled, what will be the radius of the new circle. (Take π = $\frac{22}{7}$)
Solution:

Given, Diameter of a circle = 42 cm

$\therefore$ Radius of circle = $\frac{42}{2} = 21$ cm

Perimeter of the circle = $2\pi r$

$= 2 \times \frac{22}{7} \times 21 = 132$ cm

If the perimeter of the circle doubled
= 2 × 132 = 264 cm

$$\text{Radius} = \frac{C}{2\pi} = \frac{264}{2 \times \frac{22}{7}}$$

$$= \frac{264 \times 7}{2 \times 22} = 42 \text{ cm}$$

Question 17.
The perimeter of a square and the circumference of a circle are equal. If the length of each side of the square is 22 cm, find:
(i) perimeter of the square.
(ii) circumference of the circle.
(iii) radius of the circle.
Solution:

(*i*) Side of square = 22 cm

Perimeter of square = 4 × Side

= 4 × 22 = 88 cm

(*ii*) Circumference of circle

Given, Perimeter of square = Circumferene of circle

= 88 cm

(*iii*) Circumference of circle = 88 cm

$\therefore$ Radius $= \frac{C}{2\pi} = \frac{88 \times 7}{2 \times 22} = \frac{616}{44} = 14$ cm

Question 18.
Find the radius of the circle whose circumference is equal to the sum of the circumferences of the circles having radii 15 cm and 8 cm.
Solution:

For circle with radius = 15 cm

Circumference of circle = $2\pi r$

$= 2 \times \frac{22}{7} \times 15 \text{ cm} = \frac{660}{7} \text{cm}$

For circle with radius = 8 cm

Circumference of circle = $2\pi r$

$= 2 \times \frac{22}{7} \times 8 \text{ cm} = \frac{352}{7} \text{cm}$

Sum of the circumferences of these two circles

$= \frac{660}{7} \text{cm} + \frac{352}{7} \text{cm} = \frac{1012}{7} \text{cm}$

If the required radius = R cm

Its circumference = $2\pi R$

$= 2 \times \frac{22}{7} \times R \text{ cm} = \frac{44}{7} R \text{ cm}$

Given, $\frac{44}{7} R = \frac{1012}{7}$

$\Rightarrow R = \frac{7}{44} \times \frac{1012}{7} \text{ cm} = 23 \text{ cm}$

$\therefore$ Required radius = 23 cm

Question 19.
Find the diameter of a circle whose circumference is equal to the sum of circumference of circles with radii 10 cm, 12 cm and 18 cm.
Solution:

Let the radius of the circle = R cm

$\therefore$ $2\pi R = 2\pi \times 10 + 2\pi \times 12 + 2\pi \times 18$

On dividing each terms by 2π, we get :

$R = 10 + 12 + 18 = 40$ cm

$\therefore$ Radius of the circle obtained = 40 cm

And, its diameter = 2 × Radius

= 2 × 40 cm = 80 cm

Question 20.
The circumference of a circle is eigth time the circumference of the circle with radius 12 cm. Find its diameter.
Solution:

Radius of the given circle = 12 cm

Circumference of the given circle = $2\pi r$

$= 2 \times \frac{22}{7} \times 12 = \frac{528}{7}$ cm

Circumference of the required circle = 5 ×

$\frac{528}{7} = \frac{2640}{7}$ cm

If the radius of the required circle = R cm

Its circumference = $2\pi R$

$= 2 \times \frac{22}{7} \times R = \frac{44}{7} \times R$ cm

Given : $\frac{44}{7} \times R = \frac{2640}{7}$

$\Rightarrow R = \frac{2640}{7} \times \frac{7}{44} = 60$ cm

$\therefore$ Required radius = 60 cm

Question 21.
The radii of two circles are in the ratio 3 : 5, find the ratio between their circumferences.
Solution:

The ratio of the radii of the circles = 3 : 5
Let radius of the first circle = $3x$
and radius of second circle = $5x$
$\therefore$ Circumference of first circle = $2\pi r$
$= 2\pi \times 3x = 6\pi x$
and circumference of second circle = $2\pi r$
$= 2\pi \times 5x = 10\pi x$
$\therefore$ Ratio between their circumference
$= 6\pi x : 10\pi x$
$= 16 : 10$
$= 3 : 5$

Question 22.
The circumferences of two circles are in the ratio 5 : 7, find the ratio between their radii.
Solution:

The ratio of the circumference of the circle
= 5 : 7
Let circumference of first ratio = $5x$

$\therefore$ $2\pi r = 5x \quad \Rightarrow r = \dfrac{5x}{2\pi}$

and the circumference of second ratio = $7x$

$\therefore$ $2\pi r = 7x \quad \Rightarrow r = \dfrac{7x}{2\pi}$

Ratio between their radii $= \dfrac{5x}{2\pi} : \dfrac{7x}{2\pi}$
$= 5 : 7$

Question 23.

The perimeters of two squares are in the ratio 8:15, find the ratio between the lengths of their sides.

Solution:

Let the perimeter of first square = $8x$

$\therefore$ Side of the first square = $\frac{\text{Perimeter}}{4} = \frac{8x}{4}$

and the perimeter of second square = $7x$

$\therefore$ Side of the second square = $\frac{\text{Perimeter}}{4}$

$= \frac{15x}{4}$

Now, the ratio between the sides of the square = $\frac{8x}{4} : \frac{15x}{4}$

$= 8 : 15$

Question 24.

The lengths of the sides of two squares are in the ratio 8:15, find the ratio between their perimeters.

Solution:

Let the side of first square = 8x

∴Perimeter of first square = 4 x Side = 4 x 8x = 32 x

and the side of second squares = 15x

∴Perimeter of second square = 4 x Side = 4 x 15s = 60s

Now, the ratio between their perimeter = 32x: 60x= 8: 15

Question 25.

Each side of a square is 44 cm. Find its perimeter. If this perimeter is equal to the circumference of a circle, find the radius of the circle.

Solution:

The side of a square = 44 cm

$\therefore$ Its perimeter = $4 \times$ Side

$= 4 \times 44 = 176$ cm

Since, It is given that, Circumference of a circle = Perimeter of a square

$\therefore$ Circumference of a circle = 176 cm

Let, the radius of the circle = r

$\Rightarrow 2\pi r = 176$ cm

$$r = \frac{176 \times 7}{2 \times 22} = 28 \text{ cm}$$

$\therefore$ The radius of the circle = 28 cm

EXERCISE 20 (B)

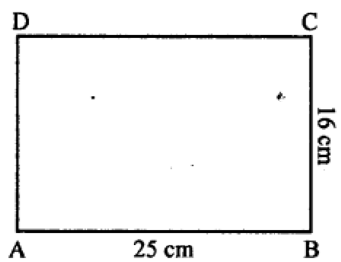
Question 1.

Find the area of a rectangle whose length and breadth are 25 cm and 16 cm.

Solution:

Length of rectangle = 25 cm

Breadth of rectangle = 16 cm



Area of rectangle = $l \times b$ or AB × BC

$= 25 \times 16\ cm^2 = 400\ cm^2$

$\therefore$ Area of rectangle = $400\ cm^2$

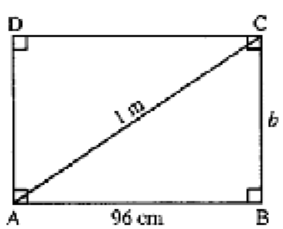
Question 2.

The diagonal of a rectangular board is 1 m and its length is 96 cm. Find the area of the board.

Solution:

Length of diagonal (AB) = 96 cm

Diagonal (AC) = 1 m = 100 cm



In right-angled triangle ABC,

By applying Pythagoras Theorem,

$(AC)^2 = (AB)^2 + (BC)^2$

$= (100)^2 = (96)^2 + BC^2$

$10000 = 9216 + BC^2$

$10000 - 9216 = BC^2$

$\sqrt{784} = BC$

$\therefore$ BC = 28 cm

Area of rectangular board

$= l \times b$ or AB × BC

$= 96 \times 28 = 2688\ cm^2$

Question 3.

The sides of a rectangular park are in the ratio 4 : 3. If its area is 1728 m^2, find
(i) its perimeter
(ii) cost of fencing it at the rate of ₹40 per meter.

Solution:

Ratio in the sides of a rectangle = 4 : 3
Area = 1728 m^2
Let length = $4x$, and breadth = $3x$
$\therefore$ Area = $l \times b$
$1728 = 4x \times 3x$
$\Rightarrow 12x^2 = 1728$

$$\Rightarrow x^2 = \frac{1728}{12}$$

$\Rightarrow x^2 = 144 = (12)^2$
$\therefore x = 12$
$\therefore$ Length = $4x = 4 \times 12 = 48$ m
Breadth = $3x = 3 \times 12 = 36$ m
(*i*) Now perimeter = $2(l + b)$
$= 2(48 + 36)$ m
$= 2 \times 84 = 168$ m
(*ii*) Rate of fencing = ₹40 per metre
Total cost = 168×40 = ₹6720

Question 4.

A floor is 40 m long and 15 m broad. It is covered with tiles, each measuring 60 cm by 50 cm. Find the number of tiles required to cover the floor.

Solution:

Length of floor (l) = 40 m
Breadth of floor (b) = 15 m
$\therefore$ Area of floor = $l \times b = 40 \times 15 = 600\ m^2$

Length of one tile = 60 cm = $\frac{6}{10}$ m

and breadth = 50 cm = $\frac{5}{10}$ m

$\therefore$ Area of one tile = $\frac{6}{10} \times \frac{5}{10}$

$= \frac{30}{100} = \frac{3}{10}\ m^2$

$\therefore$ Number of tiles = $\frac{\text{Total area of floor}}{\text{Area of one tile}}$

$$= \frac{600}{\frac{3}{10}} = \frac{600 \times 10}{3} = 2000$$

Question 5.
The length and breadth of a rectangular piece of land are in the ratio 5 : 3. If the total cost of fencing it at the rate of ₹24 per meter is ₹9600, find its :
(i) length and breadth
(ii) area
(iii) cost of levelling at the rate of ₹60 per m^2.
Solution:

Ratio in length and breadth of a rectangular piece of land = 5 : 3

Cost of fencing = ₹9600

and rate = ₹24 per m

$$\text{Perimeter} = \frac{\text{Total cost of fencing}}{\text{Rate per m}} = \frac{9600}{24} = 400 \text{ m}$$

Let length = $5x$

Then breadth = $3x$

$\therefore$ Perimeter = $2(l + b)$

$400 = 2 \times (5x + 3x)$

$\therefore$ $16x = 400$

$$x = \frac{400}{16} = 25$$

(*i*) $\therefore$ Length of land = $5x = 5 \times 25 = 125$ m

and breadth = $3x = 3 \times 25 = 75$ m

(*ii*) Area = $l \times b$

$= 125 \times 75 = 9375 \text{ m}^2$

(*iii*) Cost of levelling at rate ₹60 per m^2

= ₹$60 \times 9375 \text{ m}^2$ = ₹5,62,500

Question 6.
Find the area of the square whose perimeter is 56 cm.
Solution:

Perimeter of square = 56 cm

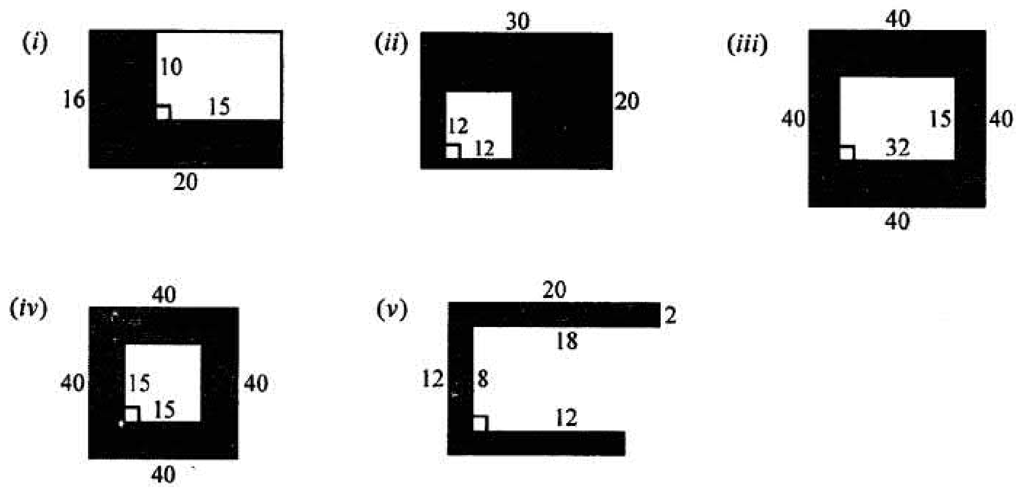
$\Rightarrow$ $4 \times \text{side} = 56$ cm

$\Rightarrow$ $\text{Side} = \frac{56}{4}$ cm

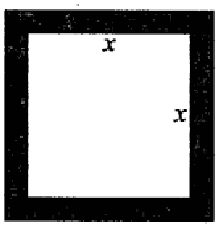
$\Rightarrow$ Side = 14 cm

$\therefore$ Area of square = $(\text{Side})^2 = (14)^2$

$= 14 \times 14 \text{ cm}^2 = 196 \text{ cm}^2$



Question 7.
A square lawn is surrounded by a path 2.5 m wide. If the area of the path is 165 m^2 find the area of the lawn.
Solution:



Area of path = 165 m^2
Width of path = 2.5 m
Let side of square lawn = x m
$\therefore$ Outer side = $x + 2 \times 2.5$
= $(x + 5)$ m
$\therefore$ Area of path = $(x + 5)^2 - x^2$
$\Rightarrow x^2 + 10x + 25 - x^2 = 165$
$\Rightarrow 10x = 165 - 25 = 140$
$$\Rightarrow x = \frac{140}{10} = 14 \text{ m}$$
$\therefore$ Side of lawn = 14 m
and area of lawn = $(14)^2$ m^2 = 196 m^2

Question 8.
For each figure, given below, find the area of shaded region : (All measurements are in cm)

Solution:

(*i*) Outer length = 20 cm

and breadth = 16 cm

$\therefore$ Outer area = $l \times b$

$= 20 \times 16 \text{ cm}^2 = 320 \text{ cm}^2$

Inner length = 15 cm

and Inner breadth = 10 cm

$\therefore$ Inner area = $15 \times 10 = 150 \text{ cm}^2$

$\therefore$ Area of shaded region = Area of whole region – Area of unshaded region

$= 320 - 150 \text{ cm}^2 = 170 \text{ cm}^2$

(*ii*) Outer length = 30 cm

and Outer breadth = 20 cm

$\therefore$ Outer area = $l \times b$

$= 30 \times 20 = 600 \text{ cm}^2$

Inner length = 12 cm and inner breadth = 12 cm

Inner area = $l \times b = 12 \times 12 = 144 \text{ cm}^2$

Area of shaded portion = Area of outer figure – Area a of inner figure

$= 600 - 144 = 456 \text{ cm}^2$

(*iii*) Area of shaded portion = Area of outer region – Area of unshaded region

$= 40 \times 40 - 32 \times 15$

$= 1600 - 480 \text{ cm}^2 = 1120 \text{ cm}^2$

(*iv*) Area of shaded region = Area of outer region – Area of inner region

$= 40 \times 40 - 15 \times 15$

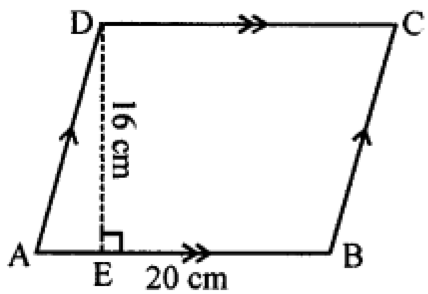
$= 1600 - 225 = 1375 \text{ cm}^2$

(*v*) Area of shaded portion

$= 2 \times 20 + 2 \times 8 + 2 \times (12 + 2)$

$= 40 + 16 + 28 \text{ cm}^2 = 84 \text{ cm}^2$

Question 9.
One side of a parallelogram is 20 cm and its distance from the opposite side is 16 cm. Find the area of the parallelogram.
Solution:



Area of parallelogram = Base × Height
= AB × DE
= 20 × 16 cm = 320 cm^2
∴ Area of parallelogram = 320 cm^2

Question 10.
The base of a parallelogram is thrice it height. If its area is 768 cm^2, find the base and the height of the parallelogram.
Solution:

Area of the parallelogram = 768 cm^2
Let the height of the parallelogram = x
Then base = $3x$
∴ Area = Base × Height
⇒ $768 = 3x \times x$
⇒ $768 = 3x^2$

⇒ $x^2 = \frac{768}{3} = 256$ cm

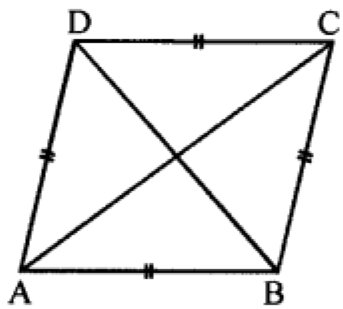
∴ $x = \sqrt{16 \times 16} = 16$ cm
∴ Base = 3 × 16 = 48 cm

and height = x = 16 cm

Question 11.
Find the area of the rhombus, if its diagonals are 30 cm and 24 cm.
Solution:

Given, diagonal $(d_1) = 30$ cm

Other diagonal $(d_2) = 24$ cm



If AC and BD are the diagonals of a rhombus its

$$\text{Area} = \frac{1}{2} \times \text{Product of it diagonals}$$

$$= \frac{1}{2} \times AC \times BD$$

$$= \frac{1}{2} \times d_1 \times d_2$$

$$= \frac{1}{2} \times 30 \times 24 \text{ cm}^2$$

$$= 15 \times 24 = 360 \text{ cm}^2$$

$\therefore$ Area of rhombus $= 360$ cm^2

Question 12.
If the area of a rhombus is 112 cm² and one of its diagonals is 14 cm, find its other diagonal.
Solution:

Area of rhombus = 112 cm^2

One diagonal = 14 cm

Let second diagonal = x

$$\text{Then, area} = \frac{\text{Product of diagonal}}{2}$$

$$\Rightarrow 112 = \frac{14 \times x}{2}$$

$$\Rightarrow x = \frac{112 \times 2}{14} = \frac{224}{14}$$

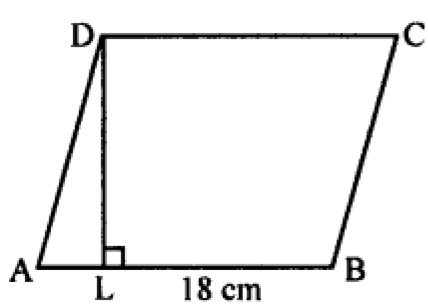
$$\Rightarrow x = 16$$

$\therefore$ Second diagonal = 16 cm

Question 13.
One side of a parallelogram is 18 cm and its area is 153 cm². Find the distance of the given side from its opposite side.
Solution:

Area of parallelogram ABCD = 153 cm^2

Side (Base) AB = 18 cm

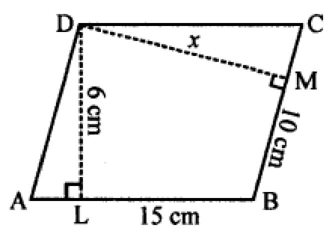


$\therefore$ Distance DL between AB and DC (altitude)

$$= \frac{\text{Area}}{\text{Base}} = \frac{153}{18} = \frac{17}{2} \text{cm} = 8.5 \text{ cm}$$

Question 14.
The adjacent sides of a parallelogram are 15 cm and 10 cm. If the distance between the longer sides is 6 cm, find the distance between the shorter sides.
Solution:

In parallelogram ABCD
AB = DC = 15 cm
BC = AD = 10 cm



Distance between longer sides AB and DC is 6 cm
i.e., perpendicular DL = 6 cm
DM ⊥ BC
Area of parallelogram = Base × Altitude
$= AB \times DL = 15 \times 6 = 90 \text{ cm}^2$
Again let DM = x cm

$\therefore$ Area of parallelogram ABCD = BC × DM
$= 10 \times x = 10x \text{ cm}^2$
$\therefore 10x \text{ cm}^2 = 90 \text{ cm}^2$

$$\Rightarrow x = \frac{90}{10} = 9 \text{ cm}$$

Question 15.
The area of a rhombus is 84 cm2 and its perimeter is 56 cm. Find its height.
Solution:

$\because$ Area of rhombus = 84 cm^2
Perimeter = 56 cm

$$\therefore \text{Its side} = \frac{56}{4} = 14 \text{ cm}$$

$$\therefore \text{Height} = \frac{\text{Area of rhombus}}{\text{Base}} = \frac{84}{14} = 6 \text{ cm}$$

Question 16.
Find the area of a triangle whose base is 30 cm and height is 18 cm.
Solution:

Base of triangle = 30 cm

Height of triangle = 18 cm

$$\therefore \text{Area} = \frac{1}{2}\text{base} \times \text{height}$$

$$= \frac{1}{2} \times 30 \times 18 = 270 \text{ cm}^2$$

Question 17.
Find the height of a triangle whose base is 18 cm and area is 270 cm^2.
Solution:

Base of triangle = 18 cm

Area of triangle = 270 cm^2

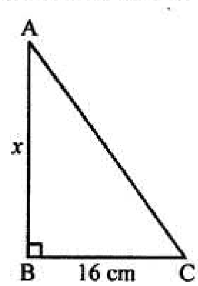
$$\therefore \text{Height} = \frac{\text{Area} \times 2}{\text{Base}}$$

$$= \frac{270 \times 2}{18} = \frac{540}{18} = 30 \text{ cm}$$

Question 18.
The area of a right-angled triangle is 160 cm^2. If its one leg is 16 cm long, find the length of the other leg.
Solution:

Area of the right angled triangle = 160 cm^2

Let base (one side) = 16 cm

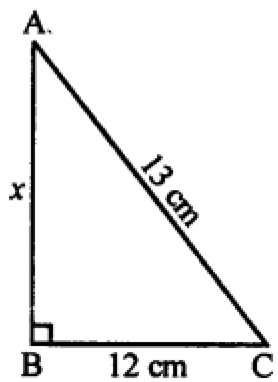


$\therefore$ Altitude (second side)

$$= \frac{\text{Area} \times 2}{\text{Base}} = \frac{160 \times 2}{16} = \frac{320}{16} = 20 \text{ cm}$$

Question 19.
Find the area of a right-angled triangle whose hypotenuse is 13 cm long and one of its legs is 12 cm long.
Solution:

In right angled ΔABC,
Base BC = 12 cm
and hypotenuse AC = 13 cm



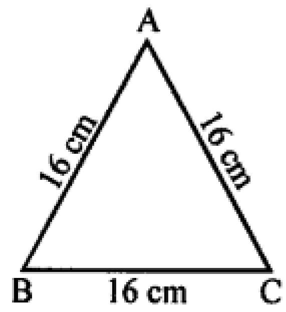
Applying Pythagoras Theorem,
$(AC)^2 = (AB)^2 + (BC)^2$
$(13)^2 = (AB)^2 + (12)^2$
$169 = (AB)^2 + 144$
$(AB)^2 = 169 - 144$
$(AB)^2 = 25$
$\therefore$ $AB = \sqrt{25}$
$= \sqrt{5 \times 5} = 5$ cm

Now, area of $\Delta ABC = \frac{1}{2}$ base $\times$ altitude

$= \frac{1}{2} \times 12 \times 5 = 30 \text{ cm}^2$

Question 20.
Find the area of an equilateral triangle whose each side is 16 cm. (Take $\sqrt{3}$= 1.73)
Solution:

Side of the equilateral triangle = 16 cm



$$\therefore \text{ Area} = \frac{\sqrt{3}}{4}(a)^2$$

$$= \frac{\sqrt{3}}{4} \times 16 \times 16$$

$$= 1.73 \times 4 \times 16 = 110.72 \text{ cm}^2$$

Question 21.
The sides of a triangle are 21 cm, 17 cm and 10 cm. Find its area.
Solution:

Let a = 21 cm, b = 17 cm and c = 10 cm

$\therefore a + b + c$

= 21 cm + 17 cm + 10 cm = 48 cm

$$s = \frac{a+b+c}{2} = \frac{48}{2} = 24 \text{ cm}$$

Area of the triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{24(24-20)(24-17)(24-10)}$$

$$= \sqrt{24 \times 3 \times 7 \times 14}$$

$$= \sqrt{2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 2 \times 7}$$

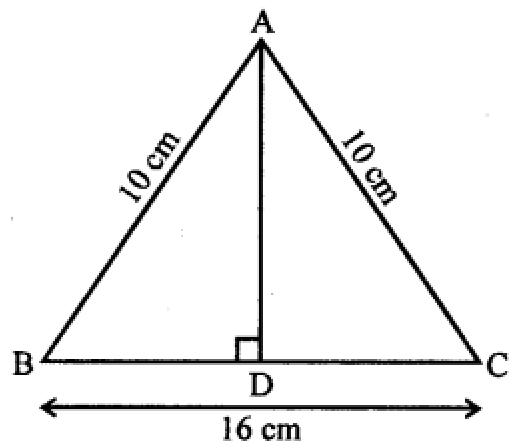
$$= 2 \times 2 \times 3 \times 7 = \text{cm}^2 = 84 \text{ cm}^2$$

Question 22.
Find the area of an isosceles triangle whose base is 16 cm and length of each of the equal sides is 10 cm.
Solution:

In isosceles ΔABC

Base $BC = 16$ cm

and $AB = AC = 10$ cm



Let $AD \perp BC$ and $BD = \frac{1}{2} BC = \frac{16}{2}$

$\therefore$ $BD = 8$ cm

In right ΔABD

$AB^2 = AD^2 + BD^2$ (Pythagoras Theorem)

$(10)^2 = AD^2 + (8)^2$

$100 = AD^2 + 64$

$100 - 64 = AD^2$

$36 = AD^2$

$AD = \sqrt{36} = \sqrt{6 \times 6}$

$\therefore$ $AD = 6$ cm

Now, area of triangle $= \frac{\text{Base} \times \text{Altitude}}{2}$

$= \frac{16 \times 6}{2} = 48 \text{ cm}^2$

Question 23.
Find the base of a triangle whose area is 360 cm^2 and height is 24 cm.
Solution:

Area of triangle = 360 cm^2

and height (h) = 24 cm

$$\therefore \text{ Base} = \frac{\text{Area} \times 2}{\text{Height}}$$

$$= \frac{360 \times 2}{24} = \frac{720}{24} = 30 \text{ cm}$$

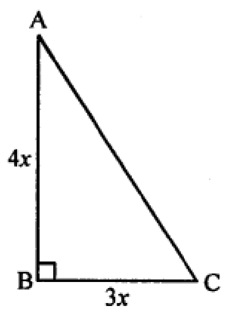
Question 24.
The legs of a right-angled triangle are in the ratio 4 :3 and its area is 4056 cm^2. Find the length of its legs.
Solution:

Area of right angled triangle = 4056 cm^2

Legs of a right angled triangle are in the ratio *i.e.* 4 : 3

Let one leg (base) = $3x$

Then second leg (altitude) = $4x$



$$\text{Area} = \frac{1}{2} \times \text{Base} \times \text{Altitude}$$

$$= \frac{1}{2} \times 3x \times 4x = 6x^2$$

$\therefore 6x^2 = 4056$

$$x^2 = \frac{4056}{6} = 676$$

$$x = \sqrt{676} = \sqrt{26 \times 26}$$

$\therefore x = 26$ cm

$\therefore$ One leg (base) = $3x = 3 \times 26 = 78$ cm

and second leg (altitude) $4x = 4 \times 26$

$= 104$ cm

Question 25.
The area of an equilateral triangle is (64 x $\sqrt{3}$) cm²– Find the length of each side of the triangle.
Solution:

Area of equilateral triangle = $64\sqrt{3}$ cm^2

Let each side = a

Then, $\frac{\sqrt{3}}{4}a^2 = 64\sqrt{3}$

$$a^2 = \frac{64\sqrt{3}\times 4}{\sqrt{3}} = 256$$

$a = (16)^2$

$\therefore$ $a = 16$ cm

$\therefore$ Each side = 16 cm

Question 26.
The sides of a triangle are in the ratio 15 : 13 : 14 and its perimeter is 168 cm. Find the area of the triangle.
Solution:

Perimeter of the triangle = 168 cm

Sum of ratios of sides = 15 + 13 + 14 = 42

Let the first side = $\frac{168\times 15}{42}$ = 60 cm

Second side = $\frac{168\times 13}{42}$ = 52 cm

Third side = $\frac{168\times 14}{42}$ = 56 cm

Now, $s = \frac{a+b+c}{2}$

$$= \frac{60+52+56}{2} = \frac{168}{2} = 84$$

$\therefore$ Area = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{84(84-60)(84-52)(84-56)}$$

$$= \sqrt{84\times 24\times 32\times 28}$$

$$= \sqrt{2\times 2\times 3\times 7\times 2\times 2\times 2\times 3\times 2 \times 2\times 2\times 2\times 2\times 2\times 2\times 7}$$

$= 2\times 2\times 2\times 2\times 2\times 2\times 3\times 7 = 1344$ cm^2

Question 27.
The diameter of a circle is 20 cm. Taking π = 3.14, find the circumference and its area.
Solution:

Diameter of circle (d) = 20 cm

$\therefore$ Circumference = $d\pi = 20 \times 3.14 = 62.8$ cm

Radius (r) = $\frac{d}{2}$ = 10 cm

$\therefore$ Area of a circle = πr^2

$= 3.14 \times 10 \times 10 = 314 \text{ cm}^2$

Question 28.
The circumference of a circle exceeds its diameter by 18 cm. Find the radius of the circle.
Solution:

Let c be the circumference and d be the diameter of the circle.

$\therefore c = d + 18$

$\Rightarrow d\pi = d + 18 \Rightarrow d\pi - d = 18$

$d(\pi - 1) = 18$

$\Rightarrow d\left(\frac{22}{7} - 1\right) = 18$

$\Rightarrow d\left(\frac{15}{7}\right) = 18$

$\Rightarrow d = \frac{18 \times 7}{15} = \frac{126}{15} = 8.4$ cm

$\therefore$ Radius = $\frac{d}{2} = \frac{8.4}{2} = 4.2$ cm

Question 29.
The ratio between the radii of two circles is 5 : 7. Find the ratio between their :
(i) circumference
(ii) areas
Solution:

(i) The ratio of the radii of the circles

$= 5 : 7$

Let radius of first circle $= 5x$

and radius of second circle $= 7x$

$\therefore$ Circumference of first circle $= 2\pi r$

$= 2\pi \times 5x = 10\pi x$

and circumference of second circle

$= 2\pi \times 7x = 14\pi x$

$\therefore$ Ratio between their circumference

$= 10\pi x : 14\pi x$

$= 10 : 14 = 5 : 7$

(ii) Area of first circle $= \pi r^2$

$$= \frac{22}{7} \times 5x \times 5x = \frac{550}{7}x^2$$

and area of second circle $= \pi r_2^2$

$$= \frac{22}{7} \times 7x \times 7x = \frac{1078}{7}x^2$$

Ratio between their areas

$$= \frac{550}{7}x^2 : \frac{1078}{7}x^2$$

$= 550 : 1078$ (Dividing by 22)

$= 25 : 49$

Question 30.

The ratio between the areas of two circles is 16 : 9. Find the ratio between their :

(i) radii

(ii) diameters

(iii) circumference

Solution:

(*i*) Let the radius of first circle = r_1

and radius of second circle = r_2

Given that ratio of the areas of circles

= 16 : 9

$$\Rightarrow \frac{\pi r_1^2}{\pi r_2^2} = \frac{16}{9}$$

$$\Rightarrow \frac{\pi r_1^2}{\pi r_2^2} = \frac{4^2}{3^2}$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{4}{3}$$

(*ii*) Let the diameter of first circle = d_1

and diameter of second circle = d_2

Since, we know that diameter = 2 × radius

$\therefore$ $d_1 = 2 \times r_1 = 2 \times 4x = 8x$

and $d_2 = 2 \times r_2 = 2 \times 3x = 6x$

Now, the ratio between the diameter of two circles = $d_1 : d_2$

$= 8x : 6x = 4 : 3$

(*iii*) Now, consider the ratio of circumference of the circles

$$= \frac{2\pi r_1}{2\pi r_2} = \frac{r_1}{r_2} = \frac{4}{3}$$

$\therefore$ The ratio between the circumference of two circles = 4 : 3

Question 31.
A circular racing track has inner circumference 528 m and outer circumference 616 m. Find the width of the track.
Solution:

Outer circumference = 616 m

$$\text{Radius (R)} = \frac{C}{2\pi} = \frac{616 \times 7}{2 \times 22} \text{ m} = 98 \text{ m}$$

Inner circumference = 528 m

$$\therefore \text{ Inner radius } (r) = \frac{528 \times 7}{2 \times 22} \text{ m} = 84 \text{ m}$$

$\therefore$ Width of track = R − r

= 98 − 84 = 14 m

Question 32.
The inner circumference of a circular track is 264 m and the width of the track is 7 m. Find:
(i) the radius of the inner track.
(ii) the radius of the outer circumference.
(iii) the length of the outer circumference.
(iv) the cost of fencing the outer circumference at the rate of ₹50 per m.
Solution:

Inner circumference of the circular track
= 264 m

(*i*) $\therefore$ Inner radius $(r) = \dfrac{C}{2\pi}$

$$= \frac{264 \times 7}{2 \times 22} = \frac{1848}{44} = 42 \text{ cm}$$

(*ii*) Width of the track = 7 m

$\therefore$ Outer radius (R) = 42 + 7 = 49 m

(*iii*) Outer circumference = 2πR

$$= 2 \times \frac{22}{7} \times 49 = 308 \text{ m}$$

(*iv*) Rate of fencing = ₹50 per metre

$\therefore$ Total cost of fencing outer circumference
= ₹50 × 308 = ₹15,400

Question 33.
The diameter of every wheel of a car is 63 cm. How much distance will the car move during 2000 revolutions of its wheel.
Solution:

Diameter of car wheel (d) = 63 cm

$\therefore$ Circumference $= \pi d = \frac{22}{7} \times 63 = 198$ cm

Distance covered in 2000 revolutions

$= 2000 \times 198$ cm

$= \frac{2000 \times 198}{100} = 3960 \text{ m} = 3.96 \text{ km}$

Question 34.
The diameter of the wheel of a car is 70 cm. How many revolutions will it make to travel one kilometre?
Solution:

Diameter of car wheel (d) = 70 cm

$\therefore$ Circumference $= \pi d = \frac{22}{7} \times 70$ cm

$= 220 \text{ cm} = \frac{220}{100} \text{ m}$

No. of revolutions in 1 km

$= 1 \text{ km} \div \frac{220}{100} \text{ m}$

$= 1 \times 1000 \times \frac{100}{220} \text{ m} = \frac{5000}{11} = 454\frac{6}{11} \text{ km}$

Question 35.
A metal wire, when bent in the form of a square of largest area, encloses an area of 484 cm². Find the length of the wire. If the same wire is bent to a largest circle, find:
(i) radius of the circle formed.
(ii) area of the circle.
Solution:

Area of the square made wire = 484 cm^2

$\therefore$ Length (side) $= \sqrt{\text{Area}} = \sqrt{484} = 22$ cm

(i) Perimeter of wire = $4 \times$ Side

$= 4 \times 22 = 88$ cm

$\therefore$ Circumference of circular wire = 88 cm

$\therefore$ Radius $(r) = \frac{C}{2\pi} = \frac{88 \times 7}{2 \times 22}$ cm = 14 cm

(*ii*) $\therefore$ Area of the circle = πr^2

$= \frac{22}{7} \times 14 \times 14 = 616 \text{ cm}^2$

Question 36.
A wire is along the boundary of a circle with radius 28 cm. If the same wire is bent in the form of a square, find the area of the square formed.
Solution:

Radius of circular wire = 28 cm

$\therefore$ Circumference = $2\pi r$

$= 2 \times \frac{22}{7} \times 28$ cm = 176 cm

$\therefore$ Perimeter of the square formed by this wire
= 176 cm

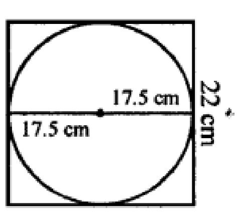
$\therefore$ Side $(a) = \frac{176}{4} = 44$ cm

Area of square so formed = $a^2 = (44)^2 \text{ cm}^2$
$= 1936 \text{ cm}^2$

Question 37.
The length and the breadth of a rectangular paper are 35 cm and 22 cm. Find the area of the largest circle which can be cut out of this paper.
Solution:

Length of rectangular paper (l) = 35 cm

Breadth of rectangular paper (b) = 22 cm



$\therefore$ Area = 35 × 22 = 770 cm^2

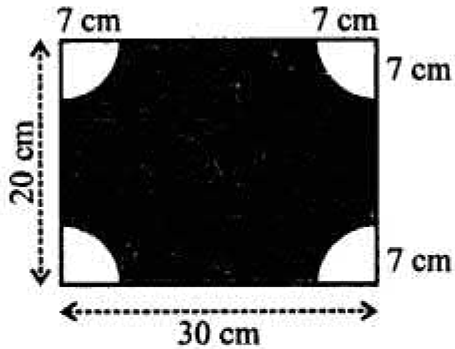
The largest circle which can be cut from the rectangular paper will have radius of 17.5 cm

$\therefore$ Area of a circle = πr^2

$$= \frac{22}{7} \times 17.5 \times 17.5 = 962.50 \text{ cm}^2$$

Question 38.

From each comer of a rectangular paper (30 cm x 20 cm) a quadrant of a circle of radius 7 cm is cut. Find the area of the remaining paper i.e., shaded portion.



Solution:

Length of paper (l) = 30 cm

and breadth (b) = 20 cm

∴ Area of rectangular paper = $l \times b$

= 30 × 20 = 600 cm

Radius of each quadrant at the corner

= 7 cm

Area of 4 quadrants $= 4 \times \frac{1}{4}\pi r^2$

$= \pi r^2 = \frac{22}{7} \times 7 \times 7 = 154 \text{ cm}^2$

∴ Area of remaining paper

$= 600 - 154 = 446 \text{ cm}^2$

CHAPTER - 21
DATA HANDLING

EXERCISE 21 (A)

Question 1.
Consider the following numbers :
68, 76, 63, 75, 93, 83, 70, 115, 82, 105, 90, 103, 92, 52, 99, 73, 75, 63, 77 and 71.
(i) Arrange these numbers in ascending order.
(ii) What the range of these numbers?
Solution:
(i) When the above data are written in ascending order. We get,
52, 63, 63, 68, 70, 71, 73, 75, 75, 76, 77, 82, 83, 90, 92, 93, 99, 103, 105, 115
(ii) Range of given numbers = Largest number – Smallest number
= 115-52 = 48

Question 2.
Represent the following data in the form of a frequency distribution table :
16, 17, 21, 20, 16, 20, 16, 18, 17, 21, 17, 18, 19, 17, 15, 15, 19, 19, 18, 17, 17, 15, 15, 16, 17, 17, 19, 18, 17, 16, 15, 20, 16, 17, 19, 18, 19, 16, 21 and 17.
Solution:
The frequency distribution for these data will be as shown below :

Numbers	Tally marks	Frequency
15	𝍸	5
16	𝍸 II	7
17	𝍸 𝍸 I	11
18	𝍸	5
19	𝍸 I	6
20	III	3
21	III	3
Total		40

Question 3.

A die was thrown 20 times and following scores were recorded.
2, 1, 5, 2, 4, 3, 6, 1, 4, 2, 5, 1, 6, 2, 6, 3, 5, 4, 1 and 3.
Prepare a frequency table for the scores.

Solution:
The frequency table for the scores will be as shown below :

No. of thrown dies	Tally marks	Frequency
1	IIII	4
2	IIII	4
3	III	3
4	III	3
5	III	3
6	III	3
Total		20

Question 4.

Following data shows the weekly wages (in ₹) of 10 workers in a factory.
3500, 4250, 4000, 4250, 4000, 3750, 4750, 4000, 4250 and 4000
(i) Prepare a frequency distribution table.
(ii) What is the range of wages (in ₹)?
(iii) How many workers are getting the maximum wages?

Solution:
(i) The frequency table for the wages of 10 workers will be as shown below :

Weekly wages in (₹)	Tally marks	Frequency
3500	I	1
3750	I	1
4000	IIII	4
4250	III	3
4750	I	1
Total		10

(ii) Range of wages (₹) = ₹4750 – ₹3500 = ₹1250
(iii) One

Question 5.
The marks obtained by 40 students of a class are given below :
80, 10, 30, 70, 60, 50, 50, 40, 40, 20, 40, 90, 50, 30, 70, 10, 60, 50, 20, 70, 70, 30, 80, 40,20, 80, 90, 50, 80, 60, 70, 40, 50, 60, 90, 60, 40, 40, 60 and 60
(i) Construct a frequency distribution table.
(ii) Find how many students have marks equal to or more than 70?
(iii) How many students obtained marks below 40?
Solution:
(i) The frequency distribution table will be shown as below :

Numbers	Tally marks	Frequency
10	II	2
20	III	3
30	III	3
40	~~IIII~~ II	7
50	~~IIII~~ I	6
60	~~IIII~~ II	7
70	~~IIII~~	5
80	IIII	4
90	III	3
Total		40

(ii) Students have marks equal to or more than 70 = 5 + 4 + 3 = 12
(iii) Students obtained marks below 40 = 2 + 3 + 3 = 8 students

Question 6.
Arrange the following data in descending order:
3.3, 3.2, 3.1, 3.7, 3.6, 4.0, 3.5, 3.9, 3.8, 4.1, 3.5, 3.8, 3.7, 3.9 and 3.4.
(i) Determine the range.
(ii) How many numbers are less than 3.5?
(iii) How many numbers are 3.8 or above?
Solution:
Descending order : 4.1, 4.0, 3.9, 3.9, 3.8, 3.8, 3.7, 3.7, 3.6, 3.5, 3.5, 3.4, 3.3, 3.2, 3.1
(i) Range = 4.1 – 3.1 = 1
(ii) Number less than 3.5 = 4
i.e., 3.4, 3.3, 3.2, 3.1
(iii) Number are 3-8 or above = 6
i.e., 3.8, 3.8, 3.9, 3.9, 4.0, 4.1

EXERCISE 21 (B)

Question 1.
Find the mean of 53, 61, 60, 67 and 64.
Solution:
Mean of 53, 6i, 60, 67 and 64

$$\therefore \text{ Mean} = \frac{53+61+60+67+64}{5} \qquad \text{(Here } n = 5)$$

$$= \frac{305}{5} = 61$$

Question 2.
Find the mean of first six natural numbers.
Solution:
First six natural numbers are : 1, 2, 3, 4, 5, 6

$$\therefore \text{ Mean} = \frac{1+2+3+4+5+6}{6} \qquad \text{(Here } n = 6)$$

$$= \frac{21}{6} = 3 \cdot 5$$

Question 3.
Find the mean of first ten odd natural numbers.
Solution:
First ten odd natural numbers are 1, 3, 5, 7, 9, 11, 13, 15, 17, 19

$$\therefore \text{ Mean} = \frac{1+3+5+7+9+11+13+15+17+19}{10} \qquad \text{(Here } n = 10)$$

$$= \frac{100}{10} = 10$$

Question 4.
Find the mean of all factors of 10.
Solution:

The factor of 10 are 2 and 5

$$\therefore \text{ Mean} = \frac{2+5}{2} \qquad \text{(Here } n = 2)$$

$$= \frac{7}{2} = 3\cdot5$$

Question 5.
Find the mean of x + 3, x + 5, x + 7, x + 9 and x + 11.
Solution:
Mean of x + 3, x + 5, x + 7, x + 9 and x + 11

$$\therefore \text{ Mean} = \frac{(x+3)+(x+5)+(x+7)+(x+9)+(x+11)}{5} \qquad \text{(Here } n = 5)$$

$$= \frac{5x+35}{5} = \frac{5(x+7)}{5} = x + 7$$

Question 6.
If different values of variable x are 19.8,15.4,13.7,11.71,11.8, 12.6,12.8,18.6,20.5 and 2.1, find the mean.
Solution:
19. +15.4 +13.7 +11.71 +11.8 +12.6 + 12.8 +18.6 + 20.5 +21.1

$$\therefore \text{ Mean} = \frac{19\cdot8+15\cdot4+13\cdot7+11\cdot71+11\cdot8+12\cdot6+12\cdot8+18\cdot6+20\cdot5+21\cdot1}{10}$$

$$= \frac{158\cdot01}{10} = 15\cdot801$$

Question 7.
The mean of a certain number of observations is 32. Find the resulting mean, if each observation is,
(i) increased by 3
(ii) decreased by 7
(iii) multiplied by 2
(iv) divided by 0.5
(v) increased by 60%
(vi) decreased by 20%

Solution:

(*i*) Increased by = 3

Observed mean = 32

If increased by 3

$\therefore$ Resulting mean = 32 + 3 = 35

(*ii*) Observed mean = 32

Decreased by = 7

$\therefore$ Resulting mean = 32 – 7 = 25

(*iii*) Observed mean = 32

Multiplied by = 2

$\therefore$ Resulting mean = 32 × 2 = 64

(*iv*) Observed mean = 32

Divided by = 0·5

$$\therefore \text{ Resulting mean} = \frac{32}{0.5} = 64$$

(*v*) Observed mean = 32

Increased by = 60%

$$\therefore \text{ Resulting mean} = 32 + \frac{60}{100}$$

$$= \frac{3200 + 60}{100} = \frac{3260}{100} = 32{\cdot}6$$

(*vi*) Observed mean = 32

Decreased by = 20%

$$\therefore \text{ Resulting mean} = 32 - \frac{20}{100}$$

$$= \frac{3200 - 20}{100} = \frac{3180}{100} = 31.8$$

Question 8.
The pocket expenses (per day) of Anuj, during a certain week, from monday to Saturday were ₹85.40, ₹88.00, ₹86.50, ₹84.75, ₹82.60 and ₹87.25. Find the mean pocket expenses per day.
Solution:
The pocket expenses (per day) during a certain week are : ₹85.40, ₹88.00, ₹86.50, ₹84.75, ₹82.60 and ₹87.25

$$\therefore \text{ Mean of pocket expenses per day} = \frac{85.40 + 88.00 + 86.50 + 84.75 + 82.60 + 87.25}{6} \qquad (\text{Here } n = 6)$$

$$= \frac{514.5}{6} = ₹85.75$$

∴Anuj expenses per day = ₹85.75

Question 9.
If the mean of 8, 10, 7, x + 2 and 6 is 9, find the value of x.
Solution:
The mean 8, 10, 7, x + 2 and 6 is 9

$$\therefore \frac{8 + 10 + 7 + x + 2 + 6}{6} = 9$$

$$\Rightarrow \frac{x + 33}{6} = 9 \Rightarrow x + 33 = 9 \times 6$$

$$\Rightarrow x = 54 - 33 \Rightarrow x = 21$$

Question 10.
Find the mean of first six multiples of 3.
Solution:
The six multiples of 3 are 3, 6, 9, 12, 15, 18

$$\therefore \text{ Mean} = \frac{3 + 6 + 9 + 12 + 15 + 18}{6} \qquad (\text{Here } n = 6)$$

$$= \frac{63}{6} = 10.5$$

Question 11.
Find the mean of first five prime numbers.
Solution:

The first five prime numbers are 2, 3, 5, 7, 11

$$\therefore \text{ Mean} = \frac{2+3+5+7+11}{5} \qquad \text{(Here } n = 5\text{)}$$

$$= \frac{28}{5} = 5\frac{3}{5} = 5{\cdot}6$$

Question 12.

The mean of six numbers :x-5,x- 1, x, x + 2, x + 4 and x + 12 is 15. Find the mean of first four numbers.

Solution:

The mean of six numbers are x – 5, x – 1,x,x + 2,x + 4 and x + 12 is 15

$$\therefore \text{ Mean} = \frac{(x-5)+(x-1)+(x)+(x+2)+(x+4)+(x+12)}{6}$$

$$= \frac{x-5+x-1+x+x+2+x+4+x+12}{6}$$

$$= \frac{12+6x}{6} = 15$$

$\Rightarrow 12 + 6x = 90$

$\Rightarrow 6x = 90 - 12$

$\Rightarrow 6x = 78$

$\Rightarrow x = \frac{78}{6} = 13$

$x = 13$

$\therefore$ The six numbers are (13 – 5), (13 – 1), 13, (13 + 2), (13 + 4), (13 + 12)

i.e. 8, 12, 13, 15, 17, 25

$$\text{Now, mean of first four numbers} = \frac{8+12+13+15}{4} = \frac{48}{4} = 12$$

Question 13.

Find the mean of squares of first five whole numbers.

Solution:

First five whole numbers are 0, 1, 2, 3, 4

Then square the whole prime numbers

$= (0)^2, (1)^2, (2)^2, (3)^2, (4)^2$

$= 0, 1, 4, 9, 16$

$$\therefore \text{ Mean} = \frac{0+1+4+9+16}{5} \quad \text{(Here } n = 5)$$

$$= \frac{30}{5} = 6$$

Question 14.

If the mean of 6, 4, 7, p and 10 is 8, find the value of p.

Solution:

The mean of 6, 4, 7, p and 10 is 8

$$\therefore \text{ Mean} = \frac{6+4+7+p+10}{5} = 8$$

$\Rightarrow 27 + p = 40$

$\Rightarrow p = 40 - 27$

$\Rightarrow p = 13$

Question 15.

Find the mean of first six multiples of 5.

Solution:

Six multiples of 5 are :

5, 10, 15, 20, 25 and 30

5, 10, 15, 20, 25 and 30

$$\therefore \text{ Mean} = \frac{5+10+15+20+25+30}{6} \quad \text{(Here } n = 6)$$

$$= \frac{105}{6} = 17.5$$

Question 16.

The rainfall (in mm) in a city on 7 days of a certain week is recorded as follows

Day :	Mon	Tue	Wed	Thus	Fri	Sat	Sun
Rainfall (in mm) :	0·5	2·7	2·6	0·5	2	5·8	1·5

Find the total and average (mean) rainfall for the week.

Solution:
The rainfall in a city on 7 days are 0.5, 2.7, 2.6, 0.5, 2, 5.8, 1.5

$$\text{Average rainfall} = \frac{0{\cdot}5 + 2{\cdot}7 + 2{\cdot}6 + 0{\cdot}5 + 2 + 5{\cdot}8 + 1{\cdot}5}{7} \qquad \text{(Number of days = 7)}$$

$$= \frac{15{\cdot}6}{7} = 2{\cdot}2 \text{ mm}$$

Question 17.
The mean of marks scored by 100 students was found to be 40, later on it was discovered that a score of 53 was misread as 83. Find the correct mean.
Solution:
Mean of 40 observations = 100
Total sum of 40 observations = 100 × 40 = 4000
Incorrect total of 40 observation is = 4000
Correct total of 40 observations = 4000 – 83 + 53 = 3970
∴ Correct mean = $\frac{3970}{100}$ = 39.70

Question 18.
The mean of five numbers is 27. If one number is excluded, the mean of remaining numbers is 25. Find the excluded number.
Solution:
Mean of 5 observations = 27
Total sum of 5 observations = 27 × 5 = 135
On excluding an observation, the mean of remaining 6 observations = 25
⇒ Total of remaining 4 observations = 25 x 4 = 100
⇒ Included observation = Total mean of 5 observations – Total mean of 4 observations
= 135- 100 = 35

Question 19.
The mean of 5 numbers is 27. If one new number is included, the new mean is 25. Find the included number.
Solution:
Mean of 5 observations = 27
Total sum of 5 observations = 27 x 5 = 135
On including an observation the mean of 6 observation = 25 x 6 = 150
⇒ Included observations = Total Mean of 6 observations – Total mean of 5 observations
= 150- 135 = 15

Question 20.

Mean of 5 numbers is 20 and mean of other 5 numbers is 30. Find the mean of all the 10 numbers taken together.

Solution:

The mean of 5 number = 20

Then, mean of other 5 number = 30

$$\therefore \text{ Mean} = \frac{20+30}{2} = \frac{50}{2} = 25$$

Question 21.

Find the median of:

(i) 5,7, 9, 11, 15, 17,2, 23 and 19

(ii) 9, 3, 20, 13, 0, 7 and 10

(iii) 18, 19, 20, 23, 22, 20, 17, 19, 25 and 21

(iv) 3.6, 9.4, 3.8, 5.6, 6.5, 8.9, 2.7, 10.8, 15.6, 1.9 and 7.6.

Solution:

(*i*) 5, 7, 9, 11, 15, 17, 2, 23 and 19

Arranging in ascending order : 2, 5, 7, 9, 11, 15, 17, 19, 23

Here, number of terms = 9 which is odd

$$\text{Median} = \frac{n+1}{2} = \frac{9+1}{2}\text{th term} = 5\text{th term} = 11$$

Hence, median = 11

(*ii*) 9, 3, 20, 13, 0, 7 and 10

Arranging in ascending order : 0, 3, 7, 9, 10, 13, 20

Here, number of terms = 7 which is odd

$$\therefore \text{ Median} = \frac{n+1}{2} = \frac{7+1}{2}\text{th term} = 4\text{th term} = 9$$

Hence, median = 9

(*iii*) 18, 19, 20, 23, 22, 20, 17, 19, 25 and 21

Arranging in ascending order : 17, 18, 19, 19, 20, 20, 21, 22, 23, 25

Here, numbers of terms = 10 which is even.

$$\therefore \text{ Median} = \frac{1}{2}\left\{\frac{n}{2}\text{th term} + \left(\frac{n}{2}+1\right)\text{th term}\right\}$$

$$= \frac{1}{2}\left\{\frac{10}{2}\text{th term} + \left(\frac{10}{2}+1\right)\text{th term}\right\}$$

$$= \frac{1}{2}\{5\text{th term} + 6\text{th term}\}$$

$$= \frac{1}{2}\{20+20\} = \frac{1}{2} \times 20 = 20$$

Hence, median = 20

(iv) 3.6, 9.4, 3.8, 5.6, 6.5, 8.9, 2.7, 10.8, 15.6, 1.9 and 7.6

Arranging in ascending order : 1.9, 2.7, 3.6, 3.8, 5.6, 6.5, 7.6, 8.9, 9.4, 10.8, 15.6

Here, number of terms = 11 which is odd

$\therefore$ Median $= \frac{n+1}{2} = \frac{11+1}{2} =$ 6th term

= 6th term = 6.5

Hence, median = 6.5

Question 22.

Find the mean and the mode for the following data :

Term	18	22	26	30	34	38
Frequency	3	5	10	2	8	2

Solution:

We prepare the table given below :

Term (x_i)	Frequency (f_i)	($f_i x_i$)
18	3	54
22	5	110
26	10	260
30	2	60
34	8	276
38	2	76
Total	30	832

$$\text{Mean} = \frac{\Sigma f_i x_i}{f_i} = \frac{832}{30} = 27.73$$

Since, the frequency of Number 26 is maximum.

$\therefore$ Mode = 26

Question 23.

Find the mode of:

(i) 5, 6, 9, 13, 6, 5, 6, 7, 6, 6, 3

(ii) 7, 7, 8, 10, 10, 11, 10, 13, 14

Solution:

(i) Arranging the Numbers in ascending order : 3, 5, 5, 6, 6, 6, 6, 6, 7, 9, 13
Mostly repeated term = 6
∴ Mode = 6
(ii) Arranging the Numbers in ascending order = 7, 7, 8, 10, 10, 10, 11, 13, 14
Mostly repeated term =10
∴ Mode = 10

Question 24.
Find the mode of :

(i)

x	15	16	17	18	19	20	21	22	23
f	6	7	9	13	10	12	8	0	4

(ii)

Height (cm)	37	38	39	40	41
Number of plants	46	89	93	90	153

Solution:
(i) Since, the frequency of number 18 is maximum
∴Mode = 18
(ii) Since, the frequency of number 41 is maximum
∴ Mode = 41

Question 25.
The heights (in cm) of 8 girls of a class are 140,142,135,133,137,150,148 and 138 respectively. Find the mean height of these girls and their median height.
Solution:
Arranging in ascending order : 133, 135, 137, 138, 140, 142, 148, 150
Here, number of girls = 8 which is even

$$\therefore \text{ Median} = \frac{1}{2}\left\{\frac{n}{2}\text{th term} + \left(\frac{n}{2}+1\right)\text{th term}\right\}$$

$$= \frac{1}{2}\left\{\frac{8}{2}\text{th term} + \left(\frac{8}{2}+1\right)\text{th term}\right\}$$

$$= \frac{1}{2}\{4\text{th term} + 5\text{th term}\}$$

$$= \frac{1}{2}\{138 + 140\}\text{ cm} = \frac{1}{2} \times 278 = 139 \text{ cm}$$

$$\therefore \quad \text{Mean} = \frac{133+135+137+138+140+142+148+150}{8}$$

$$= \frac{1123}{8}\text{cm} = 140.375 \text{ cm}$$

Question 26.
Find the mean, the median and the mode of:
(i) 12, 24, 24, 12, 30 and 12
(ii) 21, 24, 21, 6, 15, 18, 21, 45, 9, 6, 27 and 15.
Solution:

(*i*) 12, 24, 24, 12, 30 and 12

$$\therefore \text{ Mean} = \frac{12+24+24+12+30+12}{6}$$

$$= \frac{114}{6} = 19$$

Numbers are 12, 24, 24, 12, 30 and 12

Mostly repeated term = 12

$\therefore$ Mode = 12

Now, Arranging the numbers in ascending order = 12, 12, 12, 24, 24, 30

Here, number of terms 6 which is even

$$\therefore \text{ Median} = \frac{1}{2}\left\{\frac{n}{2}\text{th term} + \left(\frac{n}{2}+1\right)\text{th term}\right\}$$

$$= \frac{1}{2}\left\{\frac{6}{2}\text{th term} + \left(\frac{6}{2}+1\right)\text{th term}\right\}$$

$$= \frac{1}{2}\{3\text{th term} + 4\text{th term}\}$$

$$= \frac{1}{2}\{12 + 24\} = \frac{1}{2} \times 36 = 18$$

$\therefore$ Median = 18

(*ii*) 21, 24, 21, 6, 15, 18, 21, 45, 9, 6, 27 and 15

$$\therefore \text{ Mean} = \frac{21+24+21+6+15+18+21+45+9+6+27+15}{12}$$

$$= \frac{228}{12} = 19$$

Numbers are 21, 24, 21, 6, 15, 18, 21, 45, 9, 6, 27 and 15

Mostly repeated term = 21

$\therefore$ Mode = 21

Arranging the terms in ascending order : 6, 6, 9, 15, 15, 18, 21, 21, 21, 24, 27, 45

Here, number of terms = 12 which is even

$$\therefore \text{ Median} = \frac{1}{2}\left\{\frac{n}{2}\text{th term} + \left(\frac{n}{2}+1\right)\text{th term}\right\}$$

$$= \frac{1}{2}\left\{\frac{12}{2}\text{th term} + \left(\frac{12}{2}+1\right)\text{th term}\right\}$$

$$= \frac{1}{2}\{6\text{th term} + 7\text{th term}\}$$

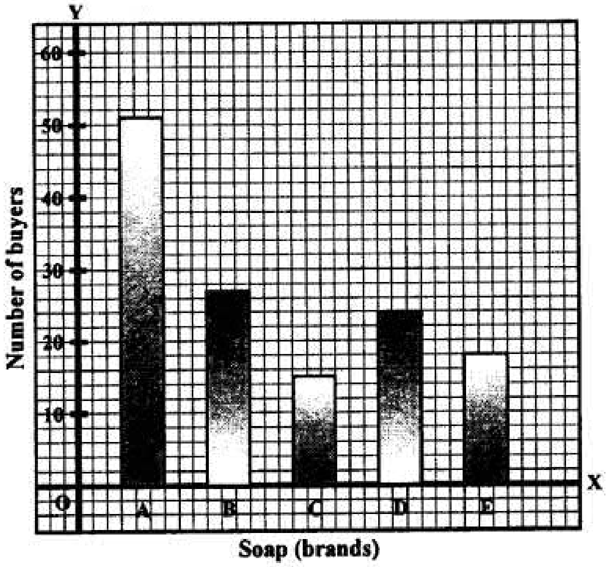
$$= \frac{1}{2}\{18 + 21\} = \frac{1}{2} \times 39 = 19.5$$

Question 27.

The following table shows the market positions of some brands of soap. Draw a suitable bar graph :

Soap (brands) :	A	B	C	D	E
No. of buyers :	51	27	15	24	18

Solution:

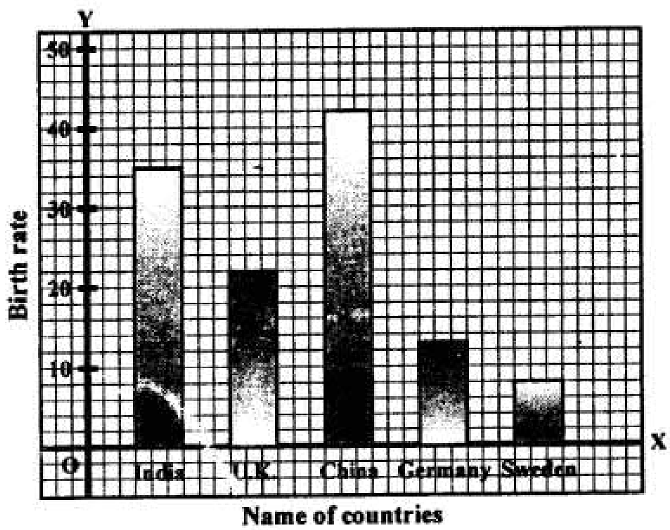


Question 28.

The birth rate per thousand of different countries over a particular period of time is shown below.

INDIA	U.K.	CHINA	GERMANY	SWEDEN
35	22	42	13	8

Solution:



CHAPTER - 22
PROBABILITY

EXERCISE 22 (A)

Question 1.
A coin is tossed once. Find the probability of
(i) getting a head
(ii) not getting a head
Solution:
Total number of possible outcomes are Head (H) and Tail (T) i.e. 2
(i) P (Getting a head) $=\frac{1}{2}$
(ii) P (Not getting a head) = $\frac{1}{2}$

Question 2.
A coin is tossed 80 times and the head is obtained 38 times. Now, if a coin tossed once, what will the probability of getting:
(i) a tail
(ii) ahead
Solution:
(*i*) $\because$ Total number of possible outcomes = 80
and, the number of favourable outcomes of getting a tail = 80 – 38 = 42

$$\therefore \text{ Probability of getting a tail} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{42}{80} = \frac{21}{40}$$

(*ii*) $\because$ Total number of possible outcomes = 80
and, the number of favourable outcomes of getting head = 38

$$\therefore \text{ Probability of getting a head} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{38}{80} = \frac{19}{40}$$

Question 3.
A dice is thrown 20 times and the outcomes are noted as shown below :

Outcomes	1	2	3	4	5	6
No. of times	2	3	4	4	3	4

Now a dice is thrown at random, find the probability of getting :
Solution:

$\because$ Total number of outcomes = 20

(i) P (getting 3) $= \frac{4}{20} = \frac{1}{5}$

(ii) A number less than 3 (1, 2) will appear = 2 + 3 = 5 times

$\therefore$ Probability $= \frac{5}{20} = \frac{1}{4}$

(iii) A number greater than 3 (4, 5, 6) will appear = 4 + 3 + 4 = 11 times

$\therefore$ Probability $= \frac{11}{20}$

Question 4.
A survey of 50 boys showed that 21 like tea while 29 dislike it. Out of these boys, one boy is chosen at random. What is the probability that the chosen boy
(i) likes tea
(ii) dislikes tea
Solution:

Total number of boys = 50

Number of boys like tea = 21

Number of boys dislike tea = 29

(i) Probability of boys like tea $= \frac{21}{50}$

(ii) Probability of boys dislike tea $= \frac{29}{50}$

Question 5.
In a cricket match, a batsman hits a boundary 12 times out of 80 balls he plays, further, if he plays one ball more, what will be the probability that:
(i) he hits a boundary
(ii) he does not hit a boundary
Solution:

(*i*) Total number of balls = 80

Hits boundaries = 12 times

$$\therefore \text{ P (Hitting a boundary)} = \frac{12}{80} = \frac{3}{20}$$

$$(ii) \text{ P (of not hitting a boundary)} = \frac{68}{80} = \frac{17}{20}$$

Question 6.
There are 8 marbles in a bag with numbers from 1 to 8 marked on each of them. What is the probability of drawing a marble with number
(i) 3
(ii) 7
Solution:
Total number of marbles = 8
(i) Probability (of getting a marble with number 3) = $\frac{1}{8}$
(ii) Probability (of getting a marble with number 7) = $\frac{1}{8}$

Question 7.
Two coins are tossed simultaneously 100 times and the outcomes are as given below:

Outcomes	Two heads (H, H)	Exactly one head (H T or T H)	No head (T T)
No. of times	21	55	24

If the same pair of coins is tossed again at random, find the probability of getting :
(i) two heads
(ii) exactly one head
(iii) no head.

Solution:

(i) Here, total number of trials = 100 times

Number of heads got (H, H) = 21

$$\therefore P(E) = \frac{\text{Number of trials in which two heads occurs}}{\text{Total number of trials}} = \frac{21}{100}$$

(ii) Total number of trials = 100 times

Number of exactly one head = 55

$$\therefore P(E) = \frac{55}{100} = \frac{11}{20}$$

(iii) Total number of trials = 100 times

Number of head = 24

$$\therefore \text{Probability} = \frac{24}{100} = \frac{16}{25}$$

Question 8.

A bag contains 4 white and 6 black balls,- all of the same shape and same size. A ball is drawn from the bag without looking into the bag. Find the probability that the ball drawn is :
(i) a black ball
(ii) a white ball
(iii) not a black ball

Solution:

Number of white balls = 4

Number of black balls = 6

Number of total balls or possible events = 6 + 4 = 10 balls

(i) Probability (a black ball)

Number of black balls = 6

Number of total balls = 10

$$\therefore \text{Probability} = \frac{6}{10} = \frac{3}{5}$$

(*ii*) P (a white ball)

Number of white balls = 4

Number of total balls = 10

$\therefore$ Probability = $\frac{4}{10} = \frac{2}{5}$

(*iii*) P (not a black ball)

$$= \frac{\text{Number of white balls}}{\text{Number of total balls}} = \frac{4}{10} = \frac{2}{5}$$

(*ii*) P (a white ball)

Number of white balls = 4

Number of total balls = 10

$\therefore$ Probability = $\frac{4}{10} = \frac{2}{5}$

(*iii*) P (not a black ball)

$$= \frac{\text{Number of white balls}}{\text{Number of total balls}} = \frac{4}{10} = \frac{2}{5}$$

Question 9.

In a single throw of a dice, find the probability of getting a number:
(i) 4
(ii) 6
(iii) greater than 4

Solution:

$\because$ Total number of outcomes = 6

(*i*) Event of getting number 4 = 1

Total number of outcomes = 6

$\therefore$ Probability = $\frac{1}{6}$

(*ii*) Event of getting number 6 = 1

Total number of outcomes = 6

$\therefore$ P = $\frac{1}{6}$

(*iii*) Greater than 4 (*i.e.* 5, 6) will appear 2 times

$\therefore$ Probability (of getting greater than 4) = $\frac{2}{6}$

$= \frac{1}{3}$

Question 10.
Hundred identical cards are numbered from 1 to 100. The cards are well shuffled and then a card is drawn. Find the probability that the number on the card drawn is :
(i) 50
(ii) 80
(iii) 40
Solution:

Here, total number of cards = 100

(*i*) Card drawn with number = 50
Favourable outcomes = 1
Total number of outcomes = 100

$$\therefore\ P = \frac{1}{100}$$

(*ii*) Card drawn with number = 80
Favourable outcomes = 1
Total number of outcomes = 100

$$\therefore\ P = \frac{1}{100}$$

(*iii*) Card drawn with number = 40
Favourable outcomes = 1
Total number of outcomes = 100

$$\therefore\ P = \frac{1}{100}$$

EXERCISE 22 (B)

Question 1.
Suppose S is the event that will happen tommorow and P(S) = 0.03.
(i) State in words, the complementary event S'.
(ii) Find P(S')
Solution:
Given, P(S) = 0.03
(i) The event will not happen tommorow.
(ii) P(S') = 1 – P(S)
P(S') = 1 – 0.03 [∵ P(S) + P(S') = 1]
P(S') = 0.97

Question 2.
Five Students A, B, C, D and E are competing in a long distance race. Each student's probability of winning the race is given below:
A → 20 %, B → 22 %, C → 7 %, D → 15% and E → 36 %
(i) Who is most likely to win the race ?
(ii) Who is least likely to win the race ?
(iii) Find the sum of probabilities given.
(iv) Find the probability that either A or D will win the race.
(v) Let S be the event that B will win the race.
(a) Find P(S)
(b) State, in words, the complementary event S'.
(c) Find P(S')
Solution:

Given Probabilities of five students A, B, C, D and E such as

P(A) = 20%, P(B) = 22%, P(C) = 7%

P(D) = 15% and P(E) = 36%

(*i*) The mostly chance of winning the race is of Student E.

[∵ P(E) = 36% maximum]

(*ii*) The least chances of winning the race is of Student C.

[∵ P(C) = 7% minimum]

(*iii*) The sum of the probabilities

= P(A) + P(B) + P(C) + P(D) + P(E)

= 20% + 22% + 7% + 15% + 36%

= 100%

(*iv*) Favourable outcomes that either A or D will win = 20% + 15% = 35%

$$\text{P(either A or D will win)} = \frac{35}{100} = \frac{7}{20}$$

(*v*) (a) Favourable outcomes that B will win = 22%

$$P(S) = \frac{22}{100} = \frac{11}{50}$$

(b) S' = B will not win the race.

(c) P(S') = 1 – P(S)

$$= 1 - \frac{11}{50} = \frac{50-11}{50} = \frac{39}{50}$$

Question 3.
A Ticket is randomly selected from a basket containing3 green, 4 yellow and 5 blue tickets. Determine the probability of getting:
(i) a green ticket
(ii) a green or yellow ticket.
(iii) an orange ticket.
Solution:

Number of green tickets = 3

Number of yellow tickets = 4

Number of blue tickets = 5

Total Number of tickets = 3 + 4 + 5 = 12

(*i*) P (getting a green ticket) = $\frac{3}{12} = \frac{1}{4}$

(*ii*) Total Number of green and yellow tickets
$= 3 + 4 = 7$ tickets

P (getiing a green or yellow ticket) = $\frac{7}{12}$

(*iii*) Since, Basket contains green, yellow and blue tickets only.

$\therefore$ Number or orange tickets = 0

$\therefore$ P (gettting an orange ticket) = $\frac{0}{12} = 0$

Question 4.
Ten cards with numbers 1 to 10 written on them are placed in a bag. A card is chosen from the bag at random. Determine the probability of choosing:
(i) 7
(ii) 9 or 10
(iii) a number greater than 4
(iv) a number less than 6

Solution:

Total Number of outcomes = 10 *i.e.* 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

(*i*) P (of getting a number 7) = $\frac{1}{10}$

(*ii*) P (of getting 9 or 10) = $\frac{2}{10} = \frac{1}{5}$

(*iii*) Numbers greater than 4 are 5, 6, 7, 8, 9 and 10 = 6

P (of getting number greater than 4)

$$= \frac{6}{10} = \frac{3}{5}$$

(*iv*) Numbers less than 6 are 1, 2, 3, 4, 5 = 5

P(of getting a number less than 6)

$$= \frac{5}{10} = \frac{1}{2}$$

Question 5.

A carton contains eight brown and four white eggs. Find the probability that an egg selected at random is :

(i) brown

(ii) white

Solution:

Number of brown eggs = 8

Number of white eggs = 4

Total Number of eggs = 8 + 4 = 12

(*i*) P (of getting a brown egg) = $\frac{8}{12} = \frac{2}{3}$

(*ii*) P (of getting a white egg) = $\frac{4}{12} = \frac{1}{3}$

Question 6.

A box contains 3 yellow, 4 green and 8 blue tickets. A ticket is chosen at random. Find the probability that the ticket is :
(i) yellow
(ii) green
(iii) blue
(iv) red
(v) not yellow

Solution:

Number of yellow tickets = 3

Number of green tickets = 4

Number of blue tickets = 8

Total Number of tickets = 3 + 4 + 8 = 15

(*i*) P (getting a yellow ticket) = $\frac{3}{15} = \frac{1}{5}$

(*ii*) P (getting a green ticket) = $\frac{4}{15}$

(*iii*) P (getting a blue ticket) = $\frac{8}{15}$

(*iv*) Since, Basket contains yellow, green and blue tickets only.

$\therefore$ Number or red tickets = 0

$\therefore$ P (gettting an red ticket) = $\frac{0}{15} = 0$

(*v*) Total number of green and blue tickets = 4 + 8 = 12 tickets

P (not getting yellow ticket) = P(getting either green or blue ticket) = $\frac{12}{15} = \frac{4}{5}$

OR

P (not getting a yellow ticket) = $1 - \frac{1}{5} = \frac{5-1}{5} = \frac{4}{5}$

Question 7.
The following table shows number of males and number of females of a small locality in different age groups.

If one of the persons, from this locality, is picked at random, what is the probability that
(a) the person picked is a male ?
(b) the person picked is a female ?
(c) the person picked is a female aged 21-50 ?
(d) the person is a male with age upto 50 years?
Solution:

$\because$ Total number of persons = Number of males + Number of females

$$= 26 + 20 = 46$$

(a) An event when the person picked is male = 8 + 12 + 6 = 26

$$\therefore \text{ Required Probability} = \frac{26}{46} = \frac{13}{23}$$

(b) An event when the person picked is female = 6 + 10 + 4

$$\therefore \text{ Required Probability} = \frac{20}{46} = \frac{10}{23}$$

(c) An event when the person picked is a female aged 21-50 = 10

$$\therefore \text{ Required Probability} = \frac{10}{46} = \frac{5}{23}$$

(d) An event when the person picked is a male aged upto 50 years = 20

$$\therefore \text{ Required Probability} = \frac{20}{46} = \frac{10}{23}$$